MAC 1143 PRECALCULUS EXAM 1 A NAME _____

SHOW ALL WORK ON SEPARATE PAPER. (You may keep this test!)
TURN IN ALL WORKSHEETS. CALCULATORS ARE REQUIRED ON THIS TEST.

1. Find all vertical, horizontal, and slant asymptotes. Sketch the graph describing the "window" of your calculator. Give all intercepts and symmetries.

$$f(X) = \frac{X}{X^2 - 9}$$

2. Sketch the graphs of $g(X) = \frac{2X^2 + 5X + 3}{X + 3}$ and $h(X) = \frac{2X^2 + 5X - 3}{X + 3}$.

(Describe your window!) Give all asymptotes.
How do these similar functions differ?

3. The Parks and Wildlife Commission introduces 100,000 fish into a large man-made lake . The population of fish, in thousands, is given by

$$N = \frac{20(5 + 3t)}{1 + 0.04t}, \quad t \geq 0, \text{ where t is in years.}$$

a) Find the population when t = 5 years, and when t = 25 years.

b) What is the limiting number of fish in the lake as time increases?

In 4 - 5, write the partial fraction decomposition for the rational expressions.

4. $\frac{X^2 + 12X + 12}{X^3 - 4X}$

5. $\frac{4X^2 - 2}{(X - 1)^3}$

2

In 6 - 8, identify the conic by name, sketch the graph, and give vertices, foci, and center (if relevant!).

6. $Y^2 - 12Y + 8X + 20 = 0$

7. $4X^2 + 25Y^2 + 24X - 50Y - 39 = 0$

8. $\frac{(Y - 2)^2}{16} - \frac{(X + 1)^2}{9} = 1$

In 9 - 11, find the equation of the specified conic. Sketch the graph.

9. Parabola with vertex at (0, 2) and focus at (-6, 2).

10. Ellipse with vertices at (2, 4) and (2, -6); and foci at (2, 2) and (2, -4).

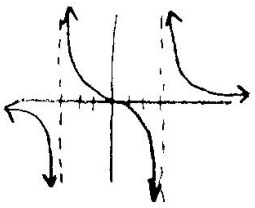
11. Hyperbola with ~~foci~~ vertices at (±2, 3) and passing through (4, 1).

12. A cross section of a large parabolic antenna is given by

$$X = \frac{Y^2}{400} .$$

Sketch the graph. At what coordinates should the receiving and transmitting equipment be positioned?

1. $f(x) = \frac{x}{x^2-9}$



Vertical asymptotes: $x = \pm 3$
Horizontal asymp: $y = 0$
x intercepts: $(0,0)$
Symmetry: Origin
Window: Zoom STANDARD

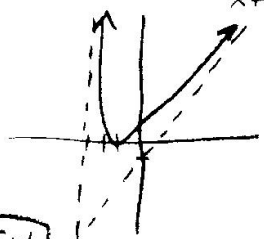
2. $g(x) = \frac{2x^2+5x+3}{x+3}$ $\quad h(x) = \frac{2x^2+5x-3}{x+3}$

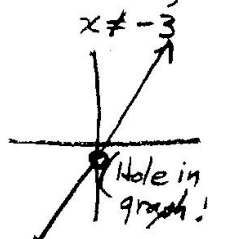
$-3 \mid 2 \;\; 5 \;\; 3$
$\quad\quad \downarrow \;\; -6 \;\; 3$
$\quad 2 \;\; -1 \;\; 6$

$g(x) = 2x - 1 + \frac{6}{x+3}$

$-3 \mid 2 \;\; 5 \;\; -3$
$\quad\quad \downarrow \;\; -6 \;\; 3$
$\quad 2 \;\; -1 \;\; 0$

$h(x) = 2x - 1, \; x \neq -3$





Be sure window is large enough!

(Degenerate case!)

Slant Asymptote: $y = 2x - 1$

Vertical asymptote: $x = -3$

WINDOW: Try y from -20 to +20.

3. $y = \frac{20(5+3x)}{1+0.04x}$

x: 0 to 1,000,000
y: 0 to 2050

a) When $x = 5$, $y = 333$ or 333,333 fish

$x = 25$, $y = 800$ or 800,000 fish

b) Limiting number (use large x!)

$x = 100{,}000$ $\quad y = 1500$ or 1,500,000 fish

4. $\frac{x^2+12x+12}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2}$

$x^2+12x+12 = A(x-2)(x+2) + Bx(x+2) + C(x)(x-2)$

Let $x=0$: $12 = A(-2)(2) + 0 + 0$

$A = -3$

$x=2$: $4+24+12 = 0 + B(2)(4) + 0$

$40 = 8B$

$B = 5$

$x=-2$ $\quad 4-24+12 = 0 + 0 + C(-2)(-4)$

$-8 = 8C$

$C = -1$

4. $\frac{-3}{x} + \frac{5}{x-2} - \frac{1}{x+2}$

5. $\frac{4x^2-2}{(x-1)^3} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$

$4x^2-2 = A(x-1)^2 + B(x-1) + C$

Let $x=1$: $4-2 = 0+0+C$

$C = 2$

$4x^2-2 = Ax^2 - 2Ax + A + Bx - B + C$

x^2: $4 = A$

x: $0 = -2A + B$

$0 = -8 + B \quad B = 8$

$\frac{4}{x-1} + \frac{8}{(x-1)^2} + \frac{2}{(x-1)^3}$

6. $y^2 - 12y + 8x + 20 = 0$ PARABOLA

$y^2 - 12y + \_\_\_ = -8x - 20 + \_\_\_$

$y^2 - 12y + 36 = -8x - 20 + 36$

$(y-6)^2 = -8x + 16$

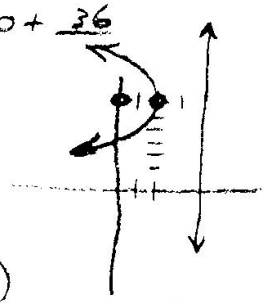
$(y-6)^2 = -8(x-2)$

$V(2,6)$ Opens Left.

$4p = -8$

$p = -2$ $\quad F(0,6)$

Directrix $x = 4$

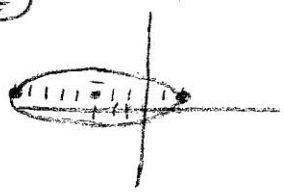


7. $4x^2+24x+25y^2-50y=39$ ELLIPSE

$4(x^2+6x+\ )+25(y^2-2y+\ )=39$

$4(x^2+6x+\underline{9})+25(y^2-2y+\underline{1})=39+36+25$

$\frac{4(x+3)^2}{100}+\frac{25(y-1)^2}{100}=\frac{100}{100}$



$\frac{(x+3)^2}{25}+\frac{(y-1)^2}{4}=1$

$a^2=25 \quad b^2=4$

$a=5 \quad b=2$

$C(-3,1)$

$a^2=b^2+c^2$

$25=4+c^2$

$c^2=21$

$c=\pm\sqrt{21}$

C: $(-3,1)$

V: $(2,1)\ (-8,1)$

F: $(-3\pm\sqrt{21},1)$

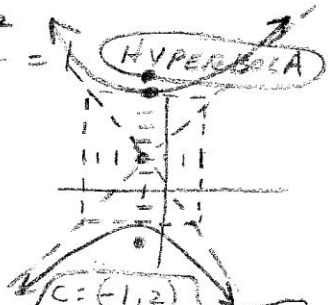
8. $\frac{(y-2)^2}{16}-\frac{(x+1)^2}{9}=$ HYPERBOLA

$a=4 \quad b=3$

$c^2=a^2+b^2$

$c^2=16+9=25$

$c=5$



C: $(-1,2)$

V: $(-1,6)\ (-1,-2)$

F: $(-1,7)\ (-1,-3)$

9. PARABOLA $y^2=4px$

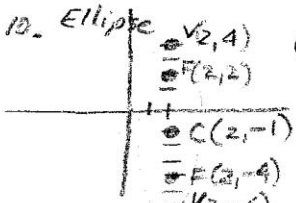
$(y-k)^2=4p(x-h)$

$V(h,k)=(0,2)$

$p=-6$

$(y-2)^2=-24(x)$

10. Ellipse



V(2,4)

F(2,2)

C(2,−1)

F(2,−4)

V(2,−6)

C = midpoint

(2,4) (2,−6)

C(2,−1)

$a=5$

$c=3$

$a^2=b^2+c^2$

$25=b^2+9$

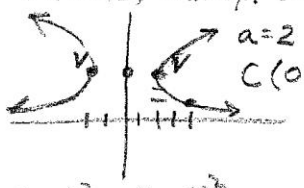
$b^2=16 \quad b=4$

$\frac{(x-2)^2}{b^2}+\frac{(y+1)^2}{a^2}=1$

$\frac{(x-2)^2}{16}+\frac{(y+1)^2}{25}=1$

11. HYPERBOLA

$V(\pm 2,3)$ through $(4,1)$



$a=2$

$C(0,3)$

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

$\frac{x^2}{4}-\frac{(y-3)^2}{b^2}=1$

$\frac{x^2}{4}-\frac{(y-3)^2}{b^2}=1$ (4,1)

$\frac{16}{4}-\frac{(-2)^2}{b^2}=1$

$4-\frac{4}{b^2}=1$

$3=\frac{4}{b^2}$

$b^2=4/3$

$\frac{x^2}{4}-\frac{(y-3)^2}{4/3}=1$

$\frac{x^2}{4}-\frac{3(y-3)^2}{4}=1$

12. $x=\frac{y^2}{400}$

$y^2=400x$ opens Right

$V(0,0) \quad 4p=400$

$p=100$

F(100,0)