MAT 1143 PRECALCULUS EXAM 3 A NAME ______________________

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. *SHOW ALL STEPS!!*
Explain your calculations and procedures for partial credit.
TURN IN ALL WORKSHEETS. CALCULATORS ARE REQUIRED ON THIS TEST.

1. Use trigonometric identities to simplify:

$$\frac{\cos x}{1 + \sin x} + \frac{1 + \sin x}{\cos x}$$

2. Simplify in such a way that the expression is not in fractional form:

$$\frac{\tan^2 x}{\csc x + 1}$$

3. Find all solutions in the interval $[0, 2\pi)$.

$$6 \cos^2 x + \cos x - 1 = 0$$

4. Find all solutions in the interval $[0, 2\pi)$.

$$2 \sin^2 x = \sin x$$

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5. Given that $\cos \theta = 3/8$ and $\tan \theta < 0$, find (in exact form) $\sin 2x$.

6. Express the following in terms of one function of one angle:

a) $\sin 36^\circ \cos 4^\circ - \cos 36^\circ \sin 4^\circ$ = ____________

b) $\cos 36^\circ \cos 4^\circ - \sin 36^\circ \sin 4^\circ$ = ____________

7. Express the following in terms of one function of one angle (or constant!):

a) $1 - \sin^2 48^\circ$ = ____________

b) $1 - 2 \sin^2 48^\circ$ = ____________

8. Express the following in terms of one function of one angle (or constant!):

a) $\cos^2 48^\circ - \sin^2 48^\circ$ = ____________

b) $\cos^2 48^\circ + \sin^2 48^\circ$ = ____________

9. Use your graphics calculator to find the roots of the function for [0, 360). Sketch and describe the viewing rectangle:

$$f(x) = \sin 4x - \sin 2x$$

10. Verify the following trig identity:

$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

11. Verify the identity by classroom approved techniques:

$\cos^4 X + \sin^2 X = \sin^4 X + \cos^2 X$

12. Verify the identity by classroom approved techniques:

$\cot t + \tan t = (\csc t)(\sec t)$

13. The lengths of two equal sides of an isosceles triangle are 20 feet. The angle between the two sides is $\theta$.

a) Express the area of the figure as a function of $\theta/2$.

b) Express the area of the figure as a function of $\theta$.

MAT 1143 EXAM 3A Solutions

1. $\dfrac{\cos x}{1+\sin x} + \dfrac{1+\sin x}{\cos x}$

$= \dfrac{\cos^2 x + (1+\sin x)^2}{\cos x(1+\sin x)}$

$= \dfrac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{\cos x(1+\sin x)}$

$= \dfrac{2+2\sin x}{\cos x(1+\sin x)}$

$= \dfrac{2\cancel{(1+\sin x)}}{\cos x\cancel{(1+\sin x)}} = \dfrac{2}{\cos x}$

2. $\dfrac{\tan^2 x\,(\csc x - 1)}{(\csc x+1)(\csc x-1)}$

$= \dfrac{\tan^2 x(\csc x - 1)}{\csc^2 x - 1}$

$= \dfrac{\tan^2 x(\csc x - 1)}{\cot^2 x}$

$= \dfrac{\tan^2 x(\csc x - 1)}{\frac{1}{\tan^2 x}}$

$= \tan^4 x(\csc x - 1)$

3. $6\cos^2 x + \cos x - 1 = 0$

$(2\cos x + 1)(3\cos x - 1) = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = \frac{1}{3}$

Q II, III $\quad$ Q I, IV

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\quad$ $x = 1.23$

$2\pi - 1.23 = 5.05$

4. $2\sin^2 x = \sin x$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

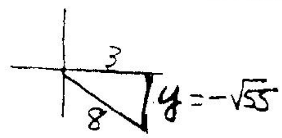
$\sin x = 0 \quad \sin x = \frac{1}{2}$

$x = 0, \pi \quad x = \frac{\pi}{6}, \frac{5\pi}{6}$

Q I, II.

5. $\cos\theta = \frac{3}{8}$ $\quad \tan\theta < 0$

$\theta$ in Q IV.




$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\cdot\frac{-\sqrt{55}}{8}\cdot\frac{3}{8}$

$= \dfrac{-3\sqrt{55}}{32}$

$3^2 + y^2 = 8^2$

$y^2 = 64 - 9$

$y^2 = 55$

$y = \pm\sqrt{55}$

6a) $\sin 36^\circ \cos 4^\circ - \cos 36^\circ \sin 4^\circ$

$= \sin(36-4) = \sin 32^\circ$

b) $\cos 36^\circ \cos 4^\circ - \sin 36^\circ \sin 4^\circ$

$= \cos(36+4) = \cos 40^\circ$

7a) $1 - \sin^2 48^\circ = \cos^2 48^\circ$

b) $1 - 2\sin^2 48^\circ = \cos 2(48^\circ) = \cos 96^\circ$

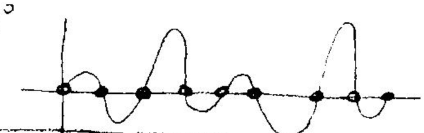
8a) $\cos^2 48^\circ - \sin^2 48^\circ = \cos 2(48^\circ) = \cos 96^\circ$

b) $\cos^2 48^\circ + \sin^2 48^\circ = 1$

9 $f(x) = \sin 4x - \sin 2x$

$x$: 0 to $360^\circ$

$y$: $-3$ to 3



ROOTS: $0^\circ, 30^\circ, 90^\circ, 150^\circ, 180^\circ, 210^\circ, 270^\circ, 330^\circ$

10. $\cos 3x = \cos(2x + x)$

LHS $= \cos 2x\cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$

$= 2\cos^3 x - \cos x - 2\cos x\sin^2 x$

$= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

$= 4\cos^3 x - 3\cos x = $ RHS

11. $\cos^4 x + \sin^2 x = (\cos^2 x)^2 + \sin^2 x$

LHS $= (1-\sin^2 x)^2 + \sin^2 x$

$= 1 - 2\sin^2 x + \sin^4 x + \sin^2 x$

$= \sin^4 x + 1 - \sin^2 x$

$= \sin^4 x + \cos^2 x = $ RHS

12. $\cot x + \tan x =$

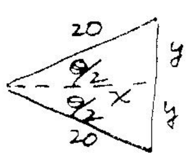
LHS $= \dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x}$

$= \dfrac{\cos^2 x + \sin^2 x}{\sin x\cos x}$

$= \dfrac{1}{\sin x\cos x} = \dfrac{1}{\sin x}\cdot\dfrac{1}{\cos x}$

$= \csc x \cdot \sec x = $ RHS

13.




$A = \frac{1}{2}bh = \frac{1}{2}(2y)(x)$

$= xy$

$= \left(20\cos\frac{\theta}{2}\right)\cdot\left(20\sin\frac{\theta}{2}\right)$

a) $= 400\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

b) $A = 200\sin\theta$

$\sin\frac{\theta}{2} = \frac{y}{20}$

$\cos\frac{\theta}{2} = \frac{x}{20}$