CALCULUS II EXAM 4C Dr. RAPALJE

Show all work as necessary on separate paper. (1 Free)

(7) 1. Find the vertex, focus, directrix and graph of $y^2+2y+6x=17$

(7) 2. Find the equation of the parabola with focus $(0,7)$ and with directrix $y=3$. Sketch the graph and find the vertex.

3. Find the coordinates of the vertices, foci, and graphs for:

(14) a) $\frac{(x+2)^2}{9}+\frac{(y-4)^2}{25}=1$ b) $\frac{(x+2)^2}{9}-\frac{(y-4)^2}{25}=1$

4. Graph, find center and vertices:

(14) a) $16x^2+9y^2+96x+36y+36=0$

b) $x^2-4y^2-14x-32y-11=0$

(7) 5. Find the equation for the conic with foci at $(3,1)$ and $(3,9)$ and with major axis length 12.

(7) 6. Find the equation for the conic with vertices at $(2,\pm 3)$ and foci at $(2,\pm 5)$

(14) 7. For $17x^2-12xy+8y^2=80$, a) show that the correct angle of rotation is such that $\cos\theta=\frac{1}{\sqrt{5}}$ and $\sin\theta=\frac{2}{\sqrt{5}}$. b) Find the rotated equation and graph the conic.

8. Given $\sin\theta=\frac{2}{\sqrt{5}}$ and $\cos\theta=\frac{1}{\sqrt{5}}$ for a rotated system.

(14) a) Find the (x,y) equation of the line $\bar{x}=\sqrt{5}$.

b) Find the (x,y) coordinates if $(\bar{x},\bar{y})$ is $(-4\sqrt{5},0)$.

(7) 9. Find a power series for $f(x)=\frac{3}{2x-5}$ centered at $c=0$. Find the radius of convergence.

10. Find Taylor/Maclaurin series expansion for

a) $\frac{(\arctan x)}{x}$, centered at $x=0$

(5ea) b) e^{-x^2}, centered at $x=0$.

c) $\ln(x+1)$, centered at $x=0$.

E.C. d) Find $\int_0^1 e^{-x^2}\,dx$ within .001.

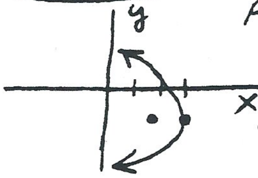
CALCULUS II Exam 4C Solutions   Dr. RAPALJE

1. $y^2+2y+6x=17$

$y^2+2y+1 = -6x+17+1$

$(y+1)^2 = -6(x-3)$

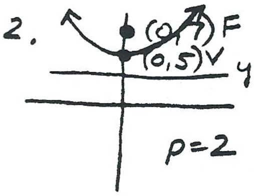
$V(3,-1)$ $4p=6$ (Left) $p=3/2$



$F(3/2,-1)$

Dir: $x = 9/2$

2.



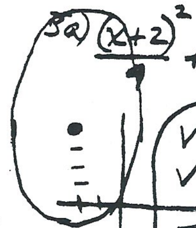
$F(0,7)$, $V(0,5)$, $y=3$

$p=2$

$(x-0)^2 = 4p(y-5)$

$x^2 = 8(y-5)$

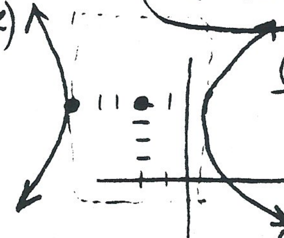
3a) $(x+2)^2 + \frac{(y-4)^2}{25} = 1$ $C(-2,4)$, $a=5$, $b=3$, $a^2=b^2+c^2$, $25=9+c^2$, $c=4$



$V(-2,-1)$, $V(-2,9)$

$F(-2,0)$, $F(-2,8)$

3b)



$\frac{(x+2)^2}{9} - \frac{(y-4)^2}{25} = 1$

$c^2=a^2+b^2$

$c^2 = 9+25$ $c=\sqrt{34}$

$V(-5,4)$, $V(1,4)$

$F(-2\pm\sqrt{34}, 4)$

4a) $16x^2+96x+9y^2+36y=-36$

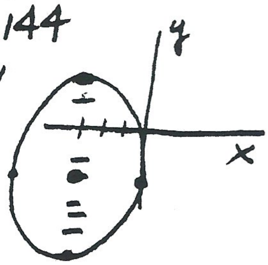
$16(x^2+6x+9)+9(y^2+4y+4) = -36+144+36$

$16(x+3)^2+9(y+2)^2 = 144$

$\frac{(x+3)^2}{9} + \frac{(y+2)^2}{16} = 1$

$C(-3,-2)$

$V(-3,2)$ $(-3,-6)$



4b) $x^2-14x-4y^2-32y=11$

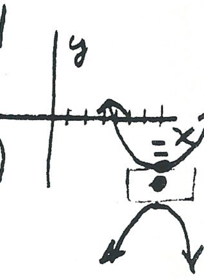
$x^2-14x+49-4(y^2+8y+16) = 11+49-64$

$(x-7)^2-4(y+4)^2 = -4$

$\frac{(y+4)^2}{1} - \frac{(x-7)^2}{4} = 1$

$C(7,-4)$

$V(7,-3)$ $(7,-5)$



5. $F(3,1)$ $(3,9)$ up + down.

$C(3,5)$ $c=4$

$2a=12$

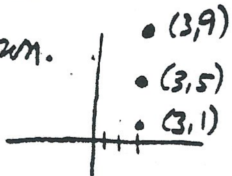
$a=6$ Ellipse

$a^2=b^2+c^2$

$36=b^2+16$

$b^2=20$

$\frac{(x-3)^2}{20} + \frac{(y-5)^2}{36} = 1$



6.

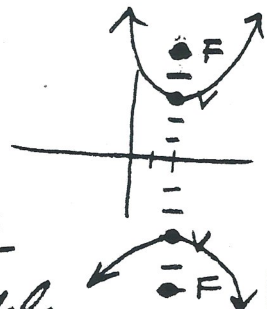
$a=3$

$c=5$

Hyperbola

$a^2+b^2=c^2$

$9+b^2=25$, $b=4$



$\frac{y^2}{9} - \frac{(x-2)^2}{16} = 1$

7a) $17x^2-12xy+8y^2=80$

$\cot 2\theta = \frac{17-8}{-12} = -\frac{3}{4}$

$\cos 2\theta = -3/5$

$\sin\theta = \sqrt{\frac{1+3/5}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt5}$ $\cos\theta = \sqrt{\frac{1-3/5}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt5}$

b) $\bar{A} = 17\cdot\frac15 - 12\cdot\frac25 + 8\cdot\frac45 = 5$

$\bar{C} = 17\cdot\frac45 + 12\cdot\frac25 + 8\cdot\frac15 = 20$

$5\bar{x}^2 + 20\bar{y}^2 = 80$

$\frac{\bar{x}^2}{16} + \frac{\bar{y}^2}{4} = 1$

8a) $\bar{x} = \sqrt5 = x\cos\theta + y\sin\theta$

$\sqrt5 = x\frac{1}{\sqrt5} + y\cdot\frac{2}{\sqrt5}$

$x+2y=5$

b) $x = \bar{x}\cos\theta - \bar{y}\sin\theta$

$= -4\sqrt5\cdot\frac{1}{\sqrt5} = -4$

$y = \bar{x}\sin\theta + \bar{y}\cos\theta$

$= -4\sqrt5\cdot\frac{2}{\sqrt5} = -8$

9. $f(x) = \frac{3}{2x-5} = \frac{\frac32}{x-5/2}$ $a=3/2$ $b=5/2$

$= -\frac35 \sum_{n=0}^{\infty}\left(\frac{2x}{5}\right)^n$

10a) $\frac{\arctan x}{x} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{(2n+1)}$

b) $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!}$

$e^{-x^2} = \sum_{n=0}^{\infty}\frac{(-1)^n (x)^{2n}}{n!}$

c) $\frac{1}{1+x} = \sum_{n=0}^{\infty}(-1)^n x^n$

$\ln(1+x) = \int\frac{1}{1+x}dx = \sum_{n=0}^{\infty}\frac{(-1)^n x^{n+1}}{n+1}$

10d) $\int_0^1 e^{-x^2}dx$

$= \int_0^1 \sum \frac{(-1)^n x^{2n}}{n!} = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)\,n!}\Big|_0^1$

$= \frac11 - \frac13 + \frac{1}{5\cdot2} - \frac{1}{7\cdot6} + \frac{1}{9\cdot24} - \frac{1}{11\cdot120}$

$< .001$

$=$