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FORMULAS for EXAM 4

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \qquad P = A\left(1 + \frac{r}{n}\right)^{-nt}$$

$$A = Pe^{rt}$$

$$\binom{n}{r} = {}_nC_r = \frac{n!}{r!\,(n-r)!}$$

ARITHMETIC SEQUENCE

$$a_n = a_1 + (n-1)d$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

GEOMETRIC SEQUENCE

$$a_n = a_1 r^{n-1}$$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

$$S_\infty = \frac{a_1}{1-r}$$

when $-1 < r < 1$

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MAC 1140 EXAM 4 B NAME _____

Chapters 5 and 11

SHOW ALL WORK ON THIS TEST OR ON SEPARATE PAPER. Circle answers. TURN IN ALL WORKSHEETS. CALCULATORS ARE REQUIRED ON THIS TEST.

Simplify each of the following:

1a) $\frac{12!}{6!} =$ b) $\binom{12}{2} =$ c) $\binom{12}{10} =$

2. $\frac{(n+2)!}{n!}$ 3. Expand $(x-2y)^6$

4. If you earn \$.10 the first day, \$.20 the second day, \$.30 the third day, \$.40 the 4th day and so on for 30 days,
   a) How much do you earn on the 30th day?
   b) How much total did you earn in 30 days?

5. If you earn \$.01 the first day, \$.02 the second day, \$.04 the third day, \$.08 the 4th day, etc.
   a) How much do you earn on the 30th day?
   b) How much did you earn (total) in 30 days?

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6. Find the sum:

$$\sum_{i=1}^{100} 5i$$

7. Find the sum

$$\sum_{i=1}^{6} 3^i$$

8a) $\log_5 \sqrt[3]{5} =$

b) $\log_2 \frac{1}{8} =$

9a) $\log_3 3^{12} =$

b) $\ln e^{3x} =$

10. Use the change of base rule to approximate

a) $\log_5 20$

b) $\log_{20} 5$

11a) Sketch the graph $y = \log_4(2x^2 - x)$

From this graph or analytically,

b) $\log_4(2x^2 - x) = 0$

c) $\log_4(2x^2 - x) > 0$

d) $\log_4(2x^2 - x) < 0$

Find the exact value and round to nearest thousandth.

12. $6e^{3x-2} = 30$

13. $500(1 + .05)^{x/4} = 1000$

14. Find the compound amount if $10,000 is invested for 20 years at 8% interest compounded
a) annually b) quarterly c) continuously.

15. Find the present value of $10,000, 20 years from now at 8% interest compounded quarterly.

16. How long will it take $50,000 to grow to $80,000 if the interest rate is 8% compounded continuously.

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MAC 1140 Exam 4B Solutions

1a) $\frac{12!}{6!} = \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7\cdot 6!}{6!}$ $= \boxed{665280}$

b) $\binom{12}{2} = {}_{12}C_2$ or $\frac{12\cdot 11}{2\cdot 1} = \boxed{66}$

c) $\binom{12}{10} = \binom{12}{2} = \boxed{66}$

2. $\frac{(n+2)!}{n!} = \frac{(n+2)(n+1)n!}{n!}$ $= \boxed{(n+2)(x+1)}$ or $\boxed{n^2+3n+2}$

3. $(x-2y)^6 = x^6 - 6(x)^5(2y) + \binom{6}{2}x^4(2y)^2 - \binom{6}{3}x^3(2y)^3 + \binom{6}{4}x^2(2y)^4 - 6x(2y)^5 + (2y)^6$

$= x^6 - 12x^5y + 15x^4\cdot 4y^2 - 20x^3 8y^3 + 15x^2\cdot 16y^4 - 6x\cdot 32y^5 + 64y^6$

$= \boxed{x^6 - 12x^5y + 60x^4y^2 - 160x^3y^3 + 240x^2y^4 - 192xy^5 + 64y^6}$

4. $a_1 = .10$ $a_2 = .20$ $a_3 = .30$ $a_4 = .40$

Arithmetic $a = .10$ $d = .10$

a) $a_{30} = (.10)(30)$ $\boxed{= \$3.00}$

b) $S_{30} = \frac{30}{2}(.10 + 3.00)$ $= 15(3.10)$ $\boxed{= \$46.50}$

5. $a_1 = .01$ $a_2 = .02$ $a_3 = .04 \ldots$

Geometric $a_1 = .01$, $r = 2$

a) $a_{30} = .01\cdot 2^{29} = \boxed{\$5368709.12}$

b) $S_n = \frac{.01(1-2^{30})}{1-2} = \boxed{\$10,737,418.23}$

#6, 7 Also use sum seq(

6. Sum Seq(5x, x, 1, 100, 1)

7. Sum Seq(3^x, x, 1, 6, 1)

(6) $\sum_{i=1}^{100} 5i = 5 + 10 + 15 + \ldots$

Arithmetic $a_1 = 5$ $d = 5$

$a_n = a_1 + (n-1)d$

$a_{100} = 5 + 99\cdot 5 = 500$

$S_{100} = \frac{100}{2}(5 + 500)$

$= \boxed{25250}$

(7) $\sum_{i=1}^{6} 3^i = 3 + 9 + 27 + 81 + 243 + 729$

$= \frac{a_1(1-r^n)}{1-r} = \frac{3(1-3^6)}{1-3} = \boxed{1092}$

8a) $\log_5 \sqrt[3]{5} = \log_5 5^{1/3} = \boxed{1/3}$

b) $\log_2 \frac{1}{8} = \log_2 2^{-3} = \boxed{-3}$

9a) $\log_3 3^{12} = \boxed{12}$

b) $\ln e^{3x} = \boxed{3x}$

10a) $\log_5 20 = x$

$5^x = 20$

$\ln 5^x = \ln 20$

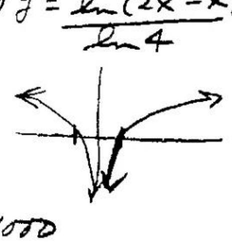
$x \ln 5 = \ln 20$

$x = \frac{\ln 20}{\ln 5} = \boxed{1.861}$

b) $\log_{20} 5 = \frac{\ln 5}{\ln 20} = \boxed{.537}$

11. $y = \log_4(2x^2 - x)$

a) $y = \frac{\ln(2x^2-x)}{\ln 4}$



b) $\log_4(2x^2 - x) = 0$

$4^0 = 2x^2 - x$

$0 = 2x^2 - x - 1$

$(2x+1)(x-1)$

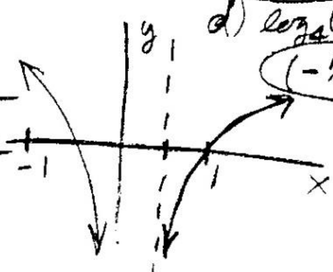
$\boxed{x = -1/2 \quad x = 1}$

c) $\log_4(2x^2 - x) > 0$

$\boxed{(-\infty, -1/2) \cup (1, \infty)}$

d) $\log_4(2x^2 - x) < 0$

$\boxed{(-1/2, 0) \cup (\tfrac{1}{2}, 1)}$



12. $6e^{3x-2} = 30$

$e^{3x-2} = 5$

$\ln e^{3x-2} = \ln 5$

$3x - 2 = \ln 5$

$3x = 2 + \ln 5$

$\boxed{x = \frac{2 + \ln 5}{3}}$

$\boxed{= 1.203}$

13. $500(1.05)^{x/4} = 1000$

$(1.05)^{x/4} = 2$

$\ln(1.05)^{x/4} = \ln 2$

$\frac{x}{4}\ln 1.05 = \ln 2$

$\frac{x}{4} = \frac{\ln 2}{\ln 1.05}$

$\boxed{x = \frac{4\ln 2}{\ln 1.05}} = \boxed{56.827}$

14. $A = P\left(1+\frac{r}{n}\right)^{nt}$ $\quad n=4,\ t=20$

a) $A = 10{,}000\left(1+.08\right)^{20}$
$= \$46{,}609.57$

b) $A = 10{,}000\left(1+.02\right)^{80}$
$= \$48{,}754.39$

c) $A = Pe^{rt}$
$= 10{,}000\,e^{20(.}$
$= 10{,}000\,e^{1.6}$
$\$49{,}530.32$

15. $P = \dfrac{A}{\left(1+\frac{r}{n}\right)^{nt}}$

$= \dfrac{10{,}000}{\left(1+.02\right)^{80}}$

$= \$2051.10$

16. $A = Pe^{rt}$

$80{,}000 = 50000\,e^{.08t}$

$1.6 = e^{.08t}$

$\ln 1.6 = \ln e^{.08t}$

$\ln 1.6 = .08t$

$t = \dfrac{\ln 1.6}{.08}$

$= 5.876$ yrs.