Categorical syllogisms – a set theory do-over

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Abstract

One models the usual S,P,M terms which appear in the 3 statements of a categorical syllogism as sets. This way the two premises and the conclusion of a categorical syllogism become statements about sets. Since sets are not "distributed", do not appear in "figures", do not have "moods", etc., such concepts will not be used in this do-over of the categorical syllogisms. One will examine all the 36 pairs of categorical premises (PCP) and partition them into 5 subsets: two subsets whose PCPs do not entail any logical conclusion (LC), and three subsets whose PCPs do each entail at least one LC and thus generate a valid categorical argument (VCA). This way one obtains three classes of VCAs, each class containing some of the valid syllogisms (VS). By definition a VS is a VCA whose LC is one of the categorical operators (or quantifiers) A,O,E,I applied to the ordered pair (S,P). The LCs of the VCA\VS set have one of the formats A(P,S), O(P,S) or I(S',P'), where S',P',M' are the complementary sets of S,P,M in a universal set U. Using a P↔S relabeling one transforms a VCA\VS PCP which entails an A(P,S) or O(P,S) LC, into a VS PCP which has an A(S,P) or O(S,P) LC, and is thus a VS. One may argue that such VCAs are of no interest since they are, up to a P↔S relabeling, identical to usual VSs. One may also argue that LCs of the format I(S',P') are of no practical interest, (maybe because the I(S',P') LC introduces the S', P' terms which do not appear in the (English) wording of the 36 PCPs). Such arguments, (outside of set theory), would play an equivalent role to the valid syllogism rules – whose only role is to eliminate the VCA\VS set as being invalid syllogisms, even if they are valid categorical arguments. Using conversions, obversions and contrapositions, or, equivalently, set relabelings involving all the six sets S,P,M,S',P',M', one can show that the VCAs inside each of the three VCA classes are transformed into each other, i.e., are equivalent to one another. Existential import (ei) means that one adds, to a PCP made of two universal premises, the supposition that one of the sets S,P,M,S',P',M' is non empty.

Keywords: *categorical premises • valid syllogisms • valid extended syllogisms • cylindrical Venn diagram • Karnaugh map • categorical operators*

1. Introduction

By definition a categorical syllogism is made of a pair of categorical premises (PCP) to which one tacks a 3rd statement, an (S,P)-conclusion, i.e., one of the categorical operators A,O,E,I applied to the order pair (S,P). If the conclusion is truly entailed by the PCP one has a valid syllogism (VS), otherwise the syllogism is invalid. We are interested in all possible PCPs, and we'll say that any PCP which entails a logical conclusion (LC) generates a valid categorical argument (VCA). As it turns out, the VCA\VS set includes those PCPs whose entailed LCs are one of the statements I(S',P'), O(P,S) or A(P,S). By embedding the VS set into the VCA set one can naturally dispense with the four syllogistic figures, term distribution, and other rules of valid syllogisms meant to eliminate all valid syllogistic arguments, VCA, except the VS. There are 36 distinct pairs of categorical premises (PCP) partitioned into 5 subsets: 1. Both premises are particular. 2. One premise is universal and one particular and they act one on M, the middle term, and the other premise acts on M', the complement of M in a universal set U. (S',P',M' are the complementary sets of S,P,M in U.) 3. One universal premise and one particular premise both acting on either M or M'. 4. Two universal premises both acting on either M or M'. 5. Two universal premises acting one on M and one on M'. It is easy to see that the PCP from the first two subsets do not entail any logical conclusions (LC). Any PCP from the subsets 3) to 5) entails at least one LC and thus generates a valid syllogistic argument (VCA). A valid syllogism (VS) is a VCA whose LC consists of one of the categorical operators A,O,E,I applied to the ordered pair (S,P). Each VCA (and VS) LC is easily found either via a “tree like method” (which eliminates, (i.e., closes), any subset (i.e., branch), emptied by a universal premise), or, by looking at the cylindrical Venn diagram below. The VCA are partitioned into three equivalence classes, each class being generated by the PCPs from subsets 3) to 5), respectively. Inside each VCA equivalence class, via a relabeling transformation of the sets S,P,M, S',P',M', (or, equivalently, via obversion, contraposition and conversion), any of the VCA can be recast (or reformulated) as any other VCA (or VS) from the same class. The syllogistic figures, the term distribution, other rules of valid syllogisms are not used in the set theoretical treatment of the VCAs. Out of the 36 distinct PCP (or just pairs), only 19 pairs entail at least one LC and thus generate VCAs, out of which 8 are distinct VS, and 6 are distinct existential import (ei) VS. (If syllogistic figures are used, then one counts 15 VS and 9 ei VS, but this means that, e.g., the same content VS, Ferio/Festino/Ferison/Fresison, receives four different names and counts as 4 distinct VS, when in reality one deals with one PCP, E(M,P)I(M,S), and one LC: O(S,P). Note that for any PCP in subset 3) one starts the (very short) tree with the non-empty intersection of the two sets appearing in the particular premise: in Ferio/Festino/Ferison/Fresison's case, Ø≠MS:=M∩S=MSP+MSP'=MSP' since the premise E(M,P) says MP= Ø. Thus the LC is MSP'≠Ø or O(S,P). (One denotes by + the union of disjoint subsets.) The VCA\VS subset contains 6 VCAs and 7 ei VCAs.

2. The Cylindrical Venn diagram (the Karnaugh map for n=3)

S'P'M	SP'M	SPM	S'PM
S'P'M'	SP'M'	SPM'	S'PM'

Fig. 1

For easier drawing, the universal set U is graphed as a rectangle – but please imagine that the left and right borders of the rectangle are glued together, so that S'PM and S'P'M are adjacent, and S'PM' and S'P'M' are adjacent, too – as in the usual 3-circle Venn diagram. On this “cylindrical Venn diagram” - or Karnaugh map with n=3, no inference rules and no axioms are needed to prove any of the syllogistic conclusions. Since the 8 subsets of Figure 1 are the “special/elementary” subsets one refers to all the time, one calls them just subsets; no other set will be a “subset”. Note that is not necessary to replace Venn's circles (John Venn 1880) by squares (Alan Marquand 1881), but it is much easier to see the LC entailed by any PCP on a cylindrical Venn diagram/Karnaugh map for n=3, than on a 3-circle Venn diagram. It took me about a year to “invent” the cylindrical Venn diagram, only to find out - after I sent to the publisher the 1st version of this paper – that Alan Marquand invented it in 1881, and then, in 1952 and 1953, Edward Veitch and Maurice Karnaugh used Karnaugh (-Veitch) maps for n=3, n=4, etc., for finding the optimal design of digital circuits. After more than 130 years, these maps, apparently, never made it into logic textbooks – which are still using the 3-circle Venn diagrams.

3. Notations

The “emptying operators” A and E appear in universal premises (All..., No...), and the “element laying” operators I and O appear in particular (Some..., Some... not) premises. As known, A(M,P) means “All M is P”, i.e., the set P'M :=P' ∩M is empty. Thus, in Fig. 1, A(M,P) acts on the M row, by emptying (two “horizontally adjacent” subsets) P'M = SP'M + S'P'M. (One denotes by + the union of disjoint subsets.) Compare the above to A(P,M), which means “All P is M”, i.e., the set PM'=P∩M' =Ø. Therefore A(P,M) acts on the M' row, by emptying, two other horizontally adjacent subsets: PM' = SPM' + S'PM'. It follows that A(M,P) and A(P,M) empty subsets not only on different rows, but also on totally different/complementary columns. Using set properties, or, obversion and conversion, one writes A(M,P)=E(M,P'), A(P,M)=E(P,M'), O(M,P)=I(M,P'), O(P,M)=I(P,M'), etc. One orders all six possible P-premises, (resp. all six possible S-premises), as vector components: $\mathbb{P}_i$ ={E(M,P'), E(M,P), E(M',P), I(M,P'), I(M,P), I(M',P)}, resp., $\mathbb{S}_i$ ={E(M,S'), E(M,S), E(M',S), I(M,S'), I(M,S), I(M',S)}. All the possible PCP are the components of the direct product of these two vectors $L_{ij} = \mathbb{P}_i \otimes \mathbb{S}_j$, i,j = 1,..,6. One always lists a PCP with the P-premise first, and the S-premise second. (P won't necessarily be the "predicate of the conclusion"; it's "just a set called P".) The intersections of the sets appearing in each premise are either empty (universal premises) or ≠Ø (particular premises). Note that four P-premises (resp. S-premises) out of six act on M and only two act on M', and that only by adding A(M',*) and O(M',*) to the list of premises one can get to 8 P-premises and 8 S-premises, with 4 P-premises, (resp. 4 S-premises) acting on the M set and 4 P-premises, (resp. 4 S-premises) acting in similar ways on the M' set. Without the mentioned "M' premises addition", there are 36 distinct PCPs partitioned into five subsets: 1. Both premises are particular. 2. One premise is universal and one particular and they act one on M, the middle term, and the other premise acts on M'. 3. One universal premise and one particular premise both acting on either M or M'. 4. Two universal premises both acting on either M or M'. 5. Two universal premises acting one on M and one on M'. It is easy to see that the PCP from the first two subsets do not entail any LC. (Subset 1) contains 9 PCPs, and subset 2) contains 8 PCPs.) Any of the other 19 PCPs from the subsets 3) to 5) entails at least one LC and thus generates at least one VCA. A valid syllogism (VS) is a VCA whose LC consists of one of the categorical operators A,O,E,I applied to the ordered pair (S,P). Each VCA (and VS) LC is easily found either via a "tree like method" which eliminates, i.e., closes any subset, i.e., branch emptied by a universal premise, or, by simply looking at the cylindrical Venn diagram above. The VCA are partitioned into three equivalence classes, each class being generated by the PCPs from subsets 3), 4), and 5), respectively. Inside each VCA equivalence class, via a relabeling transformation of the sets S,P,M, S',P',M', (or, equivalently, via obversion, contraposition and conversion), any of the VCA can be recast (or reformulated) as any other VCA (or VS) from the same class. The syllogistic figures, the term distribution, other rules of valid syllogisms are not used in the treatment of VCA, out of which 8 are distinct VS, and 6 are distinct existential import (ei) VS. (If syllogistic figures are used, then one counts 15 VS and 9 ei VS, but this means that, e.g., the same content VS, Ferio/Festino/Ferison/Fresison, recei VCA four different names and counts as 4 distinct VS, when in reality one deals with one PCP, E(M,P)I(M,S), and one LC: O(S,P). Note that **for any of the 10 PCPs in subset 3) one starts the (very short) tree with the non-empty intersection of the two sets appearing in the particular premise.** In Ferio/Festino/Ferison/Fresison's case, Ø≠MS:=M∩S=MSP+MSP' =MSP' since the premise E(M,P) says MP= Ø. Thus the LC is MSP'≠Ø or O(S,P). The VCA\VS subset contains 6 VCA and 7 ei VCA, whose LCs are one of I(S',P'), O(P,S), A(P,S).

For mnemonic reasons, one denotes A(M,*) by A_1 and A(*,M) by A_2, where * stands for either S or P, and the same for the O categorical operator. By always listing the P-premise first, one can shorten Barbara's premises to A_1A_2, Darapti's premises to A_1A_1, etc.

4. Examples of VCA recasting for Darapti's class

Subset 4) of two universal, i.e., emptying, premises both acting on either M or M' contains only five VCA, (based 4 on ei on M, and one on ei on M'), out of which two are ei VS. To determine the LC for **for any of the 5 PCPs in subset 4) one always starts the tree with M (or M'; whichever appears in both premises)** and one continues removing from M the empty subsets. For example, in Darapti's case, A_1A_1=E(M,P')E(M,S'), start with M=MS+MS'=MS =MSP+MSP'=MSP. The LC is M=MSP, aka A(M, MSP), or I(S,P) if M≠Ø. Thus:

1. A_1A_1=E(M,P')E(M,S'): A(M, SPM) → I(S,P) (M≠Ø), Darapti, [All M is P, All M is S → M=SPM]

2. EA_1=E(M,P)E(M,S'): A(M, SP'M) → O(S,P) (M≠Ø), Felapton/Fesapo, [No M is P, All M is S → M=SP'M]

3. EE=E(M,P)E(M,S): A(M, S'P'M) → I(S',P') (M≠Ø),[No M is P, No M is S → M=S'P'M]

4. A_2A_2=E(M',P)E(M',S): A(M', S'P'M') → I(S',P') (M'≠Ø), [All P is M, All S is M → M'=S'P'M']

5. A_1E=E(M,P')E(M,S): A(M, S'PM) → O(P,S) (M≠Ø), [All M is P, No M is S → M=S'PM]

The pair A_1A_1 and its ei conclusion I(S,P) can be recast as any of the other four pairs and their respective ei conclusions via these relabelings of the S,P,M,S',P',M' sets:

1↔2: P'↔P; 1↔3: S'↔S, P'↔P; 1↔4: M↔M', S'↔S, P'↔P; 1↔5: S'↔S

For example, in "set language" A_1A_1 means that M is contained in the SP intersection; A_2A_2 means that S and P are included in M, while their relative position inside M is undetermined. But this also means that S' and P' both include M', which turns A_2A_2 into an "A_1A_1 situation" - only now we have to use the S',P',M' variables in the "new" A_1A_1: All M' is P', All M' is S', with the Darapti like conclusion: I(S',P') if M'≠Ø, which would have been the entailed conclusion of A_2A_2 all along, without any recasting as A_1A_1. In "set language" EA_1 means that M and P are disjoint and M is included in S. The relative position of S and P is unknown. But the relative position of S, P' and M is perfectly known: M is contained in the SP' intersection. We have again an "A_1A_1 situation" in the S,P',M variables: All M is P', All M is S, with its modified Darapti conclusion I(S,P')=O(S,P) if M≠Ø; which would have been the Felapton/Fesapo conclusion anyhow, without any A_1A_1 recasting of EA_1. It seems that the only purpose of the (S,P)-conclusion restriction and most of the "rules of valid syllogisms" is to separate the VS from the VCA\VS. Both A_2A_2 and EE premises entail the I(S',P') ei conclusion when M≠Ø, with or without any recasting into the "VS approved" pairs A_1A_1, or EA_1. Moreover, M is undistributed in both A_2A_2 premises, and the EE premises are both negative. For more details about equivalences between valid categorical arguments please see Section 6 below.

5. Conclusions' shape

As one already saw for the PCPs in subsets 3) and 4), any entailed LC refers precisely to one subset (out of 8), and falls in one of the following two categories:

(α) one, (or even two – for PCPs in subset 5)), of the sets S, P, M, S', P', M' is reduced, via two universal, (aka emptying), premises to only one of its 4 subsets

(β) one of the 8 subsets in Figure 1 is shown to be ≠ Ø (possibly via an existential import (ei) supposition).

When ei is used, the conclusion is reached in two stages: first one of S, P, M, S' , P', or M' is reduced to just one subset out of 4 (stage **(α)**), then, the ei makes/declares that subset ≠Ø.

The above **(α)** and **(β)** express the fact that a PCP entailing an LC pinpoints to just one subset out of 8. Note that there is a "tension" between the "one subset out of 8 conclusion" to which a PCP pinpoints, and the "Aristotle's requirement" that the conclusion of a valid syllogism, LC, should not contain the middle term. The latter condition means that the LC refers to a column containing two subsets – one included in M, the other in M'. The difference in information between a PCP that pinpoints to just one subset out of 8 and the standard expression for an LC which refers to a column and thus pinpoints to two subsets out of 8, is a consequence of the requirement that the middle term should not appear in the conclusion. One can say that the LC summarizes the "new knowledge" obtained from the pair of premises, and that to list the "column LC" together with all the other information the premises provide would necessarily mean to relist one or both premises together with the "column LC" - and this is exactly what we do not want to do, as per "Aristotle's requirement" (Striker 2009: 20): "A syllogism is an argument in which, certain things being posited, something **other than what was laid down** results by necessity because these things are so." One way to keep all the information a PCP provides, without completely relisting the premises would be to spell out the "column LC" together with the subset the LC is "bound" to. For example, since a VS requires an "(S,P) conclusion", i.e., that, in the conclusion, one of the operators A,O,E,I be applied to the ordered pair (S, P), all VS conclusions are in fact necessarily bound to SPM, or SP'M, or SP'M'! (One can check below, Section 6, that there is no PCP pinpointing to the SPM' subset as being the LC.) Any VCA, bound to any other subset, has no name. (But, for example, LCs "bound to SPM" are A(S,P) (Barbara), I(S,P) (Barbari, Bramantip, Darapti, Darii/Datisi, Disamis/Dimaris), A(P,S). The last one, originates from the VCA A_2A_1: P=SPM, S'= S'P'M'. Then one gets A(P, SPM), and thus A(P,S), which has no name, even if the conclusion is bound to SPM, because A_2A_1 empty the set P except for SPM, and this does not fit the VS requirement for an “(S,P) conclusion”. But the ei, (P≠Ø), conclusion, I(S,P), gi VCA the VS Bramantip.

When one premise is universal and the other one is particular, then the LC, (entailed if and only if both premises act on the same set – either M or M' but not both) , is reached in one stage: one of the 8 subsets in Figure 1, uniquely determined, turns out to be ≠Ø. (The particular premise will have available only one subset, not two, to lay an element on, since the other horizontally adjacent subset was “just” emptied by the universal premise: only this arrangement can make both premises TRUE **and** the syllogistic argument valid. See Fact #1 below.) A standard or column LC will still refer to an entire column and not just one subset.

Note that any subset relabeling, such as, for example, P'↔M, S↔S', does not change the immediate neighbours of any of the subsets, and does not change the conclusions of any of the premises' pairs: the conclusion of “All P is M, All M is S” $=A_2A_1$, on a new, "relabeled Figure 1”, will still be P=SPM, S'= S'P'M'.

Fact #1 For any pair of premises, {P-premise, S-premise}, both acting on the same row, there will always be one and only one subset “acted upon twice”; for any pair {P-premise, S-premise}, acting on two rows, there will always be one and only one column whose two subsets are both acted upon.

Proof: Cf. Fig. 1, two of the sets S, P, S', P', unless they are complementary sets, always have one and only one common column. Consider first the “M-row operators” A_1, O_1, E, I. In a P-premise, the operators A_1, O_1 act on the two P' columns and the E,I operators act on the two P columns. In an S-premise, the operators A_1, O_1 act on the two S' columns and the E,I operators act on the two S columns. Thus a pair (P-premise, S-premise), both acting on the M row, may act either on {P', S'}, or on {P', S}, or on {P, S'}, or on {P, S}, in which cases, respectively, either the subset S'P'M, or SP'M, or S'PM, or SPM is acted upon twice, and, respectively, either the subset SPM, or S'PM, or SP'M, or S'P'M is **not** acted upon at all. Thus two universal premises acting on the same row will empty 3 subsets, (of M or M'), and one universal and one particular premise acting on the same row will always place a set element on precisely one subset.

Since the A_2, O_2 operators - which act on the M' row - behave similarly to the E,I operators which act on M row - i.e., in a P-premise, the operators A_2, O_2 act on the two P columns, (exactly as E,I do on the M row), and in an S-premise, the operators A_2, O_2 act on the two S columns, (exactly as E,I do on the M row), it follows, as above, that a “2-row acting” pair of premises will always “act upon a column twice” either emptying both column's subsets, (and this is the only interesting case!), or possibly laying set elements in both column's subsets, or emptying one of the column's subset and laying a set element on the other column's subset – all these latter variants correspond to pairs of premises that do not entail any LC. (See below the paragraphs (i) and (ii2).) The four 2-row acting pairs of universal premises will thus empty one column, plus two other subsets, located on two different rows, on each side of that emptied column.(See the paragraph (ii1) below.) QED. (An examination of the 36 cases below makes the proof of Fact #1 clear, too.)

6. A more detailed discussion of the matrix L_{ij} , i,j = 1,..,6

The matrix $L_{ij} = \mathbb{P}_i \otimes \mathbb{S}_j$, i,j = 1,6 naturally splits into four 3 by 3 sub matrices: $L^{(1)}:=L_{ij}$, i,j = 1,2,3, contains only, (and they are the only ones), pairs of two universal premises; $L^{(2)}:=L_{ij}$, i=4,5,6, j=1,2,3, contains pairs of one particular P-premise, [gotten from replacing in $L^{(1)}$ the universal P-premise with the corresponding, (and contradictory), particular P-premise], and one universal S-premise (left unchanged from $L^{(1)}$); $L^{(3)}:=L_{ij}$, i=1,2,3, j=4,5,6, contains pairs of one universal P-premise, (unmodified from $L^{(1)}$), and one particular S-premise, [gotten from replacing in $L^{(1)}$ the universal S-premise with the corresponding, (and contradictory), particular S-premise]; and the sub-matrix $L^{(4)}:=L_{ij}$, i,j = 4,5,6 which contains only, (and they are the only ones), pairs of two particular premises.

(i) $\mathbf{L^{(4)}}$**:** The pairs of premises in the sub-matrix $L^{(4)}:=L_{ij}$, i,j = 4,5,6, do not entail any LC. The two particular premises will "lay set elements" either on three subsets of the same row (M or *M'*), or on 4 subsets on different rows. Since, any conclusion of such a pair would just relist one or two of its premises, there is no way to satisfy Aristotle's requirement, (Striker 2009: 20), that "A syllogism is an argument in which, certain things being posited, something **other than what was laid down** results by necessity because these things are so." Thus, per Aristotle's insight, these pairs will not generate any valid syllogism, VS; this means nine pairs of premises on the no conclusion/discarded list.

(ii) $\mathbf{L^{(1)}}$**:** contains two sorts of universal premises pairs:

(ii0) The 5 "1-row acting" pairs of universal premises. Four pairs act on the M row only, $L_{11}=A_1A_1$, $L_{12}=A_1E$, $L_{21}=EA_1$, $L_{22}=EE$, and, one pair acts on the M' row only, $L_{33}=A_2A_2$. As the Fact #1 has shown, the M subsets SPM, or S'PM, or SP'M, or S'P'M are **not** emptied by $L_{11}=A_1A_1$, $L_{12}=A_1E$, $L_{21}=EA_1$, $L_{22}=EE$, respectively, and the S'P'M' subset of M' is **not** emptied by $L_{33}=A_2A_2$. Again, as per Aristotle's insight, only existential imports on M, resp., M', will count and produce 5 VS, each respectively "bound" on one of the above **not** emptied subsets. (Two out of five VCA are the VS Darapti and Felapton/Fesapo, bound on SPM and SP'M, respectively.) Thus the 5 "1-row acting" pairs of universal premises each produces one ei VCA, since we get one conclusion if ei is used each time one of the sets M*,* or M'*,* is reduced, via two "1-row acting" universal premises, to only one of its 4 subsets.

(ii1) The 4 "2-row acting" pairs of universal premises. They have to contain A_2 as a premise - since this is the only universal operator acting on the 2nd row M'. These 4 pairs are: $L_{13}=A_1A_2$, $L_{23}=EA_2$, $L_{31}=A_2A_1$, $L_{32}=A_2E$. They empty four subsets on two different rows and three different columns, located, cf. Fact #1, as follows: two empty subsets are on the same column, and the other two empty subsets are on different rows and on different sides of the empty column. These pairs are responsible for 12 different conclusions. **To determine the LC for subset 5) of the PCPs, one always starts two trees: one for each of the two letters - other than M and M' - which appear in the premises.** For A_1A_2=A(M,P)A(S,M)=E(M,P')E(M',S) start with S=SM'+SM=SM =SPM+SP'M =SPM and P'=MP'+M'P'=M'P'=S'P'M'+SP'M'=S'P'M'. The LCs are A(S,P)=A(P',S') (Barbara), and via ei on S, I(S,P) (Barbari), plus, via ei on P', I(S',P').

1. Thus the pair of premises $L_{13}=A_1A_2$=A(M,P) A(S,M)= E(M,P')E(M',S) empties the column SP' and the subsets S'P'M and SPM', and, out of the 3 columns SP', S'P' and SP, occupied by the sets S and P', (whose intersection is SP'), only the subsets SPM out of S, and S'P'M' out of P' "survive". LCs are therefore aplenty: A(S, SPM), A(P', S'P'M'), E(S,P'), from which it follows A(S, P), A(P', S'), E(S, P'), A(S, M), A(P', M'). But the last two conclusions are exactly the premises – so they do not count, (as new knowledge), and the first three, via set theory, (or contraposition and obversion), are equivalent: A(S, P)=A(P',S')= E(S,P'). We'll keep just A(S,P) as the only one universal conclusion, out of the three independent conclusions entailed by the "Barbara pair of premises" $L_{13}=A_1A_2$. The other two independent conclusions involve ei: on S, i.e., supposing $S\neq\emptyset$, one gets I(S,P), Barbari, and, via ei on P', one gets the no name I(S',P'), for a total of three independent conclusions entailed by the pair $L_{13}=A_1A_2$=A(M,P)A(S,M). Any other conclusions, such as I(S,M) or I(P,M) are not independent: they follow directly from the premises and $S\neq\emptyset$. Moreover, P'= S'P'M' follows from S=SPM: if we list, (now, for simplicity, on one row), from left to right, the adjacent/neighbouring subsets that were not emptied by Barbara's premises, they are SPM, S'PM, S'PM', S'P'M'. This reads, from left to right, (resp. from right to left), precisely as $S \subseteq M \subseteq P$, and, resp., $P' \subseteq M' \subseteq S'$ – which is also how the transitivity of the inclusions A(S, M), A(M,P), or the Euler diagrams, would have represented Barbara's premises.

2. Analogously, the premises A_2A_1=A(P,M) A(M,S)= E(M',P)E(M,S'), empty 4 subsets out of 6 from the columns S'P, S'P' and SP, occupied by the sets S' and P, (whose intersection is S'P). Only the subsets SPM out of P and S'P'M' out of S' will again "survive". Thus, same "survivors" but now as parts of other "big sets" S', P instead of S,P'. The independent conclusions are the no name A(P,S), and, via ei on P, I(S,P) - Bramantip. Via ei on S', one gets (again) a no name I(P', S'). One can also see, that via a simple relabeling transformation, $M\to M$, $S\to P$, $P\to S$, A_2A_1 becomes A_1A_2: A_2A_1 =A(P,M)A(M,S)→ A(S,M)A(M,P)=E(M,P')E(M',S). One can also see, that via another relabeling transformation, $M\to M'$,

$S \to S'$, $P \to P'$, A_2A_1 also becomes A_1A_2: A_2A_1 =A(P,M) A(M,S)→A(P', M') A(M',S')=E(M,P')E(M',S), [or one may use contraposition on A(P', M') to get A(M,P), and on A(M',S') to get A(S,M)]. Both relabeling transformations map the premises and the conclusions of A_2A_1 onto the premises and the conclusions of A_1A_2. [See next section for all the relabeling transformations between the VCA generated by the 4 "2-row acting" pairs of universal premises.]

3. The EA_2 = E(M,P) E(M',S) and A_2E= E(M',P) E(M,S) are even more similar than A_1A_2 and A_2A_1 are. Each of EA_2 and A_2E, empty 4 subsets out of the 6 subsets of same 3 columns SP', SP' and SP. The two subsets that survive are: SP'M and S'PM' if the premises are EA_2, and SP'M' and S'PM if the premises are A_2E. The type (α), two entailed LCs per pair of premises, are thus, for EA_2: A(S, SP'M), A(P, S'PM'). One chooses, as independent conclusions E(S,P)(=A(S,P')=A(P, S')), (Celarent/Cesare), and, via ei on P the no name O(P,S), plus, via ei on S, O(S,P), (Celaront/Cesaro).
4. Initial conclusions for A_2E are: A(S, SP'M'), A(P, S'PM). One chooses, as independent conclusion E(S,P) (=A(S,P')=A(P, S')), (Camestres/Camenes). And, via ei on P, the no name O(P,S), plus, via ei on S, O(S,P), (Camestrop/Camenop). This way, we get again to three independent conclusions when ei is used each time one of the sets S, P, S' , P' is reduced, via two "2-row acting" universal premises, to only one of its 4 subsets.

(iii) **$L^{(2)}$ and $L^{(3)}$**. Firstly, observe that the "2-row acting", 1-particular, 1-universal pairs of premises from $L^{(2)}$: $L_{43}=O_1A_2$, $L_{53}=IA_2$, $L_{61}=O_2A_1$, $L_{62}=O_2E$, and from $L^{(3)}$: $L_{16}=A_1O_2$, $L_{26}=EO_2$, $L_{34}=A_2O_1$, $L_{35}=A_2I$, do not entail any conclusion. These 8 pairs are gotten from the 4 (ii1) pairs, by substituting a particular premise in place of an universal premise. But by doing this, the emptying, and the element laying, happen now on two different rows. Any LC would just relist the premises. Thus, as per Aristotle's insight, the 8 pairs of 1-particular, 1-universal premises, acting on 2 rows, M **and** M', span the 2nd class of pairs that do not entail any LC. This adds up to a total of 9+8=17 of such pairs. Out of the other 36-17=19 pairs, we already saw 4 pairs of premises, (ii1), that entail 3 independent conclusions per pair, and 5 pairs of premises, (ii0), that entail one conclusion per pair. The rest of 10 pairs from $L^{(2)}$ and $L^{(3)}$, originate from the 5 "1-row acting" pairs of universal premises in $L^{(1)}$, by replacing one universal premise with its contradictory particular premise, and thus, cf. Fact #1, each such pair results in one precise subset being ≠Ø, and entails exactly one LC per pair, for a total of 27 valid categorical arguments, (VCA), 14 out of which - the classically valid syllogisms, (VS), have names [even multiple names for one and the same syllogism, (or pair of premises), when the premises' terms can be switched around without changing the premises' meaning]. More precisely, the five $L^{(2)}$ pairs, (which were obtained from $L^{(1)}$'s five "1-row acting" universal pairs, by changing an universal P-premise into its contradictory, particular P-premise): $L_{41}=O_1A_1$, $L_{42}=O_1E$, $L_{51}=IA_1$, $L_{52}=IE$, $L_{63}=O_2A_2$, lead to, in order, the following (β) type, conclusions: SP'M≠Ø (or O(S,P), Bocardo), S'P'M ≠Ø (or I(S',P'), no name), SPM≠Ø (or I(S,P), Disamis/Dimaris), S'PM≠Ø (or O(P,S) no name), S'PM'≠Ø (or O(P,S) no name). For the last 5 out of 10, one substitutes the contradictory particular S-premise for the universal S-premise of the $L^{(1)}$'s five "1-row acting" universal pairs, to obtain: $L_{14}=A_1O_1$, $L_{24}=EO_1$, $L_{15}=A_1I$, $L_{25}=EI$, $L_{36}=A_2O_2$. The conclusions of these pairs are, in order: S'PM≠Ø (or O(P,S)), S'P'M ≠Ø (or I(S',P')), SPM≠Ø (or I(S,P), Darii/Datisi), SP'M≠Ø (or O(S,P), Ferio/Festino/Ferison/Fresison), SP'M'≠Ø (or O(S,P), Baroco). One can notice that IE and A_1O_1 have the same conclusion S'PM≠Ø, O_1A_1 and EI have the same conclusion SP'M≠Ø, IA_1 and A_1I have the same conclusion SPM≠Ø, O_1E and EO_1 have the same conclusion S'P'M ≠Ø (since on the M row there are only 4 subsets and one has 8 pairs of premises which place/lay at least one set element in exactly one subset of M).

7. Classes of equivalent syllogistic arguments

The premises' action is easier to follow if we uniformly express any premise as either an E or I operator, acting firstly on M, or M', as the case may be. Consider for example the pairs: A_1A_1, O_1A_1, A_1O_1. Write:

A_1A_1=E(M,P')E(M,S')

O_1A_1=I(M,P')E(M,S')

A_1O_1=E(M,P')I(M,S'). All three pairs use the same variables M,P',S'. This is because, as was observed in Fact #1's proof, A_1A_1 acts twice on S'P'M, not at all on SPM, (we'll say that Darapti is bound not on the subset on which the premises' pair acts twice, but on SPM on which it doesn't act at all, and thus allows the conclusion M= SPM, out of which, via ei, the Darapti's conclusion follows. Equally important is that A_1A_1 acts once on SP'M, and once on S'PM, the subsets next to S'P'M on the "cylindrical Venn diagram", and these are exactly the subsets assured to be ≠Ø by O_1A_1, (Bocardo), and A_1O_1, respectively.

Let's now consider another similar group of 3 pairs of premises:

EE=E(M,P)E(M,S)

IE=I(M,P)E(M,S)

EI=E(M,P)I(M,S). All three pairs use the same variables M,P,S. This is because, as was observed in Fact #1's proof, EE acts twice on SPM, not at all on S'P'M, (we'll say that the no name EE:M=S'P'M is bound not on the subset on which the premises' pair acts twice, but on S'P'M on which the pair doesn't act at all, and thus allows the conclusion M= S'P'M, out of which, via ei, the no name I(S',P') conclusion follows. Equally important is that EE acts once on SP'M, and once on S'PM, and these are exactly the subsets assured to be ≠Ø by EI, (Ferio/Festino/Ferison/Fresison), and IE, respectively.

<u>Fact #2</u>: if we relabel P'→P, S'→S, then the first group of 3 pairs of premises is transformed in the 2^{nd} group of 3 pairs of premises, and, the 3 conclusions from the 1^{st} group of pairs, via this relabeling, become the 3 conclusions of the 2^{nd} group of pairs. This happens because the subsets on which A_1A_1 acted twice, resp. not at all, are mapped into subsets on which EE acts twice, resp. not at all. The same is true about the subsets on which A_1A_1 acted once – they are transformed into subsets on which EE acts once. This way not only pairs of premises are mapped onto pairs of premises, but their conclusions are mapped into respective conclusions, too. There are 5 different groups of 3 pairs of premises each, and 4 relabeling transformations that map the first set of 3 pairs of premises to the other 4 and back to the 1^{st} groups of 3 pairs of premises. One can argue that only one set of 3 pairs of premises is independent and the rest represent just what one would have gotten by a relabeling of the variables S,P,M. The final conclusion is that the 5 pairs of two universal premises acting on the same row, A_1A_1, EE, A_1E, EA_1, A_2A_2 are equivalent, and all the other 10 pairs of premises, one universal and one particular, are equivalent, too. This is so because the two strains of 5 VCA each, which start with O_1A_1 and A_1O_1, and continue with IE and resp. EI, etc. are in fact equivalent, too: one can see this, for the above mentioned pairs, via a relabeling S↔P. Thus we have 10 pairs that generate equivalent VCA: O_1A_1, IE, O_1E, IA_1, O_2A_2, A_1O_1, EI, A_1I, EO_1, A_2O_2. The set of 4 "2-row acting" pairs of universal premises can be transformed, by relabeling, among themsel VCA, too. Thus we found 3 different classes of pairs of premises, easily characterized as being: 4 pairs of 2 universal premises acting on **two** rows, M **and** M', 5 pairs of 2 universal premises acting on **one** row, M **or** M', 10 pairs of one universal and one particular premises, acting on **one** row, M **or** M'. Thus one has 3 classes of pairs of categorical premises (PCP) which generate valid syllogisms (VCA).

Below one lists the VCA from two classes out of three, grouped by the subset they do not act upon, and to which we say that they are "bound" to. One VCA class contain 5 ei VCA and the other one contains 10 VCA, for a total of 15 VCA split in five groups of three VCA each - according to the subset they are bound to. These five VCA groups use, (or act upon), the complementary variables to the variables characterizing the subset these VCA are bound to.

1. VCA bound to the subset SPM:

A_1A_1=E(M,P')E(M,S')   M=SPM. If M≠Ø: I(S,P), Darapti

O_1A_1=I(M,P')E(M,S') SP'M ≠Ø or O(S,P), Bocardo

A_1O_1=E(M,P')I(M,S') S'PM≠Ø or O(P,S), No name

2. VCA bound to the subset SP'M:

EA_1=E(M,P)E(M,S') M=SP'M. If M≠Ø: O(S,P), Felapton/Fesapo

IA_1=I(M,P)E(M,S') SPM≠Ø or I(S,P), Disamis/Dimaris

EO_1=E(M,P)I(M,S') S'P'M≠Ø or I(S',P'), No name

3. VCA bound to the subset S'P'M:

EE=E(M,P)E(M,S) M=S'P'M. If M≠Ø: I(S',P'), No name

IE=I(M,P)E(M,S) S'PM≠Ø or O(P,S), No name

EI=E(M,P)I(M,S) SP'M≠Ø or O(S,P), Ferio/Festino/Ferison/Fresison

4. (M' row) VCA bound to the subset S'P'M':

A_2A_2=E(M',P)E(M',S) M'=S'P'M'. If M'≠Ø: I(S',P'), No name

O_2A_2=I(M',P)E(M',S) S'PM'≠Ø or O(P,S), No name

A_2O_2=E(M',P)I(M',S) SP'M'≠Ø or O(S,P), Baroco

5. VCA bound to the subset S'PM:

A_1E=E(M,P')E(M,S) M=S'PM. If M≠Ø: O(P,S), No name

O_1E=I(M,P')E(M,S) S'P'M≠Ø or I(S',P'), No name

A_1I=E(M,P')I(M,S) SPM≠Ø or I(S,P), Darii/Datisi

One sees that the 5 groups of 3 VCA each, [which include 7 distinct VS, (two of them based on ei on M)], are, modulo a relabeling of S,P,M, equivalent.

One may verify the transitivity (and thus the group properties) of the equivalences using the following relabeling maps:

1↔2: P'↔P

1↔3: S'↔S, P'↔P

1↔4: M↔M', S'↔S, P'↔P

1↔5: S'↔*S*

2↔3: S↔S'

2↔4: M↔M', S'↔S

2↔5: P'↔P, S↔S'

3↔4: M↔M'

3↔5: P↔P'

4↔5: M↔M', P'↔P

Because there are only 4 subsets per each row, (M or M'), when, by relabeling, one maps one "binding subset" into another "binding subset", one also map subsets on which the group of VCA, bound to the 1st subset, do not act, act once, or act twice, into subsets on which the 2nd group of VCA, bound to the 2nd subset, do not act, act once, or act twice, respectively. This ensures that not only the pairs of premises of the 1st group of VCA transform into the pairs of premises of the 2nd group of VCA, but the conclusions from the 1st group of VCA, transform into the conclusions of the 2nd group of VCA.

Another way to show that the 5 groups of 3 VCA each are equivalent, is to start with 3 pairs of premises written in the variables A,B,C instead of the usual S,P,M:

Group 0. All B is A, All B is C

Some B is not A, All B is C

All B is A, Some B is not C

Choosing B=M, A=P, C=S we get the pairs of premises of the 1st group of VCA.

Choosing B=M, A=P, C=S' we get the pairs of premises of the 2nd group of VCA.

Choosing B=M, A=P', C=S' we get the pairs of premises of the 3rd group of VCA.

Choosing B=M', A=P', C=S', we get the pairs of premises of the 4th group of VCA.

Finally, choosing B=M, A=P', C=S we get the pairs of premises of the 5th group of VCA.

It is as if we represented Group 0, in 5 different system of coordinates: the number of distinct premise pairs, and VCA, is at most 3 not 15. We can also notice that the 5 VCA generated by "Some B is not A, All B is C", are equivalent to the 5 VCA generated by "All B is A, Some B is not C", via the relabeling A↔C. This way one can see that the same generic wording of the premises can be represented in different ways, leading to different VCA, with different conclusions, but in fact the 5 groups are equivalent: the five VCA generated by the pairs of premises A_1A_1, EE, A_1E, EA_1, A_2A_2 are equivalent, and the ten VCA generated by the pairs of premises O_1A_1, IE, O_1E, IA_1, O_2A_2, A_1O_1, EI, A_1I, EO_1, A_2O_2 are equivalent, too.

The above equivalences show again that if a pair of premises entails an LC, from only a set theoretical point of view, the difference between a VCA and a VS is irrelevant.

Note that M is not distributed in the VCA A_2A_2: M'=S'P'M'→I(S',P'), (via ei on M'), and that A_2A_2 turns out to be equivalent to A_1A_1: M=SPM→I(S,P), (via ei on M, Darapti). Also, there are pairs of two negative premises in three of the VCA - EE, O_1E, EO_1: EE generates a VCA equivalent to Darapti, (or Felapton/Fesapo), and O_1E, EO_1 generate VCAs equivalent to Darii. Thus there are pairs of premises that entail an LC but do not satisfy the usual "valid syllogisms rules", "the middle term has to be distributed in at least one premise", and, "no valid syllogism has 2 negative premises". One can start with the premises of Darapti and Darii, (i.e., A_1A_1, and resp., A_1I), re-write them using obversion and contraposition as the premises A_2A_2, (resp. O_1E), written in other variables, get the conclusions of A_2A_2, (resp. O_1E), in those variables, then realize that those conclusions can be re-written, (via appropriate "back relabelings"), as the usual Darapti, M=SPM, and Darii, SPM≠Ø, conclusions. This way one can use no name VCA which do not satisfy the usual "rules of valid syllogisms" to "bear the burden" of inferring all the conclusions for the VS contained in the two VCA classes whose VS representati VCA are Darapti and resp. Darii.

The "2-row acting" VCA/VS:

EA_2=E(M,P)E(M',S) S=SP'M, P=S'PM' (SP'M, S'PM'="survive" as the only subsets of S, resp. P, which are not emptied by the premises EA_2.) Thus: A(S,SP'M), A(P,S'PM'). One chooses, as independent conclusions E(S,P)(=A(S,P')=A(P, S')), (Celarent/Cesare), and, via ei on P the no name O(P,S), and, via ei on S, O(S,P), (Celaront/Cesaro).

A_1A_2=E(M,P')E(M',S) S=SPM, P'=S'P'M', A(S,P) Barbara, I(S,P) Barbari (S≠Ø), I(S',P') no name (P'≠Ø)

A_2A_1=E(M',P)E(M,S') P=SPM, S'=S'P'M', A(P,S) no name, I(S,P) Bramantip (P≠Ø), I(S',P') no name ($S'\neq\emptyset$)

$A_2E=E(M',P)E(M,S)$ $S=SP'M'$, $P= S'PM$. Thus: $A(S,SP'M')$, $A(P,S'PM)$. One chooses, as independent conclusion $E(S,P)(=A(S,P')=A(P, S'))$, (Camestres/Camenes). And, via ei on P, the no name $O(P,S)$, plus, via ei on S, $O(S,P)$, (Camestrop/Camenop)

The S,P,M,S',P',M' relabeling transformations showing that A_1A_2, A_2A_1, A_2E, EA_2 are equivalent, (because not only the premises transform into one another, but their respective conclusions, too):

$A_1A_2\leftrightarrow A_2A_1$: $M\leftrightarrow M'$, $S\leftrightarrow S'$, $P\leftrightarrow P'$ (or $S\leftrightarrow P$)

$A_2E\leftrightarrow EA_2$: $M\leftrightarrow M'$ (or $S\leftrightarrow P$)

$A_1A_2\leftrightarrow EA_2$: $P\leftrightarrow P'$

$A_1A_2\leftrightarrow A_2E$: $M\leftrightarrow M'$, $P\leftrightarrow P'$

$A_2A_1\leftrightarrow A_2E$: $S\leftrightarrow S'$

$A_2A_1\leftrightarrow EA_2$: $M\leftrightarrow M'$, $S\leftrightarrow S'$

Note that the six relabelings listed 1st on each row are transitive and form a group. The two $S\leftrightarrow P$ relabelings, listed 2nd on their respective rows, can not be "composed" with any other relabelings.

Or, one can start with the "generic" pair of premises All B is A, All C is B.

Then, making the obvious choice B=M, A=P, C=S, we get A_1A_2, Barbara's premises.

But choosing B=M, A=P', C=S, we get the EA_2 premises.

And choosing B=M', A=P', C=S, we get the A_2E premises.

Finally choosing B=M, A=S, C=P, we get the A_2A_1 premises.

Thus, no matter what their initial wording is, for any pair of concrete categorical premises presented to us, one can label their 3 terms in such a way, that if the pair entails an LC, then it can be expressed as either A_1A_2, or A_1A_1, or A_1I, (or any other preferred triplet of representative VCAs from each one of the 3 classes of premises that entail LCs). After the LC of A_1A_2, or A_1A_1, or A_1I, is written down, one can do a "back relabeling" to re-express the conclusion via the most intuitive term labeling suggested by the initial premises.

8. Conclusions

Instead of the old accounting rules and restrictions imposed to separate the VS from VCA – an (S,P) conclusion, "In any valid syllogism the middle term is distributed at least once", "No valid syllogism has two negative premises", etc., the **Venn diagram**, (cylindrical or not, but on the usual "3 intersecting circles" Venn diagram, the above facts are difficult to see), **approach**, allows for simpler rules:

1. The 36 PCP fall into 5 classes: 3 classes entail an LC and 2 do not.
2. Each LC is either of type **(α)** or of type **(β)** above, and refers to just one subset, out of the 8 subsets of U.
3. Inside each of the 3 classes of PCP entailing an LC, the VCA (and VS) are all equivalent in the sense described above.
4. One may offer two, or even five, "new rules of valid syllogisms". Two negative rules: 1. No two particular premises are allowed (this coincides with one of the old rules). 2. A universal premise and a particular premise, one acting on the middle term M and the other acting on its complementary set M' are not allowed. (Note that the "old rules of valid syllogisms" were in fact meant to invalidate all but the VS.) Three positive rules - the rest of the pairs of premises are allowed since they entail at least one LC: two universal premises acting on the "same row" (either M or M'); two universal premises acting on

"two rows" (both M and M'); a universal premise and a particular premise acting on the same row (either M or M').

5. As described in Section 4, the logical consequences of the 19 out of 36 possible pairs of premises are as follows: the "(S,P) conclusions" A(S,P), E(S,P), I(S,P), O(S,P) – which are satisfied only by the VS; A(P,S) entailed only by A_2A_1; I(S',P') and O(P,S). The latter conclusions are entailed by pairs of premises which, via ei or not, generate VCA which are not VS (VCA\VS). If one could logically argue that these I(S',P'), O(P,S), A(P,S) conclusions are not to be admitted, even if logically entailed by the VCA\VS pairs of premises, then, indeed, only the VS are valid. As most of the logic textbooks do, one can restrict the valid syllogisms, by definition, to only the pairs of premises whose entailed consequences are of the "(S,P) type"; or one can use the rules of valid syllogisms to help eliminate any pair of premises which does not generate a VS.

Because of its lack of symmetry, the usual "3 intersecting circles Venn diagram" model, was used only to "verify" particular syllogisms' validity, but not to find all the possible logical conclusions from all the categorical pairs of premises. (By inflating the number of cases to consider, the syllogistic figures were a detractor of such an endeavour, too.) See, e.g., Barker (2003). See also, Quine (1982), who proposed as "an hour's pastime" exercise, the Venn diagram checking of all premises' pairs for conclusion entailment.

After I initially wrote this paper, I found out that the "cylindrical Venn diagram" was discovered long time ago: it is now called a Karnaugh(-Veitch) map for 3 sets. The "cylinder idea" is used to match "close enough" the adjacency displayed by the 8 subsets on the "3-circle Venn diagram". For the same adjacency reason a Karnaugh map for 4 sets is represented as a 4 by 4 square with "glued edges" - which thus becomes a torus. (See Marquand (1881), Veitch (1952), Karnaugh (1953), (Wikipedia.org/wiki/Karnaugh_map, and Wikipedia.org, Quine–McCluskey algorithm.) ["Close enough", means, e.g., that after Barbara's premises empty 4 subsets out of 8, the other 4 subsets left would be disconnected on a rectangular diagram, but are still connected on the cylindrical Venn diagram and moreover satisfy $S \subseteq M \subseteq P$.]

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The class of two universal premises acting on two rows

PCP (Pair of Categorical premises)	Set translation	Subset translation	Subset LC	Subset ei	CVS LC	CVS ei	VSA LC	VSA ei
Class of two universal premises acting on two rows								
A_1A_2	MP'=Ø	SP'M=Ø	S=SPM		A(S,P)			
	SM'=Ø	S'P'M=Ø		S≠Ø	Barbara	I(S,P)		
		SPM'=Ø	P'=S'P'M'			Barbari	A(P',S')=	
		SP'M'=Ø		P'≠Ø			A(S,P)	I(S',P')
							(discard)	No name
A_2A_1	PM'=Ø	SPM'=Ø	P=SPM				A(P,S)	
	MS'=Ø	S'PM'=Ø		P≠Ø		I(S,P)	No name	
		S'PM=Ø	S'=S'P'M'			Bramantip	A(S',P')=	
		S'P'M=Ø		S'≠Ø			A(P,S)	I(S',P')
							(discard)	No name
EA_2	MP=Ø	SPM=Ø	S=SP'M		E(S,P)			
	SM'=Ø	S'PM=Ø		S≠Ø	Celarent/	O(S,P)		
		SPM'=Ø	P=S'PM'		Cesare	Celaront/	E(S,P)	
		SP'M'=Ø		P≠Ø		Cesaro	(discard)	O(P,S)
								No name
A_2E	PM'=Ø	SPM'=Ø	S=SP'M'		E(S,P)			
	MS=Ø	S'PM'=Ø		S≠Ø	Camestres/	O(S,P)		
		SPM=Ø	P=S'PM		Camenes	Camestrop/	E(S,P)	
		SP'M=Ø		P≠Ø		Camenop	(discard)	O(P,S)
								No name
Total # of PCP=4,Total # of VSA in this class=12					3	4	1	4
					3 CVS	4 ei CVS	1 VSA\CVS	4 ei VSA\CVS

5 Groups of 3 pairs, each set of 3 pairs being bound on adjacent subsets.								
Group 1					CVS LC	CVS ei	VSA LC	VSA ei
A_1A_1	MP'=Ø	SP'M=Ø	M=SPM					
	MS'=Ø	S'P'M=Ø		M≠Ø		I(S,P)		
		S'PM=Ø				Darapti		
		S'P'M=Ø						
O_1A_1	MP'≠Ø	S'PM=Ø	SP'M≠Ø		O(S,P)			
	MS'=Ø	S'P'M=Ø			Bocardo			
		SP'M≠Ø						
		S'P'M≠Ø						
		(discard)						
A_1O_1	MP'=Ø	SP'M=Ø	S'PM≠Ø				O(P,S)	
	MS'≠Ø	S'P'M=Ø					No name	
		S'PM≠Ø						
		S'P'M≠Ø						
		(discard)						
Total VSA=3					1	1	1	
Group 2								
EA_1	MP=Ø	SPM=Ø	M=SP'M					
	MS'=Ø	S'PM=Ø		M≠Ø		O(S,P)		
		S'PM=Ø				Felapton/		
		S'P'M=Ø				Fesapo		
EO_1	MP≠Ø	SPM=Ø	S'P'M≠Ø				I(S',P')	
	MS=Ø	SP'M=Ø					No name	
		S'PM≠Ø						
		SPM≠Ø						
		(discard)						
IA_1	MP≠Ø	S'PM=Ø	SPM≠Ø		I(S,P)			
	MS'=Ø	S'P'M=Ø			Disamis/			
		SPM≠Ø			Dimaris			
		S'PM≠Ø						
		(discard)						
Total VSA=3					1	1	1	
Group 3								
EE	MP=Ø	SPM=Ø	M=S'P'M					
	MS=Ø	S'PM=Ø		M≠Ø				I(S',P')
		SPM=Ø						No name
		SP'M=Ø						
IE	MP≠Ø	SPM=Ø	S'PM≠Ø				O(P,S)	
	MS=Ø	SP'M=Ø					No name	
		S'PM≠Ø						
		SPM≠Ø						
		(discard)						
EI	MP=Ø	SPM=Ø	SP'M≠Ø		O(S,P)			
	MS≠Ø	S'PM=Ø			Ferio/			
		SP'M≠Ø			Festino/			
		SPM≠Ø			Ferison/			
		(discard)			Fresison			
Total VSA=3					1		1	1

Group 4 (Row M')					CVS LC	CVS ei	VSA LC	VSA ei
A_2A_2	M'P=Ø	SPM'=Ø	M'=S'P'M'					
	M'S=Ø	S'PM'=Ø		M'≠Ø				I(S',P')
		SPM'=Ø						No name
		SP'M'=Ø						
O_2A_2	M'P≠Ø	SPM'=Ø	S'PM'≠Ø				O(P,S)	
	M'S=Ø	S'PM'=Ø					No name	
		S'PM'≠Ø						
		SPM'≠Ø						
		(discard)						
A_2O_2	M'P=Ø	SPM'=Ø	SP'M'≠Ø		O(S,P)			
	M'S≠Ø	S'PM'=Ø			Baroco			
		SP'M'≠Ø						
		SPM'≠Ø						
		(discard)						
Total VSA=3					1		1	1
Group 5								
A_1E	MP'=Ø	SP'M=Ø	M=S'PM					
	MS=Ø	S'P'M=Ø		M≠Ø				O(P,S)
		SPM=Ø						No name
		SP'M=Ø						
O_1E	MP'≠Ø	SPM=Ø	S'P'M≠Ø				I(S',P')	
	MS=Ø	SP'M=Ø					No name	
		S'P'M≠Ø						
		SP'M≠Ø						
		(discard)						
A_1I	MP=Ø	SPM=Ø	SPM≠Ø		I(S,P)			
	MS≠Ø	S'PM=Ø			Darii/			
		SP'M≠Ø			Datisi/			
		SPM≠Ø						
		(discard)						
Total VSA=3					1		1	1
Grand Total								
VSA=15					5	2	5	3
10 VSA from the class of 1 universal + 1 particular premises acting on same row					5 CVS		5 VSA\CVS	
5 ei VSA from the class of 2 universal premises acting on the same row						2 ei CVS		3 ei VSA\CVS