A cylindrical Venn diagram model for categorical syllogisms

Dan Constantin Radulescu

Abstract

One shows that a set theoretical approach to categorical syllogisms is much more natural than the "figures, moods and rules of the syllogism approach". (The latter satisfies "Aristotle's requirement that the middle term M should not appear in the conclusion. Striker, 2009.) In the "set approach" one deals with the eight 3-set intersections that span a universal set U: U=MSP+MS'P+MSP'+ MS'P'+M'SP+M'S'P+M'SP'+M'S'P', where S,P,M are the usual categorical terms (interpreted now as sets) appearing in the wording of the pairs of categorical premises (PCPs) and of the logical conclusions (LCs) which the PCPs might entail; the union of disjoints sets is denoted by a + sign; MSP:= M∩S∩P, etc.; S',P',M' are the categorical terms non-S, non-P, non-M,, now interpreted as the complementary sets in U of the S,P,M, respectively. In this model of categorical syllogisms, when using a cylindrical Venn diagram, (Marquand 1881), (Veitch 1952), (Karnaugh 1953), it is self-evident that if a PCP entails an LC at all, (and thus generates a valid categorical argument (VCA), then the LC singles out one and only one of the 8 subsets of U, and affirms about it either that it is non-empty, or, that one of the sets S,P,M,S',P',M' is empty except, possibly, the subset which the LC singled out. Thus the middle term M, or its complementary M', are very much part of the LC. (Of course, to satisfy Aristotle's requirement that M should not appear in the LC, an LC of the first type, e.g., SPM≠Ø, may be re-written, with some loss of "intuitive information", as I(S,P), and an LC of the second type, e.g., S=SPM, may be re-written as A(S,P).) The valid syllogisms (VSs) are those VCAs whose LCs can be re-written in the "(S,P)-format", i.e., one of the categorical operators A,O,E,I is applied to the ordered pair (S,P). After the "middle term elimination", the LCs of the VCA\VS set are of the I(S',P'), A(P,S), or O(P,S) type. It is easy to see that there are five classes of PCPs – two do not entail LCs, and three do, thus generating three distinct VCA classes. Inside each VCA class, via a relabeling transformation of the sets S,P,M, S',P',M', any of the VCA (or VS) can be recast (or reformulated) as any other VCA from the same class. This "naming covariance" suggests that, at least from a set theoretical point of view, (i) the (S,P)-format LC restriction is not meaningful, and (ii) one may consider that there are only three distinct VCAs (and VSs), chosen as any one representative per VCA class; for example, one may choose as representatives Darapti, Darii and Barbara – all the other VCAs (not only VSs) maybe written, using appropriately chosen terms, as either a Darapti, Darii or Barbara VS. There are always relabelings transforming any VCA from the VCA\VS set into a VS. The role of VCA relabelings is similar to the role of "reduction of syllogisms" (the latter was applied only to the VSs).

Keywords: categorical syllogisms • categorical premises • cylindrical Venn diagram • Karnaugh map

1. The Cylindrical Venn diagram (the Karnaugh map for n=3)

S'P'M	SP'M	SPM	S'PM
S'P'M'	SP'M'	SPM'	S'PM'

Fig. 1

For easier drawing, the universal set U is graphed as a rectangle – but please imagine that the left and right borders of the rectangle are glued together, so that S'PM:=S'∩P∩M and S'P'M are adjacent, and S'PM' and S'P'M' are adjacent, too – as in the usual 3-circle Venn diagram. On this "cylindrical Venn diagram" - or Karnaugh map with n=3, no inference rules and no axioms are needed to prove any of the syllogistic conclusions: it is self-evident that the 36 distinct pairs of categorical premises (PCP or just pairs) split into 5 classes - 2 classes do not entail any logical conclusions (LC), but 3 classes do, and thus generate VCAs. Since the 8 subsets of Figure 1 are the "elementary" subsets of U, one calls them just subsets; no other set will be a "subset". Note that is not necessary to replace Venn's circles (John Venn 1880) by squares (Alan Marquand 1881), to arrive at the the above conclusions via diagrams, but it is surely much easier to see the LCs entailed by the PCPs on a cylindrical Venn diagram/Karnaugh map for n=3, than on a 3-circle Venn diagram. For no good reason the Marquand/Veitch/ Karnaugh maps do not show up in logic books even if they are extensively used in digital circuits engineering books. (Veitch 1952, Karnaugh 1953; I rediscovered the cylindrical Venn diagram in 2017.)

2. Notations

As known, A(M,P) means "All M is P", i.e., the set P'M :=P' ∩M is empty. Thus A(M,P) acts on the M row, by emptying (two "horizontally adjacent" subsets) P'M = SP'M + S'P'M. Compare the above to A(P,M), which means "All P is M", i.e., the set PM'=P∩M' =Ø. Thus A(P,M) acts on the M' row, by emptying, two other horizontally adjacent subsets: PM' = SPM' + S'PM'. It follows that A(M,P) and A(P,M) empty subsets not only on different rows, but also on totally different/complementary columns.

We follow three conventions concerning the pairs of categorical {P, S} premises:

1. Always list a PCP with the P-premise first, and the S-premise second. (P won't necessarily be the "predicate of the conclusion"; it's "just a set called P".)
2. Since A(M,P) ≠ A(P,M), and O(M,P) ≠ O(P,M), the operators A and O will receive an index: 1 or 2, depending on the position of M inside the ordered pair on which they act.
3. Namely, define A_1 o{*, M}:= A(M,*), and A_2 o{*, M}:= A(*, M), where * is either S, or P. This way, when A_1, (resp. A_2), is applied to an unordered pair {*, M}, it will pick up M as the first, (resp. second), set for A to act upon. One can now use a one letter indexed categorical operators to symbolize an S or P premise: the meaning of A_1A_2 will be, (using the convention to firstly list the P-premise), A(M,P) A(S,M) – the premises of the syllogism Barbara. Same notation rule will be applied to the O operator. O(M,P) means "Some M is not P", i.e., the set P'M ≠Ø, and O(S,M) will mean "Some S is not M ", i.e., the set SM' ≠Ø. Analogously, O_1 o{*, M}:= O(M,*), and O_2 o{*, M}:= O(*, M). The E and I operators do not need indices since they are symmetric. E(S,M) means SM = Ø and I(S,M) means SM ≠Ø; they act on the M row, as A_1, O_1 do. Thus, a no index, or an index 1 operator, acts on the M row.

The only operators acting on the M' row are A_2, O_2. Their respective actions on the M' row are similar to the actions of E, resp. I, on the M row: for example, A(P,M) empties PM', E(P,M) empties PM, etc. Note that giving indices to A and O replaces the use of the 4 Figures into which the two premises' terms can be arranged. Keeping up with the "4 figures", resulted, e.g., in a quadruple naming - Ferio, Festino, Ferison, Fresison, denote one and the same VS: EI:O(S,P). (The LC is written after the PCP and the column.) Getting rid of the rest of "double naming", (Celarent/Cesare, Celaront/Cesaro, Disamis/Dimaris, etc.), reduces the number of VSs, from 24 to only 14 (with only 6 out of 14 – instead of 9 out of 24 – based on ei).

One more notation:

The "emptying operators" A_1, A_2, and E appear in universal premises (All..., No...), and the "element laying" operators O_1, O_2 , and I appear in particular (Some..., Some... not) premises. We'll order all six possible P-premises, (resp. all six possible S-premises), as vector components: $\mathbb{P}_i$ ={ A_1, E, A_2, O_1, I, O_2} o{P, M}, resp. $\mathbb{S}_i$ ={ A_1, E, A_2, O_1, I, O_2} o{S, M}. All the possible PCP are the components of the direct product of these two vectors $L_{ij} = \mathbb{P}_i \otimes \mathbb{S}_j$, i,j = 1,..,6. So, the total number of distinct pairs of premises is 36. As it will be noticed below, 17 pairs do not entail any LCs, 15 do each entail exactly one LC, and each of the 4 pairs of "2-row action" universal premises, entail 3 independent LCs each, for a total number of 19 pairs of premises that entail a total of 27 LCs. Thus, according to our definitions, starting with categorical pairs of premises in the S,P,M variables, (i.e., A,O,E,I applied to the terms/sets S,P,M), one obtains 27 VCAs, out of which, 14 - the VS - have familiar names (even more than one familiar name per each VS). Counting each set of two and the one set of four equivalent VSs as just one distinct syllogism per set, i.e., disregarding figures for equivalent, (or identical content), syllogisms, one gets just 8 VSs without ei, and 6 ei VSs.

3. Examples of VCA recasting for Darapti's class

This class contains only five VCAs, (based 4 on ei on M, and one on ei on M'), out of which two are ei VSs. It is useful to use conversion, obversion, and contraposition to uniformly re-write the A and O operators as E and I operators applied to appropriate sets: e.g., write A(M,P) as E(M',P), O(S,M) as I(M',S). One obtains:

1. A_1A_1=E(M,P')E(M,S'): A(M, SPM) → I(S,P) (M≠Ø), Darapti, [All M is P, All M is S → M=SPM]

2. EA_1=E(M,P)E(M,S'): A(M, SP'M) → O(S,P) (M≠Ø), Felapton/Fesapo, [No M is P, All M is S → M=SP'M]

3. EE=E(M,P)E(M,S): A(M, S'P'M) → I(S',P') (M≠Ø),[No M is P, No M is S → M=S'P'M]

4. A_2A_2=E(M',P)E(M',S): A(M', S'P'M') → I(S',P') (M'≠Ø), [All P is M, All S is M → M'=S'P'M']

5. A_1E=E(M,P')E(M,S): A(M, S'PM) → O(P,S) (M≠Ø), [All M is P, No M is S → M=S'PM]

The pair A_1A_1 and its ei LC, I(S,P), can be recast as any of the other four pairs and their respective ei LCs, via these relabelings of the S,P,M,S',P',M' sets:

1↔2: P'↔P; 1↔3: S'↔S, P'↔P; 1↔4: M'↔M, S'↔S, P'↔P; 1↔5: S'↔S

For example, in "set language" A_1A_1 means that M is contained in the SP intersection; A_2A_2 means that S and P are included in M, while their relative position inside M is undetermined. But this also means that S' and P' both include M', which turns A_2A_2 into an "A_1A_1 situation" - only now we have to use the S',P',M' variables in the "new" A_1A_1: All M' is P', All M' is S', with the Darapti like LC: I(S',P') if M'≠Ø, which would have been the entailed LC of A_2A_2 all along, without any recasting as A_1A_1. In "set language" EA_1 means that M and P are disjoint and M is included in S. The relative position of S and P is unknown. But the relative position of S, P' and M is perfectly known: M is contained in the SP' intersection. We have again an "A_1A_1 situation" in the S,P',M variables: All M is P', All M is S, with its modified Darapti LC, I(S,P')=O(S,P) if M≠Ø; which would have been the Felapton/Fesapo LC anyhow, without any A_1A_1 recasting of EA_1. The above shows that all five ei VCAs in this class may be considered as being equivalent - there seem to be no logical motivation for the "(S,P)-format LC restriction", neither for most "rules of valid syllogisms" - M is undistributed in both A_2A_2 premises, and EE are two negative premises that entail the I(S',P') ei LC when M≠Ø, with or without any recasting into the "VS approved" pairs A_1A_1, or EA_1. Note that set relabeling in a generic VCA is justified by the fact that, if we denoted some sets, e.g., by the S,P,M letters, we would have been at liberty to denote them by the letters S',P',M' as well. In a VCA containing concrete terms, such as the usually recognized as a Celarent/Cesare type VS "No Greeks are immortals", "All Socrates and friends are Greeks", Therefore "No Socrates and friends are immortals". Replacing immortals by P', the first premise changes to: "No Greeks are non-P"="All Greeks are P", and the LC becomes "All Socrates and friends are P". One has finally to attend to the meaning which P has to have in order that the content of the VS is left unchanged: since immortals=P', it follows that P=non-immortals=mortals., and one did re-write a Celarent/Cesare VS as a Barbara VS. Any VCA\VS having an A(P,S) or O(P,S) LC becomes a VS via a P↔S relabeling, and for any VCA\VS having an I(S',P') LC, there exists a relabeling, transforming it into a VS. Not only that, but for any VS or VCA there exists a relabeling transforming it into, e.g., a Darapti, Darii or Barbara VS. For more details about equivalences between VCAs please see Section 6 below.

4. Conclusions' shape

As one can see from the below discussion of all the possible pairs of premises, each and every one of the entailed LCs refers precisely to one subset (out of 8), and falls in one of the following two categories:

(α) one, or even two, of the sets S, P, M, S', P', M' is reduced, via two universal, (aka emptying), premises to only one of its 4 subsets

(β) one of the 8 subsets in Figure 1 is shown to be ≠ Ø (possibly via an ei added premise).

When ei is used, the LC is reached in two stages: first one of S, P, M, S' , P', or M' is reduced to just one subset out of 4 (stage **(α)**), then, the ei makes/declares that subset ≠Ø.

The above **(α)** and **(β)** express the fact that a PCP entailing an LC pinpoints to just one subset out of the 8. Note that there is a "tension" between the "one subset out of 8 conclusion" to which a PCP pinpoints, and the "Aristotle's requirement" that the conclusion of a valid syllogism, LC, should not contain the middle term. The latter condition means that the LC refers to a column containing two subsets – one included in M, the other in M'. The difference in information between a PCP that pinpoints to just one subset out of 8 and the standard expression for an LC which refers to a column and thus pinpoints to two subsets out of 8, is a consequence of the requirement that the middle term should not appear in the conclusion. One can say that the LC summarizes the "new knowledge" obtained from the pair of premises, and that to list the LC together with all the other information the premises provide would necessarily mean to relist one or both premises together with the LC - and this is exactly what we do not want to do, as per "Aristotle's requirement" (Striker 2009: 20): "A syllogism is an argument in which, certain things being posited, something **other than what was laid down** results by necessity because these things are so." One way to keep all the information a PCP provides, without completely relisting the premises would be to spell out the LC together with the subset the LC is "bound" to. For example, since a VS requires an "(S,P) conclusion", i.e., that, in the conclusion, one of the operators A,O,E,I be applied to the ordered pair (S, P), all VS conclusions are in fact necessarily bound to SPM, or SP'M, or SP'M'! (One can check below, Section 5, that there is no PCP pinpointing to the SPM' subset.) Any VCA, bound to any other subset, has no name. (But, for example, LCs "bound to SPM" are A(S,P) (Barbara), I(S,P) (Barbari, Bramantip, Darapti, Darii/Datisi, Disamis/Dimaris), A(P,S). The last one, originates from the VCA A_2A_1: P=SPM, S'= S'P'M'. Then one gets A(P, SPM), and thus A(P,S), which has no name, even if the conclusion is bound to SPM, because A_2A_1 empty the set P except for SPM, and this does not fit the VS requirement for an "(S,P) conclusion". But the ei, (P≠Ø), conclusion, I(S,P), gives the VS Bramantip.

When one premise is universal and the other one is particular, then the LC, entailed if and only if both premises act on the same set – either M or M' (but not both) , is reached in one stage: one of the 8 subsets in Figure 1, uniquely determined, turns out to be ≠Ø. (The particular premise will have available only one subset, not two, to lay an element on, since the other horizontally adjacent subset was "just" emptied by the universal premise: only this arrangement can make both premises TRUE **and** the syllogistic argument valid. See Fact #1 below.) A standard LC will still refer to an entire column and not just one subset.

Note that any subset relabeling, such as, for example, P'↔M, S↔S', does not change the immediate neighbours of any of the subsets, and does not change the conclusions of any of the premises' pairs: the conclusion of "All P is M, All M is S" =A_2A_1, on a new, "relabeled Figure 1", will still be P=SPM, S'= S'P'M'.

Fact #1 For any pair of premises, {P-premise, S-premise}, both acting on the same row, there will always be one and only one subset "acted upon twice"; for any pair {P-premise, S-premise}, acting on two rows, there will always be one and only one column whose two subsets are both acted upon.

Proof: Cf. Fig. 1, two of the sets S, P, S', P', unless they are complementary sets, always have one and only one common column. Consider first the "M-row operators" A_1, O_1, E, I. In a P-premise, the operators A_1, O_1 act on the two P' columns and the E,I operators act on the two P columns. In an S-premise, the operators A_1, O_1 act on the two S' columns and the E,I operators act on the two S columns. Thus a pair (P-premise, S-premise), both acting on the M row, may act either on {P', S'}, or on {P', S}, or on {P, S'}, or on {P, S}, in which cases, respectively, either the subset S'P'M, or SP'M, or S'PM, or SPM is acted upon twice, and, respectively, either the subset SPM, or S'PM, or SP'M, or S'P'M is **not** acted upon at all. Thus two universal premises acting on the same row will empty 3 subsets, (of M or M'), and one universal and one particular premise acting on the same row will always place a set element on precisely one subset.

Since the A_2, O_2 operators - which act on the M' row - behave similarly to the E,I operators which act on M row - i.e., in a P-premise, the operators A_2, O_2 act on the two P columns, (exactly as E,I do on the M row), and in an S-premise, the operators A_2, O_2 act on the two S columns, (exactly as E,I do on the M row), it follows, as above, that a "2-row acting" pair of premises will always "act upon a column twice" either emptying both column's subsets, (and this is the only interesting case!), or possibly laying set elements in both column's subsets, or emptying one of the column's subset and laying a set element on the other column's subset – all these latter variants correspond to pairs of premises that do not entail any LC. (See below the paragraphs (i) and (ii2).) The four 2-row acting pairs of universal premises will thus empty one column, plus two other subsets, located on two different rows, on each side of that emptied column.(See the paragraph (ii1) below.) QED. (An examination of the 36 cases below makes the proof of Fact #1 clear, too.)

5. A more detailed discussion of the matrix L_{ij} , i,j = 1,..,6

The matrix $L_{ij} = \mathbb{P}_i \otimes \mathbb{S}_j$, i,j = 1,6 naturally splits into four 3 by 3 sub matrices: $L^{(1)}:=L_{ij}$, i,j = 1,2,3, contains only, (and they are the only ones), pairs of two universal premises; $L^{(2)}:=L_{ij}$, i=4,5,6, j=1,2,3, contains pairs of one particular P-premise, [gotten from replacing in $L^{(1)}$ the universal P-premise with the corresponding, (and contradictory), particular P-premise], and one universal S-premise (left unchanged from $L^{(1)}$); $L^{(3)}:=L_{ij}$, i=1,2,3, j=4,5,6, contains pairs of one universal P-premise, (unmodified from $L^{(1)}$), and one particular S-premise, [gotten from replacing in $L^{(1)}$ the universal S-premise with the corresponding, (and contradictory), particular S-premise]; and the sub-matrix $L^{(4)}:=L_{ij}$, i,j = 4,5,6 which contains only, (and they are the only ones), pairs of two particular premises.

(i) $\mathbf{L^{(4)}}$**:** The pairs of premises in the sub-matrix $L^{(4)}:=L_{ij}$, i,j = 4,5,6, do not entail any LC. The two particular premises will "lay set elements" either on three subsets of the same row (M or *M'*), or on 4 subsets on different rows. Since, any conclusion of such a pair would just relist one or two of its premises, there is no way to satisfy Aristotle's requirement, (Striker 2009: 20), that "A syllogism is an argument in which, certain things being posited, something **other than what was laid down** results by necessity because these things are so." Thus, per Aristotle's insight, these pairs will not generate any VCA; this means nine pairs of premises on the no conclusion/discarded list.

(ii) $\mathbf{L^{(1)}}$**:** contains two sorts of universal premises pairs:

(ii0) The 5 "1-row acting" pairs of universal premises. Four pairs act on the M row only, $L_{11}=A_1A_1$, $L_{12}=A_1E$, $L_{21}=EA_1$, $L_{22}=EE$, and, one pair acts on the M' row only, $L_{33}=A_2A_2$. As the Fact #1 has shown, the M subsets SPM, or S'PM, or SP'M, or S'P'M are **not** emptied by $L_{11}=A_1A_1$, $L_{12}=A_1E$, $L_{21}=EA_1$, $L_{22}=EE$, respectively, and the S'P'M' subset of M' is **not** emptied by $L_{33}=A_2A_2$. Again, as per Aristotle's insight, only existential imports on M, resp., M', will count and produce 5 VCAs, each respectively "bound" on one of the above **not** emptied subsets. (2 out of 5 are the VS Darapti and Felapton/Fesapo, bound on SPM and SP'M, respectively.) Thus the 5 "1-row acting" pairs of universal premises each produces one ei conclusion or ei VCA, since we get one conclusion if ei is used each time one of the sets M, or M', is reduced, via two "1-row acting" universal premises, to only one of its 4 subsets.

(ii1) The 4 "2-row acting" pairs of universal premises. They have to contain A_2 as a premise - since this is the only universal operator acting on the 2nd row M'. These 4 pairs are: $L_{13}=A_1A_2$, $L_{23}=EA_2$, $L_{31}=A_2A_1$, $L_{32}=A_2E$. They empty four subsets on two different rows and three different columns, located, cf. Fact #1, as follows: two empty subsets are on the same column, and the other two empty subsets are on different rows and on different sides of the empty column. These pairs are responsible for 12 different conclusions:

1. The pair of premises $L_{13} = A_1A_2 = A(M,P)\ A(S,M) = E(M,P')E(M',S)$ empties the column SP' and the subsets S'P'M and SPM', and, out of the 3 columns SP', S'P' and SP, occupied by the sets S and P', (whose intersection is SP'), only the subsets SPM out of S, and S'P'M' out of P' "survive". LCs are therefore aplenty: A(S, SPM), A(P', S'P'M'), E(S,P'), from which it follows A(S, P), A(P', S'), E(S, P'), A(S, M), A(P', M'). But the last two conclusions are exactly the premises – so they do not count, (as new knowledge), and the first three, via set theory, (or contraposition and obversion), are equivalent: A(S, P)=A(P',S')= E(S,P'). We'll keep just A(S,P) as the only one universal conclusion, out of the three independent LCs entailed by the "Barbara pair of premises" $L_{13} = A_1A_2$. The other two independent LCs involve ei: on S, i.e., supposing S≠Ø, one gets I(S,P), Barbari, and, via ei on P', one gets the no name I(P', S'), for a total of three independent conclusions entailed by the pair $L_{13} = A_1A_2 = A(M,P)A(S, M)$. Any other conclusions, such as I(S,M) or I(P,M) are not independent: they follow directly from the premises and S≠Ø. Moreover, P'= S'P'M' follows from S=SPM: if we list, (now, for simplicity, on one row), from left to right, the adjacent/neighbouring subsets that were not emptied by Barbara's premises, they are SPM, S'PM, S'PM', S'P'M'. This reads, from left to right, (resp. from right to left), precisely as $S \subseteq M \subseteq P$, and, resp., $P' \subseteq M' \subseteq S'$ – which is also how the transitivity of the inclusions A(S, M), A(M,P), or the Euler diagrams, would have represented Barbara's premises.

2. Analogously, the premises $A_2A_1 = A(P,M)\ A(M,S) = E(M',P)E(M,S')$, empty 4 subsets out of 6 from the columns S'P, S'P' and SP, occupied by the sets S' and P, (whose intersection is S'P). Only the subsets SPM out of P and S'P'M' out of S' will again "survive". Thus, same "survivors" but now as parts of other "big sets" S', P instead of S,P'. The independent conclusions are the no name A(P,S), and, via ei on P, I(S,P) - Bramantip. Via ei on S', one gets (again) a no name I(P', S'). One can also see, that via a simple relabeling transformation, M→M, S→P, P→S, which may be written simply as S↔P, A_2A_1 becomes A_1A_2: $A_2A_1 = A(P,M)\ A(M,S) \to A(S,M)A(M,P) = E(M,P')E(M',S)$. One can also see, that via another relabeling transformation, M→M', S→S', P→P', A_2A_1 also becomes A_1A_2: $A_2A_1 = A(P,M)\ A(M,S) \to A(P', M')\ A(M',S') = E(M,P')E(M',S)$. Both relabeling transformations also map the conclusions of A_2A_1 onto the conclusions of A_1A_2. [See next section for all the relabeling transformations between the VCAs generated by the four "2-row acting" pairs of universal premises.]

3. The $EA_2 = E(M,P)\ E(M',S)$ and $A_2E = E(M',P)\ E(M,S)$ are even more similar than A_1A_2 and A_2A_1 are. Each of EA_2 and A_2E, empty 4 subsets out of the 6 subsets of the same 3 columns SP', SP' and SP. The two subsets that survive are: SP'M and S'PM' if the premises are EA_2, and SP'M' and S'PM if the premises are A_2E. The type (α), two entailed LCs per pair of premises, are thus, for EA_2: A(S, SP'M), A(P, S'PM'). One chooses, as independent conclusions E(S,P)(=A(S,P')=A(P, S')), (Celarent/Cesare), and, via ei on P the no name O(P,S), plus, via ei on S, O(S,P), (Celaront/Cesaro).

4. Initial conclusions for A_2E are: A(S, SP'M'), A(P, S'PM). One chooses, as independent conclusion E(S,P)(=A(S,P')=A(P, S')), (Camestres/Camenes). And, via ei on P, the no name O(P,S), plus, via ei on S, O(S,P), (Camestrop/Camenop). This way, we get again to three independent conclusions when ei is used each time one of the sets S, P, S' , P' is reduced, via two "2-row acting" universal premises, to only one of its 4 subsets.

(iii). **$L^{(2)}$ and $L^{(3)}$**. Firstly, observe that the "2-row acting", 1-particular, 1-universal pairs of premises from $L^{(2)}$: $L_{43} = O_1A_2$, $L_{53} = IA_2$, $L_{61} = O_2A_1$, $L_{62} = O_2E$, and from $L^{(3)}$: $L_{16} = A_1O_2$, $L_{26} = EO_2$, $L_{34} = A_2O_1$, $L_{35} = A_2I$, do not entail any LCs. These 8 pairs are gotten from the four (ii1) pairs, by substituting a particular premise for an universal premise. But by doing this, the emptying, and the element laying, happen now on two different rows. Any LC would just relist the premises. Thus, as per Aristotle's insight, the 8 pairs of 1-particular, 1-universal premises, acting on 2 rows, M **and** M', span the 2nd class of pairs that do not entail any LC. This adds up to a total of 9+8=17 of such pairs. Out of the other 36-17=19 pairs, we already saw 4 pairs of premises, (ii1), that entail 3 independent conclusions per pair, and 5 pairs of premises, (ii0), which entail one conclusion per pair. The rest of 10 pairs from $L^{(2)}$ and $L^{(3)}$, originate from the 5 "1-row acting" pairs of universal premises in $L^{(1)}$, by replacing one universal premise with its contradictory particular premise, and thus, cf. Fact #1, each such pair results in one precise subset being ≠Ø, and entails exactly one LC per pair, for a total of 27 VCAs, 14 out of which - the VSs, have names [even multiple names for one and the same syllogism, (or pair of premises), when the premises' terms can be switched around without changing the premises' meaning]. More precisely, the five $L^{(2)}$ pairs, (which were obtained from $L^{(1)}$'s five “1-row acting” universal pairs, by changing an universal P-premise into its contradictory, particular P-premise): L_{41} $=O_1A_1$, $L_{42}=O_1E$, $L_{51}=IA_1$, $L_{52}=IE$, $L_{63}=O_2A_2$, lead to, in order, the following (β) type, conclusions: SP'M≠Ø (or O(S,P), Bocardo), S'P'M ≠Ø (or I(S',P'), no name), SPM≠Ø (or I(S,P), Disamis/Dimaris), S'PM≠Ø (or O(P,S) no name), S'PM'≠Ø (or O(P,S) no name). For the last 5 out of 10, one substitutes the contradictory particular S-premise for the universal S-premise of the $L^{(1)}$'s five “1-row acting” universal pairs, to obtain: $L_{14} = A_1O_1$, $L_{24}=EO_1$, $L_{15}=A_1I$, $L_{25}=EI$, $L_{36}=A_2O_2$. The conclusions of these pairs are, in order: S'PM≠Ø (or O(P,S)), S'P'M ≠Ø (or I(S',P')), SPM≠Ø (or I(S,P), Darii/Datisi), SP'M≠Ø (or O(S,P), Ferio/Festino/Ferison/Fresison), SP'M'≠Ø (or O(S,P), Baroco). One can notice that A_1O_1 and IE have the same conclusion S'PM≠Ø, O_1A_1 and EI have the same conclusion SP'M≠Ø, IA_1 and A_1I have the same conclusion SPM≠Ø, O_1E and EO_1 have the same conclusion S'P'M ≠Ø (since on the M row there are only 4 subsets and one has 8 pairs of premises which place/lay at least one set element in exactly one subset of M).

6. Classes of equivalent syllogistic arguments

Note that the set relabelings from Section 4 and the present one are somewhat conceptually similar to the classical “doctrine of reduction of syllogisms”, (Coffey 1938: 335-355). The latter shows that syllogisms from Figures 2, 3 and 4 are valid if those from Figure 1 are valid, then shows that if Barbara is valid then the other syllogisms from Figure 1 are valid, too. In the “set approach” one already knows, (from the cylindrical Venn diagram), that all the VCAs are valid – the relabelings just show that all VCAs from the same class are equivalent.

The premises' action is easier to follow if we uniformly express any premise as either an E or I operator, acting firstly on M, or M', as the case may be. Consider for example the pairs: A_1A_1, O_1A_1, A_1O_1. Write:

A_1A_1=E(M,P')E(M,S')

O_1A_1=I(M,P')E(M,S')

A_1O_1=E(M,P')I(M,S'). All three pairs use the same variables M,P',S'. This is because, as was observed in Fact #1's proof, A_1A_1 acts twice on S'P'M, not at all on SPM, (we'll say that Darapti is bound not on the subset on which the premises' pair acts twice, but on SPM on which it doesn't act at all, and thus allows the conclusion M= SPM, out of which, via ei, the Darapti's conclusion follows. Equally important is that A_1A_1 acts once on SP'M, and once on S'PM, the subsets next to S'P'M on the “cylindrical Venn diagram”, and these are exactly the subsets assured to be ≠Ø by O_1A_1, (Bocardo), and A_1O_1, respectively.

Let's now consider another similar group of 3 pairs of premises:

EE=E(M,P)E(M,S)

IE=I(M,P)E(M,S)

EI=E(M,P)I(M,S). All three pairs use the same variables M,P,S. This is because, as was observed in Fact #1's proof, EE acts twice on SPM, not at all on S'P'M, (we'll say that the no name EE:M=S'P'M is bound not on the subset on which the premises' pair acts twice, but on S'P'M on which the pair doesn't act at all, and thus allows the conclusion M= S'P'M, out of which, via ei, the no name I(S',P') conclusion follows. Equally important is that EE acts once on SP'M, and once on S'PM, and these are exactly the subsets assured to be ≠Ø by EI, (Ferio/Festino/Ferison/Fresison), and IE, respectively.

Fact #2: if we relabel P'→P, S'→S, then the first group of 3 pairs of premises is transformed in the 2nd group of 3 pairs of premises, and, the 3 conclusions from the 1st group of pairs, via this relabeling, become the 3 conclusions of the 2nd group of pairs. This happens because the subsets on which A_1A_1 acted twice, resp. not at all, are mapped into subsets on which EE acts twice, resp. not at all. The same is true about the subsets on which A_1A_1 acted once – they are transformed into subsets on which EE acts once. This way not only pairs of premises are mapped onto pairs of premises, but their conclusions are mapped into respective conclusions, too. There are 5 different groups of 3 pairs of premises each, and 4 relabeling transformations that map the first set of 3 pairs of premises to the other 4 and back to the 1st groups of 3 pairs of premises. One can argue that only one set of 3 pairs of premises is independent and the rest represent just what one would have gotten by a relabeling of the variables S,P,M,S',P',M'. The final conclusion is that the 5 pairs of two universal premises acting on the same row, A_1A_1, EE, A_1E, EA_1, A_2A_2 are equivalent, and all the other 10 pairs of premises, one universal and one particular, are equivalent, too. This is so because the two strains of 5 VCAs each, which start with O_1A_1 and A_1O_1, and continue with IE and resp. EI, etc. are in fact equivalent, too: one can see this, for the above mentioned pairs, via a relabeling S↔P. Thus we have 10 pairs that generate equivalent VCAs: O_1A_1, IE, O_1E, IA_1, O_2A_2, A_1O_1, EI, A_1I, EO_1, A_2O_2. The set of 4 "2-row acting" pairs of universal premises can be transformed, by relabeling, among themselves, too. Thus we found 3 different types of pairs of premises, easily characterized as being: 4 pairs of 2 universal premises acting on **two** rows, M **and** M', 5 pairs of 2 universal premises acting on **one** row, M **or** M', 10 pairs of one universal and one particular premises, acting on **one** row, M **or** M'. Thus one has 3 types of PCPs which generate VCAs.

Below one lists the VCAs from two classes out of three, grouped by the subset they do not act upon, and to which we say that they are "bound" to. One VCA class contains 5 ei VCAs and the other one contains 10 VCAs, for a total of 15 VCAs split in five groups of three VCAs each - according to the subset they are bound to. These five VCA groups use, (or act upon), the complementary variables to the variables characterizing the subset these VCAs are bound to.

1. VCAs bound to the subset SPM:

A_1A_1=E(M,P')E(M,S') M=SPM. If M≠Ø: I(S,P), Darapti

O_1A_1=I(M,P')E(M,S') SP'M ≠Ø or O(S,P), Bocardo

A_1O_1=E(M,P')I(M,S') S'PM≠Ø or O(P,S), No name

2. VCAs bound to the subset SP'M:

EA_1=E(M,P)E(M,S') M=SP'M. If M≠Ø: O(S,P), Felapton/Fesapo

EO_1=E(M,P)I(M,S') S'P'M≠Ø or I(S',P'), No name

IA_1=I(M,P)E(M,S') SPM≠Ø or I(S,P), Disamis/Dimaris

3. VCAs bound to the subset S'P'M:

EE=E(M,P)E(M,S) M=S'P'M. If M≠Ø: I(S',P'), No name

IE=I(M,P)E(M,S) S'PM≠Ø or O(P,S), No name

EI=E(M,P)I(M,S) SP'M≠Ø or O(S,P), Ferio/Festino/Ferison/Fresison

4. (M' row) VCAs bound to the subset S'P'M':

A_2A_2=E(M',P)E(M',S) M'=S'P'M'. If M'≠Ø: I(S',P'), No name

O_2A_2=I(M',P)E(M',S) S'PM'≠Ø or O(P,S), No name

A_2O_2=E(M',P)I(M',S) SP'M'≠Ø or O(S,P), Baroco

5. VCAs bound to the subset S'PM:

A_1E=E(M,P')E(M,S) M=S'PM. If M≠Ø: O(P,S), No name

O_1E=I(M,P')E(M,S) S'P'M≠Ø or I(S',P'), No name

A_1I=E(M,P')I(M,S) SPM≠Ø or I(S,P), Darii/Datisi

One sees that the 5 groups of 3 VCAs each, [which include 7 distinct VSs, (two of them based on ei on M)], are equivalent modulo a relabeling of S,P,M,S',P',M'.

One may verify the transitivity of the equivalences using the following relabeling maps:

1↔2: P'↔P

1↔3: S'↔S, P'↔P

1↔4: M↔M', S'↔S, P'↔P

1↔5: S'↔*S*

2↔3: S↔S'

2↔4: M↔M', S'↔S

2↔5: P'↔P, S↔S'

3↔4: M↔M'

3↔5: P↔P'

4↔5: M↔M', P'↔P

Because there are only 4 subsets per each row, (M or M'), when, by relabeling, one maps one "binding subset" into another "binding subset", one also map subsets on which the group of VCAs, bound to the 1st subset, do not act, act once, or act twice, into subsets on which the 2nd group of VCAs, bound to the 2nd subset, do not act, act once, or act twice, respectively. This ensures that not only the pairs of premises of the 1st group of VCAs transform into the pairs of premises of the 2nd group of VCAs, but the conclusions from the 1st group of VCAs, transform into the conclusions of the 2nd group of VCAs.

Another way to show that the 5 groups of 3 VCAs each are equivalent, is to start with 3 pairs of premises written in the variables A,B,C instead of the usual S,P,M:

Group 0. All B is A, All B is C

Some B is not A, All B is C

All B is A, Some B is not C

Choosing B=M, A=P, C=S we get the pairs of premises of the 1st group of VCAs.

Choosing B=M, A=P, C=S' we get the pairs of premises of the 2nd group of VCAs.

Choosing B=M, A=P', C=S' we get the pairs of premises of the 3rd group of VCAs.

Choosing B=M', A=P', C=S', we get the pairs of premises of the 4th group of VCAs.

Finally, choosing B=M, A=P', C=S we get the pairs of premises of the 5th group of VCAs.

It is as if we represented Group 0, in 5 different system of coordinates: the number of distinct premise pairs, and VCAs, is at most 3 not 15. We can further notice that the 5 VCAs generated by "Some B is not A, All B is C", are equivalent to the 5 VCAs generated by "All B is A, Some B is not C", via the relabeling A↔C. This way one can see that the same generic wording of the premises can be represented in different ways, leading to different VCAs, with different LCs, but in fact the 5 groups are equivalent: the five VCAs generated by the pairs of premises A_1A_1, EE, A_1E, EA_1, A_2A_2 are equivalent, and the ten VCAs generated by the pairs of premises O_1A_1, IE, O_1E, IA_1, O_2A_2, A_1O_1, EI, A_1I, EO_1, A_2O_2 are equivalent, too.

The above equivalences show again that the VSs are just VCAs whose LCs happen be in the "(S,P) format".

Note that M is not distributed in the VCA A_2A_2: M'=S'P'M'→I(S',P'), (via ei on M'), and that A_2A_2 turns out to be equivalent to A_1A_1: M=SPM→I(S,P), (via ei on M, Darapti). Also, there are pairs of two negative premises in three of the VCAs - EE, O_1E, EO_1: EE generates a VCA equivalent to Darapti, (or Felapton/Fesapo), and O_1E,

EO_1 generate VCAs equivalent to Darii. Thus there are pairs of premises that entail an LC but do not satisfy the usual "valid syllogisms rules", "the middle term has to be distributed in at least one premise", and, "no valid syllogism has 2 negative premises". One can start with the premises of Darapti and Darii, (i.e., A_1A_1, and resp., A_1I), re-write them using obversion and contraposition as the premises A_2A_2, (resp. O_1E), written in other variables, get the conclusions of A_2A_2, (resp. O_1E), in those variables, then realize that those conclusions can be re-written, (via appropriate "back relabelings"), as the usual Darapti, M=SPM, and Darii, SPM≠Ø, conclusions. This way one can use VCAs which do not satisfy the usual "rules of valid syllogisms" to "bear the burden" of inferring all the conclusions of the VSs from the two VCA classes which contain Darapti and resp. Darii.

The "2-row acting" VCAs:

EA_2=E(M,P)E(M',S) S=SP'M, P=S'PM' (SP'M, S'PM'="survive" as the only subsets of S, resp. P, which are not emptied by the premises EA_2.) Thus: A(S,SP'M), A(P,S'PM'). One chooses, as independent conclusions E(S,P)(=A(S,P')=A(P, S')), (Celarent/Cesare), and, via ei on P the no name O(P,S), and, via ei on S, O(S,P), (Celaront/Cesaro).

A_1A_2=E(M,P')E(M',S) S=SPM, P'=S'P'M', A(S,P) Barbara, I(S,P) Barbari (S≠Ø), I(S',P') no name (P'≠Ø)

A_2A_1=E(M',P)E(M,S') P=SPM, S'=S'P'M', A(P,S) no name, I(S,P) Bramantip (P≠Ø), I(S',P') no name (S'≠Ø)

A_2E=E(M',P)E(M,S) S=SP'M', P= S'PM. Thus: A(S,SP'M'), A(P,S'PM). One chooses, as independent conclusion E(S,P)(=A(S,P')=A(P, S')), (Camestres/Camenes). And, via ei on P, the no name O(P,S), plus, via ei on S, O(S,P), (Camestrop/Camenop)

The S,P,M,S',P',M' relabeling transformations showing that A_1A_2, A_2A_1, A_2E, EA_2 are equivalent, since not only the premises transform into one another, but their respective conclusions, too:

A_1A_2↔A_2A_1: M↔M', S↔S', P↔P' or, S↔P

A_2E↔EA_2: M↔M' or, S↔P

A_1A_2↔EA_2: P↔P'

A_1A_2↔A_2E: M↔M', P↔P'

A_2A_1↔A_2E: S↔S'

A_2A_1↔EA_2: M↔M' , S↔S'

Note that the eight relabelings are transitive but do not form a group acting on A_2E, EA_2, A_1A_2, A_2A_1. Nevertheless one can see that each of the four PCP, and their respective LCs, can be recast as any other of the four PCP, and their respective LCs, via some of the above eight relabelings.

Or, one can start with the "generic" pair of premises All B is A, All C is B.

Then, making the obvious choice B=M, A=P, C=S, we get A_1A_2, Barbara's premises.

But choosing B=M, A=P', C=S, we get the EA_2 premises.

And choosing B=M', A=P', C=S, we get the A_2E premises.

Finally choosing B=M, A=S, C=P, we get the A_2A_1 premises.

Thus, no matter what their initial wording is, for any pair of concrete categorical premises presented to us, one can label their 3 terms in such a way, that if the pair entails an LC, then it can be expressed as either A_1A_2, or A_1A_1, or A_1I, (or any other preferred triplet of representatives from each one of the 3 classes of premises that entail LCs). After the LC of A_1A_2, or A_1A_1, or A_1I, is written down, one can do a "back relabeling" to re-express the conclusion via the most intuitive term labeling suggested by the initial premises.

7. Conclusions

Instead of the old accounting rules and restrictions imposed on the classically valid syllogisms – an (S,P)-format LC, the "syllogistic figures", "In any valid syllogism the middle term is distributed at least once", "No valid syllogism has two negative premises", etc., the **Venn diagram**, (cylindrical or not, but on the usual "3 intersecting circles" Venn diagram, the above facts are difficult to see), **approach**, allows for simpler rules:

1. The 36 PCP fall into 5 classes: three classes entail an LC and two do not.
2. Each LC is either of type **(α)** or of type **(β)** above, and refers to just one subset, out of the 8 subsets of U.
3. Inside each of the 3 classes of PCP entailing an LC, the VCAs (and VSs) are all equivalent in the sense that any VCA from the class may be re-written as any other VCA in the same class via an appropriate term (aka set) relabeling. This also means that in essence there are only 3 independent VCAs (or VSs).
4. One may offer two, or even five, "new rules of valid syllogisms". Two negative rules: 1. No two particular premises are allowed (this coincides with one of the old rules). 2. A universal premise and a particular premise, one acting on the middle term M and the other acting on its complementary set M' are not allowed. (Note that the "old rules of valid syllogisms" were in fact meant to invalidate all but the VS.) Three positive rules - the rest of the pairs of premises are allowed since they each entail at least one LC: two universal premises acting on the "same row" (either M or M'); two universal premises acting on "two rows" (both M and M'); a universal premise and a particular premise acting on the same row (either M or M').
5. As described in Section 4, the logical consequences of the 19 out of 36 possible pairs of premises are as follows: the "(S,P) conclusions" A(S,P), E(S,P), I(S,P), O(S,P) – which are satisfied only by the VS; A(P,S) entailed only by A_2A_1; I(S',P') and O(P,S). The latter conclusions are entailed by pairs of premises which, via ei or not, generate VCAs which are not VSs (VCA\VS). If one could logically argue that these I(S',P'), O(P,S), A(P,S) conclusions are not to be admitted, even if logically entailed by the VCA\VS pairs of premises, then, indeed, only the VSs are valid. As most of the logic textbooks do, one can restrict the valid syllogisms, by definition, to only the pairs of premises whose entailed LCs are of the "(S,P)-format"; or one can use notions like distribution to help eliminate any pair of premises which does not generate a VS. I do not see a logical motivation for the "(S,P)-format" LC restriction, neither for the distribution notion. For any VCA from the VCA\VS set, it is easy to find a set relabeling changing it into a VS instead.

Because of its lack of symmetry, the usual "3 intersecting circles Venn diagram" model, was used only to "verify" particular syllogisms' validity, but, as far as I know, not to find all the possible logical conclusions from all the categorical pairs of premises. (By inflating the number of cases to consider, the syllogistic figures were a detractor of such an endeavour, too.) See, e.g., Barker (2003). See also, Quine (1982), who proposed as "an hour's pastime" exercise, the Venn diagram checking of all premises' pairs for conclusion entailment.

After this paper was initially written, one had to add the "finding" that the "cylindrical Venn diagram" is in fact a Karnaugh(-Veitch) map for 3 sets. The "cylinder idea" is used to match "close enough" the adjacency displayed by the 8 subsets on the "3-circle Venn diagram". For the same adjacency reason a Karnaugh map for 4 sets is represented as a 4 by 4 square with "glued edges" - which thus becomes a torus. (See Marquand (1881), Veitch (1952), Karnaugh (1953), (Wikipedia.org/wiki/Karnaugh_map.) ["Close enough", means, e.g., that after Barbara's premises empty 4 subsets out of 8, the other 4 subsets left would be disconnected on a rectangular diagram, but are still connected on the cylindrical Venn diagram and moreover satisfy $S \subseteq M \subseteq P$.]

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