An intuitive, visual presentation of categorical syllogisms

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Abstract
One proposes a very simple scheme for finding logical conclusions (LCs) from any pair of categorical premises (PCP), by using the fact that any universal, (resp. particular), premise empties, (resp. lays set elements into), two subsets out of the eight subset partition of the universal set U which models the categorical statements. This shows, (without using syllogistic moods and figures, syllogistic axioms and inference rules, nor valid syllogism rules), that any LC refers to just one of the 8 subsets of U. Via set relabeling, (instead of syllogism reduction), any PCP entailing an LC, may be recast as, e.g., one of the Barbara, Darii or Darapti valid syllogisms (VSs).

1. CYLINDRICAL VENN DIAGRAM (KARNAUGH MAP, N=3)

S'P'M	SP'M	SPM	S'PM
S'P'M'	SP'M'	SPM'	S'PM'

Fig. 1

The universal set U is graphed as a rectangle – but the left and right borders of the rectangle are glued together to generate a cylinder, so that S'PM and S'P'M are adjacent, and S'PM' and S'P'M' are adjacent, too – as in the usual 3-circle Venn diagram. (S,P,M are sets, and S',P',M' are their complementary sets in the universal set U, whose partition may be written as U=MSP+MS'P+MSP' +MS'P'+M'SP+M'S'P +M'SP'+ M'S'P', (where the union of disjoints sets is denoted by a plus sign and MSP:= M∩S∩P, etc.). Since the 8 subsets of Figure 1 are the "elementary" subsets of U, one calls them just subsets; no other set will be a "subset". On this "cylindrical Venn diagram", (or Karnaugh map with n=3), all the syllogistic conclusions are graphically obvious, much more so than on a 3-circle Venn diagram. After I thought, in 2017, that I "invented" the cylindrical Venn diagram - I found out that Alan Marquand really invented it in 1881, (a year after John Venn proposed his 3-circle Venn diagram), and then, in 1952 Edward Veitch, and, in 1953 Maurice Karnaugh, used Karnaugh (-Veitch) maps for n=3, n=4, etc., to find the optimal design for digital circuits. These cylindrical or toroidal maps make a standard appearance in engineering books, but, apparently, never made it into, (may I say any?), logic textbooks – which are still using the 3-circle Venn diagrams.

2. SETS AND CATEGORICAL PREMISES

By definition a categorical syllogism is made of a PCP to which one tacks a 3rd statement, an (S,P)-conclusion, i.e., one of the categorical operators, (or quantifiers), A,O,E,I applied to the ordered pair (S,P). If the conclusion is truly entailed by the PCP one has a VS, otherwise the syllogism is invalid. There are 36 distinct PCPs expressed via only the S,P,M terms and the categorical operators A,O,E,I applied to pairs of these 3 terms. Modeling such premises on a cylindrical Venn diagram or Karnaugh map with n=3, where the middle term M occupies one row and M', its complementary set, occupies a 2nd row, one sees that the PCPs do not act symmetrically on the M and M' rows: four P-premises (resp. S-premises) out of six act on M and only two act on M'. In the LC finding recipes below all PCPs are expressed using only the E and I operators. It Thus it is convenient to express, from the beginning, the six P-premises using only the E and I operators; one "lumps" them into one square of opposition (acting on M), (I)={E(M, P*), I(M, P*)}, P*∈{P, P'}, plus (II)={E(M', P), I(M', P)} - acting on M'. Similarly the six S-premises are "lumped" into another square of opposition, (III)={E(M, S*), I(M, S*)}, S*∈{S, S'}, and (IV)={E(M', S), I(M', S)}. A universal premise, e.g., E(M, P*) means "No M is P*" or MP*:=M∩P*=Ø and its contradictory, I(M, P*) means "Some M is P*" or MP*≠Ø, P*∈{P, P'}. The 36 distinct PCPs are naturally partitioned into 5 subsets: two subsets whose PCPs do not entail any LC, and three subsets whose PCPs do each entail at least one LC and thus generate valid categorical arguments (VCAs), accordingly also split into three VCA classes, each class containing some of the VS. (Any VS is a VCA with a special LC: E(S,P*) or I(S,P*) with P*∈{P, P'}. The VCA\VS set has LCs of one of the formats: I(S',P'), E(S',P)=A(P,S) or I(S',P)=O(P,S). From a set theoretical point of view the difference is not significant since any VCA\VS may be recast as a VS via a set relabeling; Radulescu (2017).)

The five PCP subsets are characterized as: **#1PCPs).** Two universal premises both acting on either M or M'. There are 5 such PCPs, (gotten by combining the universal premises in (I) and (III) and adding the universal PCP extracted from (II) and (IV)), with each LC being that either M or M' is reduced to just one subset out of its 4 subsets. Then one needs existential import (ei) on either M or M' in order to express the LC without any reference to M (or M'). (According to Aristotle, M or M' appearing in the LC would mean repeating the premises' content instead of stating the "new knowledge" the VCA or VS "should" bring (Striker 2009: p. 20). But in Aristotle's time, set theory was not invented yet – thus, in the present paper, the middle term will be very much part of any LC, before one removes it, with some loss of information, in order to write the LCs in their traditional format.) **#2PCPs).** One universal premise and one particular premise both acting on either M or M'. There are 10 such PCPs, (8 are obtained by combining the universal premises in (I) with the particular premises in (III) and vice versa, and the other two are obtained in the same manner from (II) and (IV)); each of their LCs is of the type: one subset of U is ≠Ø. Each of the four M subsets appears in an LC twice as being ≠Ø: for example, SP'M≠Ø, meaning O(S,P), is the LC of E(M,P)I(M,S), Ferio/Festino/Ferison/Fresison, and also the LC of I(M,P')E(M,S'), Bocardo. **#3PCPs).** Two universal premises, acting one on M and one on M'. There are 4 such PCPs, (obtained by combining the universal premises from (I) and (IV), plus the similar ones from (II) and (III). The result of each such PCP is the emptying of 4 subsets of U -two on M and two on M' – with two empty subsets out of 4 being located on the same column, and the other two being located on each side of that column but on different rows (one on the M row, one on the M' row). Consequently, the two LCs of

such a PCP are that both sets in one of the pairs of sets, (S,P), (S',P'), (S,P'), (S',P), whose intersection is a column of U, are each reduced to one subset out of 4. For example, Barbara's PCP empties the column SP' and leaves S=SPM and P'=S'P'M', (see the proof at the end of this section), which leads to the following 3 independent LCs: A(S,P), A(P',S'), and via ei on S and P', one gets I(S,P) and I(S',P'). Note that the two universal LCs are not independent since A(S,P),=A(P',S')=E(S,P'), cf. contraposition or set definitions, but I(S,P) and I(S',P') are independent ei LCs. **#4PCPs).** Both premises are particular. Obviously there are 9 such PCPs and they entail no LC. **#5PCPs).** One premise is universal and one particular, and they act one on M, and the other one on M', (4 obtained by combining a universal, (resp. particular), premise in (I) with a particular (resp. universal) premise in (IV) similarly, the other 4 are obtained from (II) and (III)). These are 8 more PCPs not entailing any LC.

To prove the above claims about the shape of the LCs one may notice that each VCA (and VS) LC is easily found either via a "tree like method" (which eliminates, (i.e., closes), any subset (i.e., branch), emptied by a universal premise), or, directly by looking at the cylindrical Venn diagram in Fig. 1. The "tree like method" is easier to apply if one first writes any premise using only the E or I statements – which we already did. For any PCP in subset #2 one starts the (very short) tree with the non-empty intersection of the two sets appearing in the particular premise: in Ferio/Festino/Ferison/Fresison's case, $\emptyset \neq MS := M \cap S = MSP + MSP' = MSP'$ since the premise E(M,P) says MP= $\emptyset$. Thus the LC is MSP'$\neq\emptyset$ or O(S,P). For any PCP in subset #1, the unique subset the LC is referring to is found by "starting a tree" with either M or M' – the set which appears in both universal premises. It will result that M (or M') equals its intersection with the complements of the other two sets appearing in the two **#1PCPs** universal premises. (For example, in Darapti's PCP case, A(M,P) A(M,S)=E(M,P') E(M,S'), write M=MP+MP'=MP=MPS+ MPS'=MPS.) For any PCP in subset #3 each of the two "LC subsets" can be found via two short trees, each starting with one of the "letter sets" other than M and M' and continuing by eliminating its subsets emptied by the two universal premises. For example, in the case of Barbara's PCP, A(M,P)A(S,M)=E(M,P') E(M',S) start with S=SM'+SM=SM=SPM+SP'M=SPM and P'=MP'+M'P'=M'P'=S'P'M' +SP'M'= S'P'M'. The LCs are A(S,P)=A(P',S') (Barbara), and via ei on S, I(S,P) (Barbari), plus, via ei on P', I(S',P'). (This explains why there are 27 distinct VCAs, generated by only 19 PCPs.) It is again clear that any LC refers to just one subset from the eight subset partition of U.

Thus the VCA are partitioned into three classes, each class being generated by the PCPs from the subsets #1 to #3 above. One may show that inside each of the three VCA classes, any VCA may be recast or reformulated, via a relabeling transformation of the sets S,P,M,S',P',M', as any other VCA in the same class, which makes all VCAs equivalent with three representatives chosen one per VCA class. For example the Darapti, Darii and Barbara representatives may be chosen. In particular, the VCA\VS set whose LCs have one of the formats A(P,S), O(P,S) or I(S',P') may be recast, via a set relabeling as VSs. Out of the 36 distinct PCP (or just pairs), only 19 pairs entail at least one LC and thus generate VCAs, out of which 8 are distinct VS, and 6 are distinct ei VS. (If syllogistic figures are used, then one counts 15 VS and 9 ei VS, but this means that, e.g., the same content VS, Ferio/Festino/ Ferison/Fresison, receives four different names and counts as 4 distinct VS, when in reality one deals with one PCP, E(M,P)I(M,S), and one LC: O(S,P). The VCA\VS subset contains 6 non ei VCAs and 7 ei VCAs.

3. CONCLUSIONS

Discarding the syllogistic moods and figures, syllogistic axioms and inference rules, and valid syllogism rules, in favour of a pure set modelling of the syllogistic terms, greatly simplifies the categorical syllogisms' presentation. Compare, e.g., with other expositions: Alvarez and Correia 2012, Mineshima, Okada, Takemura 2012, Avarez-Fontecilla 2016, (or Lukasiewicz 1957). One have shown that the middle term M always appears in any LC – since the LC always refers to just one subset out of the eight U subsets. Only by losing some information one may recast the LC as referring to a two subset column. Possible LC examples from each of the three VCA classes are S=SPM, SPM≠Ø, M=SPM , as LCs for Barbara, Darii and Darapti, respectively. With some loss of information they translate into the usual A(S,P), I(S,P), and, via existential import on M, I(S,P). Even more interesting, using just set relabeling, instead of syllogism reduction, one can show that the VCAs from the same class are equivalent: any VCA (or VS) can be recast (or reformulated) as any other VCA from the same class. E.g., Barbara, Darii and Darapti may be chosen as representatives of the three VCA classes. Finally, one may use "the old style" indirect reduction to show that Darii and Darapti are not logically independent of Barbara.

REFERENCES

Alvarez, E. and Correia, M. (2012) 'Syllogistic with indefinite terms', *History and Philosophy of Logic,* 33, 297–306.

Alvarez-Fontecilla, E. (2016) Canonical syllogistic moods in traditional Aristotelian logic, Logica Universalis, 10, 517–31.

Karnaugh, Maurice (1953), The map method for synthesis of combinational logic circuits, Transactions of the American Institute of Electrical Engineers, Part 1, 72, 593-599.

Lukasiewicz, J. (1957) Aristotle's Syllogistic (2nd ed.) Oxford.

Marquand, Allan (1881), On logical diagrams for n terms, Philosophical Magazine 12, 266-270.

Mineshima, K., Okada, M. & Takemura, R. Stud Logica (2012) 100: 753.

Radulescu, D. C., (2017 submitted), A cylindrical Venn diagram model for categorical syllogisms.

Striker, Gisela (2009) (Translation, Introduction and Commentary) Aristotle's Prior Analytics Book I. Oxford University Press (Clarendon Aristotle Series), Oxford, p. 20.

Veitch, Edward, W., A chart method for simplifying truth functions, Proceedings of the Association for Computing Machinery, pp. 127–133, 1952.