An Extension of the Rules of Valid Syllogism to Rules of Conclusive Syllogisms with Indefinite Terms

In memory of my sister Cristina Popa, 1948-2018.

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Abstract. One discusses and contrasts the Rules of Valid Syllogism (RofVS) used in Classical Syllogistics, with the Rules of Conclusive Syllogisms (RofCS) which can predict the Logical Consequence (LC) of any conclusive syllogism – a syllogism made of a pair of categorical premises (PCP) which entail a logical consequence (LC), where both PCP and LC may contain indefinite terms: the positive, S,P,M, terms, and their complementary sets, S',P',M', in the universe of discourse, U, called the negative terms. A "pattern and type" classification, splits the 32 conclusive syllogisms into four types, Barbara, Darapti or Darii and Disamis – each containing eight syllogisms, which follow only three "patterns of inclusion and intersection", namely either $S \subseteq M \subseteq P$ (Barbara), or, $M \subseteq S$, $M \subseteq P$ (Darapti), or $M \cap S \neq \emptyset$, $M \subseteq P$ (Darii). (The Disamis type syllogisms follow the Darii pattern, but switch the roles played by the P and S terms.) By using only four Rules of Conclusive Syllogisms (RofCS), (and abandoning the RofVS "two negative premises are not allowed" and "the middle term has to be distributed in at least one premise"), one can make the RofCS into a "theory" "almost equivalent" with the formulas which describe all the conclusive syllogisms, including existential import syllogisms (to which the RofVS do not make any reference). Notably, each precise LC of the Barbara, Darapti and Darii patterns pinpoints a unique partitioning subset of U: $S=S \cap M \cap P$ and $P'=S' \cap M' \cap P'$ for Barbara's pattern - which entails two LCs; $M=M \cap S \cap P$ for Darapti's pattern; $S \cap M \cap P \neq \emptyset$ for Darii's pattern.

Keywords: Categorical Premises, Conclusive Syllogism, Valid Syllogism, Rules of Conclusive Syllogisms, Rules of Valid Syllogism, Term Relabelings.

1 Preliminaries

One uses the following notations and abbreviations: U for the universe of discourse of a 3-term syllogism, made of 2^3 partitioning subsets of U, which will be simply called subsets. No other set will be called a subset except a partitioning subset of U. (Boole, [1], calls these partitioning subsets "constituents".) Juxtaposition of set names/letters will denote set intersections: for example, SM denotes the intersection, $S \cap M$, of the sets S and M. The union of sets will be denoted by a + instead of $\cup$. PCP will stand for pair of categorical premises; LC for logical consequence or conclusion. Existential Import will be shortened to ei. VS stands for valid syllogism(s), recognized as such by the Classical Syllogistics, and the RofVS stand for the Rules of Valid Syllogism as accepted by Classical Syllogistics. The RofCS stands for the Rules of Conclusive Syllogisms. DofA stands for Domain of Applicability of the RofCS or of the RofVS. ESC stands for Empty Set Constraints. A syllogism contains three categorical statements - two premises and their proposed logical consequence (LC) or conclusion. Each of the two premises contains the middle term, denoted by M, and two other terms, P and S which will appear again in the LC. The S, P, M terms are called positive terms and their complementary sets in U, S', P', M', are the negative terms; together they are the indefinite terms. The Classical Syllogistics considers premises formulable only via positive terms, and, accepts, by definition, that the valid syllogisms can have only these LC formats: A(S,P), E(S,P), I(S,P), O(S,P). By contrast, even after eliminating the middle term from it, the LC of a conclusive syllogism can have any of the eight formats E(S*,P*), I(S*,P*), where $P^* \in \{P, P'\}$, $S^* \in \{S, S'\}$.

One also uses the shorthand notations, E':=E(M',x), A':=A(M',x), I':=I(M',x), O':= O(M',x), where $x \in \{S, P\}$; these notations underline that the above statements are "acting on" M'. Following the convention that in a PCP the P-premise is always listed firstly, one may use a shorthand notation for any PCP: for example, one can write Barbara's premises as AE' and Celarent premises as EE' – without mentioning, or keeping track, of the (unnecessary) syllogistic figures.

2 Introduction

In the next Section the notions of term distribution and of affirmative and negative categorical statements are extended, in a consistent way, to premises and LCs containing negative terms. In the important Section 4, one will distinguish, e.g., between the Barbara pattern, $S\subseteq M\subseteq P$, and the Barbara type conclusive syllogisms, which one may denote by $S^*\subseteq M^*\subseteq P^*$, where $P^*\in\{P, P'\}$, $M^*\in\{M, M'\}$, $S^*\in\{S, S'\}$. One notices that once the middle term is decided upon, there are eight conclusive syllogisms of Barbara's type, all conforming to the Barbara pattern. One gives formulas, (1) - (3ii) and (1') - (3ii'), which encompass all the conclusive syllogisms and their conclusions (LCs) for each of the four types of the conclusive syllogisms (originating from the three patterns of "conclusiveness"). These formulas can replace the moods and figures of the Classical Syllogistics and extends the latter to premises and logical consequences (LCs) which contain negative terms. One also classifies and gives formulas, (4i) – (5ii), for the PCPs which do not entail any LC. And one lists four Rules of Conclusive Syllogisms (RofCS) that all the conclusive syllogisms do satisfy. Section 5 summarizes and comments on the Classical Syllogistics and the Rules of Valid Syllogism (RofVS). Section 6 verifies that, indeed, the RofCS #4, (two affirmative or two negative premises entail an affirmative LC; one affirmative and one negative premises entail a negative LC), is satisfied by all the conclusive syllogisms. The Sections 7 and 8 examine how the RofVS and the RofCS can be developed into "respectable theories". The RofCS can predict, for each conclusive syllogism or existential import (ei) conclusive syllogism, its LC – out of which the middle term was eliminated. One recognizes that the whole purpose of the RofVS and the RofCS - to be able to quickly find the LC of any PCP, can be accomplished with about the same ease, (or even easier), by using the formulas for the four types of the conclusive syllogisms. The Sections 9 and 10 examine empty set constraints (ESCs) and how many simultaneously sound conclusive syllogisms one may obtain out of three given terms.

3 The notions of distribution and of affirmative and negative categorical statements extended to indefinite terms

"A term is said to be distributed when reference is made to all the individuals denoted by it; it is said to be undistributed when they are only referred to partially, i.e., information is given with regard to a portion of the class denoted by the term, but we are left in ignorance with regard to the remainder of the class." Keynes [2]. Firstly, one may expand the definition of distribution, by agreeing that whatever distribution the two terms appearing in a categorical statement may have, then their complementary terms in U, are automatically assigned an opposite distribution. Thus since in I(M,P), the terms M and P are undistributed, the terms M', P' are distributed in the same I(M,P) statement. This is in agreement with the obversion and contraposition rules and the standard definition of distribution: from I(M,P)=O(M,P')= O(P,M') one realizes that M' and P' are distributed since M,P were not. The same definition of distribution, extended to indefinite terms, was used, e.g., by Alvarez and Correia [3] and was cited in Alvarez-Fontecilla and Lungenstrass [4] as having been used in 1932 by Wilkinson [5]. Note also that the arguments of a statement and of its contradictory one, have opposite distributions – in E(M,P) both M and P are distributed, while in the contradictory statement, I(M,P), both M and P are undistributed, (while M', P' are distributed). Similarly, in A(M,P), M is distributed and P is not, while in the contradictory statement, O(M,P), the term distributions are reversed.

One can define, for both universal and particular premises, when they are considered as affirmative statements, and when they are considered as negative statements. The universal negative premises are E(M,P), E(M,S), E(M',P'), E(M',S'), and the only particular negative premises are O(P,M), O(S,M), O(M,P), and O(M,S). Denoting $h \in \{S,P\}$, $h' \in \{S',P'\}$, the universal negative premises are E(M,h), E(M',h'), the particular negative premises are I(M',h), I(M,h'), the universal affirmative premises are E(M',h)=A(h,M), E(M,h')=A(M,h), the particular affirmative premises are I(M,h), I(M',h'). One can see that the switch $E \leftrightarrow I$ while the arguments are left unchanged transforms universal negative premises into particular affirmative premises (and vice versa), and transforms universal affirmative premises into particular negative premises (and vice versa). The switch $M \leftrightarrow M'$, (resp. $h \leftrightarrow h'$), transforms affirmative premises into negative premises and vice versa. One can define the negativity or signature, s, of a statement symbol, s(A)=s(I)=0, s(E)=s(O)=1, the signature of a term, s(M)=s(P)=s(S)=0, s(M')=s(P')=s(S')=1, and, the signature of a whole statement as the sum of the signatures modulo 2 of the statement's symbol and all of its terms. Then a statement is affirmative, (or positive), if its signature is zero, and is negative if its signature is 1. Thus, e.g., s(A(M,P))=s(E(M,P'))=s(A(P',M'))=0, s(A(M',P))=s(E(M',P'))=1; therefore the first three statements are affirmative, and the last two are negative. These definitions will be used to prove by cases the RofCS #4 – see Section 6.

4 Conclusive Syllogisms with indefinite terms and the Rules of Conclusive syllogisms

Given three concrete terms and their complementary ones, e.g., good - bad people, wise - foolish people, intelligent - dumb people, one can construct a valid syllogism of each of the Barbara, Darapti or Darii patterns in $3!2^3=48$ ways. But once one decides which term plays the role of the middle term, M, then the 3! permutations "of the role played by each term" are reduced to only 2! permutations of the remaining two "end terms" (S and P). Thus, for each pattern, and once the middle term is agreed upon, one can construct 2*8=16 PCPs out of the three given terms and their complementary ones. But, (in the usual, S, P, M term notation), there are only eight conclusive syllogisms of Barbara's type,
(1) E(M*,P*)E(M*',S*): S*=M*P*'S*, P*=M*'P*S*',
where, from now on, M*∈{M, M'}, P*∈{P, P'}, S*∈{S, S'}, and where, the precise, "one pinpointed subset" LCs - written after the column sign – were obtained via decomposing the S* and P* into subsets, (Jevons [6] style): S*=S*M*+ S*M*'=S*M*= S*M*P*'+S*M*P*= S*M*P*', etc. There are only eight, not 16, conclusive syllogisms of Barbara's type, S*⊆M*⊆P*, because, e.g., the two choices, S⊆M⊆P and P'⊆M'⊆S', correspond to the same pair of Barbara type premises. (Nevertheless, one can see that each Barbara type conclusive syllogism entails two different LCs.) Also, there are only eight conclusive syllogisms of Darapti's pattern, M⊆S, M⊆P, because the Darapti type premises are invariant to the permutation S$\leftrightarrow$P. They can all be written as
(2) E(M*,P*) E(M*,S*): M*=M*P*'S*'.

Finally, Darii's pattern is not symmetric to the S$\leftrightarrow$P permutation – thus it generates 16 distinct PCPs: One gets eight Darii type conclusive syllogisms,
(3i) E(M*,P*)I(M*,S*): S*M*P*'≠Ø,

and another eight Disamis type conclusive syllogisms,
(3ii) I(M*,P*)E(M*,S*): $P^*M^*S^{*'}\neq\emptyset$,

even if both sets follow the Darii pattern (one might have called it the Disamis pattern as well). By accepting both positive and negative terms in the premises and the conclusions, one thus gets 32 conclusive syllogisms – only eight of them are the distinct, i.e., Boolean, valid syllogisms from the Classical Syllogistics. See Copi [7], Hurley [8]. One may rewrite the precise LCs from the formulas (1)-(3ii) as LCs out of which the middle term was eliminated:

(1') E(M*,P*)E(M*',S*): A(S*,P*'), A(P*,S*'); I(S*,P*') if $S^*\neq\emptyset$, I(P*,S*') if $P^*\neq\emptyset$.

(2') E(M*,P*)E(M*,S*): I(S*',P*') if $M^*\neq\emptyset$.

(3i') E(M*,P*)I(M*,S*): O(S*,P*).

(3ii') I(M*,P*)E(M*,S*): O(P*,S*)

The formulas (1') - (3ii'), listing the LCs from which the middle term was eliminated, agree with the four Rules of Conclusive Syllogisms (RofCS) listed below:

RofCS #1: The distribution of the end terms is conserved in all non-ei and ei conclusive syllogisms, except in type (1') ei conclusive syllogisms, where ei on S*, (resp. P*) changes S*, (resp. P*), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P*, (resp. S*), remains the same as it was in the PCP. (The ei condition has to be imposed on the smallest set – the one included in the other two terms of a syllogism of types Barbara or Darapti – afterwards the smallest set, or any other term which includes it, can be dropped/eliminated to obtain an ei LC.)

RofCS #2: From two universal premises follows a universal LC, except when an ei condition on M*, S* or P* is imposed – then a particular LC follows.

RofCS #3: If the PCP contains at least one particular premise, then the LC, if any, is particular.

RofCS #4: If the two premises are affirmative or the two premises are negative, then the LC, if any, is affirmative; from one affirmative and one negative premises a negative LC follows – if any. This rule is valid even for LCs obtained after an ei condition was imposed.

The formulas (1') - (3ii') verify, by inspection, the RofCS #1, #2, and #3. But to verify the RofCS #4 one has to show that RofCS #4 is satisfied for each choice of S*, M*, P* in each of the formulas (1') - (3ii') – see

Section 5 for the proof. The precise definitions of affirmative and negative statements have already been given in the previous Section.

Moreover, the RofCS #1 and #2 unambiguously predict the LCs in the case of two universal premises, (see formulas (1') and (2')): a universal LC is predicted, cf. RofCS #2; since the distributions of the end terms is conserved, (RofCS #1), and there are only two universal statements (or quantifiers) - A and E, an universal LC in which, say, S* and P* should be distributed, can be written either as E(S*,P*) or as A(S*,P*') - but, by obversion, the two LCs are equivalent. Similarly, when at least one premise is particular, the RofCS #1 and #3 unambiguously predict the LC: according to RofCS #3, the LC is particular, and if, say, the S* and P* terms should be distributed in the LC, the two possible LC statements are, again, identical: I(S*',P*')=O(S*',P*). Thus if RofCS #4 is not satisfied, when added to the "predictive" RofCS #1, #2, and #3, it means that the examined PCP is not conclusive, or, in the case of two universal premises, (which always entail at least one LC – since formulas (1') and (2') cover all possible PCPs containing only universal premises), it means that the premises are of the Darapti type, (i.e., the same term, M*, appears in both premises), and that an existential import condition should be imposed on the middle term, and a particular LC follows – in accord with RofCS #2. See below.

Thus, one can apply – to the PCPs listed in the formulas (1) - (3ii), (or, equivalently, to the same PCPs - as listed in the formulas (1') - (3ii')) – the RofCS listed above, to deduce, the LCs from the formulas (1') - (3ii') - out of which the middle term was eliminated. By definition, the RofCS, as well as the RofVS, deal only with LCs whose term variables are $P^* \in \{P,P'\}$ and $S^* \in \{S,S'\}$. Therefore, the RofCS cannot, and do not, predict the "one (partitioning) subset of U" universal LCs from the formulas (1)-(2). In particular, the RofCS do not predict the universal LCs of the type (2) non-ei Darapti conclusive syllogisms, A(M*, S*'P*') or M*=M*S*'P*'. But the universal LC of the type E(S*,P*), (or A(S*, P*')), predicted by RofCS #1 and #2, for the type (2) PCPs, E(M*,P*)E(M*,S*), will not satisfy the RofCS #4 since two negative premises entail an affirmative LC and the distributions of the terms S* and P* are conserved (RofCS #1): for example, if the PCP is E(M,P)E(M,S), then RofCS #1 and #2 predict A(S,P')=E(S,P), which contradicts RofCS #4 . The only LCs which satisfy all the RofCS for type (2) PCPs, are the particular LCs which preserve in the LCs the distributions which the end terms, P and S, already had in the PCPs: I(S*',P*').

Thus, for the Darapti type conclusive syllogisms, the RofCS automatically detect that, in the variables S* and P*, only a particular and existential import (ei) LC is possible. So, if the middle term is unwanted, the RofCS "forces upon us" an ei LC.

One could have obtained the above results starting with an eight by eight PCP matrix obtained by pairing up the eight categorical P-premises, E(M*,P*), I(M*,P*), with the eight categorical S-premises, E(M*,S*), I(M*,S*). Besides the 32 conclusive syllogisms' PCPs listed above, the 64 PCP matrix contains the other 32 PCPs which do not entail any LC. They are split into patterns and subtypes as follows. The two particular premises pattern:

(4i) I(M*,P*)I(M*,S*),

(4ii) I(M*,P*)I(M*',S*).

The pattern of one universal premise plus one particular premise, one "acting" on M and the other on M':

(5i) E(M*,P*)I(M*',S*),

(5ii) I(M*,P*)E(M*',S*).

Note that the lack of LCs for the PCPs of types (4i)-(5ii) is an obvious consequence of the "one subset LC" paradigm, (since these PCPs do not pinpoint a unique partitioning subset of U: formula (4i), (resp. (4ii)), asserts that at least one, (resp. two), but possibly, even three, (resp. four), partitioning subsets of U are non-empty; formulas (5i) and (5ii) assert that one or two partitioning subsets of U are non-empty – contrast this with formulas (3i) and (3ii) which entail that precisely one partitioning subset of U is non-empty). To develop the RofCS into a "theory" one has to prove or postulate that all the PCPs of types (4i)-(5ii) are inconclusive. Firstly, one postulates that the (4i) PCPs are inconclusive, (and are thus excluded from the RofCS domain of applicability (DofA)). Then, one shows that if one applies the RofCS #1, #2 and #3 to the PCPs of types (4ii), (5i) and (5ii), the predicted LCs contradict RofCS #4 – therefore these types of PCPs are not part of the DofA of the RofCS, either. See Section 8. For PCPs of types (4i)-(5ii), Classical Syllogistics offers the counterexample method or the Venn diagrams method to prove that a PCP does not entail any LC. The RofVS "theory" uses the RofVS #1 and #2, (which are not true if LCs containing negative terms are accepted), to prove that those PCPs of types (4i)-(5ii) which are formulable via only positive terms do not entail any LC. See Section 7.

5 A short discussion of Classical Syllogistics

Classical Syllogistics uses premises formulable only via positive terms, uses syllogistic figures, accepts as generating valid syllogisms only those PCPs which entail a logical consequence (LC) of one of the formats A(S,P), E(S,P), I(S,P), O(S,P). Instead of using the moods and figures of the Classical Syllogistics in order to handle the premises formulable only via positive terms, one notices that one is left only with PCPs formulable via positive terms, if one cuts off, from the 64 PCP matrix mentioned above, two rows, (resp. columns), corresponding to the two P-premises, (resp. S-premises), containing negative terms, E(M',P')= A(M',P)=:A', and I(M',P')=O(M',P)=:O', (resp. E(M',S')=A(M',S)=:A', and I(M',S')=O(M',S)=:O'). Cutting off two rows and two columns eliminates 8*4 - 4=28 elements from the eight by eight PCP matrix, and one is left with a six by six, 36 PCP matrix made of premises formulable via only positive terms. (The subtracted 4 equals the number of matrix elements at the intersection of the two eliminated rows and the two eliminated columns, which, when subtracting 4*8, were subtracted twice.) Out of these 36 PCPs, eight PCPs will generate the eight Boolean valid syllogisms (VS) which become 15 VS when redundant syllogistic figures give different names to the same content VS: e.g., the same VS, “No M is P, Some M is S, Therefore Some S is not P”, is counted four times under four different names Ferio, Festino, Ferison, Fresison, according to the syllogistic figures corresponding to various permutations of the premises’ terms. Another three PCPs out of 36 will generate the existential import (ei) VS Bramantip, Darapti and Felapton/Fesapo. Thus, a total of only 11 distinct PCPs formulable via only positive terms generate all the 19, or even 24 VS and ei VS (if one adds Barbari, Celaront/Cesaro and Camestros/Camenos obtained via ei on S). About the other 25 PCPs from the 36 PCP matrix, Classical Syllogistics has to assert, either that they do not entail any LC at all, or that the entailed LCs do not have any of the correct formats, A(S,P), E(S,P), I(S,P), O(S,P), imposed by the requirement that P should be the predicate of the LC.

As part of the 36 PCP matrix of the PCPs formulable using only positive terms, one is left with five PCPs of type (4i), made of two particular premises both acting on either M or M': I(M,P*)I(M,S*), $P^* \in \{P, P'\}$, $S^* \in \{S, S'\}$, and also I(M',P)I(M',S) – since there exists only one PCP formulable via positive terms if both premises contain M'. There are only four PCPs of type (4ii): I(M,P*)I(M',S), P*∈{P, P'}, and I(M',P) I(M,S*), S*∈{S, S'}. There are four PCPs of type (5i), E(M,P*)I(M',S), P*∈{P, P'}, and E(M',P)I(M,S*), S*∈{S, S'}, and another four PCPs of type (5ii): I(M,P*) E(M',S), P*∈{P, P'}, and I(M',P)E(M,S*), S*∈{S, S'}. The total number of PCPs formulable via only positive terms which do not entail any LC is thus 17. As already mentioned, one arrives at this lack of LCs conclusion not by using the Venn diagram method of Classical Syllogistic, but rather the obvious and intuitive "one subset LC" paradigm: if the pair of categorical premises (PCP) pinpoint a unique partitioning subset of U one has an LC, otherwise there is no LC. This leaves eight PCPs about which the Classical Syllogistic has something interesting to say.

Four, out of these last eight PCPs, are dismissed by Classical Syllogistics since their LCs is O(P,S): IE (Fireo?), AO (Bacordo?), I'E' (Boraco?), AE (Falepton?), where, as before, one denotes E':=E(M', x), A':=A(M',x), I':=I(M',x), O':=O(M',x), for x*∈{S, P}. Note that the Classical Syllogistic uses the particular, ei LC, I(S,P), of the Bramantip premises, E'A, but not the universal LC, A(P,S), of the same premises, since A(P,S) does not have one of the four LC formats required by the condition that the P term be the predicate of the LC. Note that, the four PCPs entailing the O(P,S) LC, and Bramantip's PCP, entailing the A(P,S) universal LC, do not contradict any of the RofVS, and both Classical Syllogistics and the RofVS predict for these PCPs the same LCs: either O(P,S) or A(P,S). Since the RofVS do not mention ei – and thus do not apply to the ei VS, one can, for now, say that the domain of applicability (DofA) for the RofVS is made of a total of 13 PCPs: the eight Boolean VS plus the five PCPs whose LCs are A(P,S) or O(P,S). But in order to expand the RofVS up to an (almost axiomatic) "theory" equivalent to the Classical Syllogistics one has to extend the RofVS such that they can also predict the existential import (ei) LCs of the ei VS recognized by Classical Sylogistics: Bramantip, Darapti, Felapton/Fesapo, Barbari, Celaront/Cesaro, Camestros /Camenos. Section 7 describes how the RofVS "theory" does that, and how the RofVS "theory" eliminates from the RofVS DofA the other 17 PCPs mentioned above, which do not entail any LC. The Classical Syllogistics' reason for "explaining away" and "brushing over" the conclusive syllogisms having A(P,S) and O(P,S) as LCs is that a premises' transposition and a relabeling S↔P, (Quine [9] calls it relettering), would transform them, without any change in their content, into VS. [Another very good reason to dismiss the conclusive syllogisms whose LCs are A(P,S) and O(P,S), would have been that their premises, "contradict pairwise" the premises of the VS whose LCs are A(S,P) and O(S,P). Indeed, the following pairs of PCPs have premises which either contradict each other, EI and IE, OA and AO, E'I' and I'E', or are contraries to each other, EA and AE – the latter two PCPs are both sound only if M=P=S=0. Barbara's and Bramantip's premises, AE' and E'A, can be all sound only if S=P=M. Thus, when the terms take concrete values, whichever PCP generates a sound conclusive syllogism, can be written as a VS. See Section 10.]

Finally, there are four PCPs which are discarded by the Classical Syllogistics in accordance with the RofVS #1 and RofVS #2. The PCP, E'E'=E(M',P)E(M',S)= A(P,M)A(S,M), is discarded because M is undistributed in both premises; then, the PCPs, EE, OE, EO, are discarded because in each PCP both premises are negative. The RofVS #1 and RofVS #2 postulate that such PCPs, i.e., the four PCPs above, E'E', EE, OE, EO, cannot entail any of the LCs accepted by Classical Syllogistic – which can indeed be shown using Venn diagrams – see Quine [9]. But, as formula (2') shows about E'E' and EE, and formulas (3i') and (3ii') show for the EO and OE PCPs, the above four PCPs generate ei conclusive syllogisms or conclusive syllogisms, all having I(S',P') as their LC.

But since the RofVS do not refer at all to the ei conclusive syllogisms, one has to slightly generalize the RofVS, (using the RofCS as a model), such that the RofVS DofA extends to the ei VS of Classical Syllogistics for which the RofVS should predict the ei LCs admitted by Classical Syllogistics. The Rules of Conclusive Syllogisms (RofCS) are satisfied by all the 32 PCPs which entail LCs. On their above 32 PCPs DofA, the RofCS can predict both existential import (ei) LCs and non-existential import (non-ei) LCs, out of which the middle term was eliminated. If one postulates that the PCPs of type (4i) do not entail any LCs, (which is true according to the "one subset LC" paradigm used above to argue that all the PCPs of types (4i)-(5ii) do not entail any LCs), then the RofCS show that the PCPs of types (4ii), (5i) and (5ii) cannot entail any LCs – since the LCs predicted by the RofCS #1, #2 and #3, contradict the RofCS #4. Thus RofCS can replace the Classical Syllogistics extended to indefinite terms to decide if a PCP entails an LC, and to efficiently predict that LC. See Section 8. An analogous proof is given in Section 7, that if one postulates, in accord with the RofVS that "two negative premises or two premises in which the middle term is undistributed do not entail an LC", then any proposed LCs being entailed by any of the 17 PCPs mentioned above, (as not entailing any LC according to the "one subset LC" paradigm), would contradict at least one of the RofVS. Note that one can simply ignore the RofVS, as Quine [9] does. For an axiomatic treatment of the RofVS and another ("sprawling" - since syllogistic figures are considered) extension to PCPs containing negative terms, see Alvarez and Correia [3].

6 A proof of RofCS #4 - "if only one premise is negative, the conclusion must be negative, if the premises are both affirmative or both negative, the conclusion is affirmative"

One has to show that RofCS #4 is satisfied for each choice of S*, M*, P*, in each of the formulas (1') - (3ii'). For each of the four formulas, one uses the definitions introduced in Section 3, and one does a proof by cases. For conclusive syllogisms of type (1') Barbara, with premises E(M*,P*)E(M*',S*), and with the LCs A(S*,P*')=A(P*,S*')=E(S*,P*); I(S*,P*') if $S^*\neq\emptyset$, I(P*,S*') if $P^*\neq\emptyset$, the premises' signatures are (1+s(M*)+s(P*)) mod 2, (2+s(M*)+s(S*)) mod 2, the universal LC, (after middle term elimination), has the signature (1+s(P*)+s(S*)) mod 2 and the particular, ei LCs have the very same signature: (1+s(P*)+s(S*)) mod 2. It results that when s(S*)=s(P*) the LCs are negative statements, while the premises have different signatures, i.e., one premise is affirmative and one negative; thus RofCS #4 holds. When s(S*) and s(P*) have different signatures then the LC is an affirmative statement, while the premises have the same signature, i.e., the premises are either both affirmative or both negative – and RofCS #4 holds again. Instead of continuing to verify RofCS #4 in the same way for the formulas (2') - (3ii'), one can slightly simplify the verification procedure by observing that the formulas (1') - (3ii') - from whose LCs the middle term was eliminated - were already written to formally satisfy all the RofCS, including RofCS #4: For types (1') and (2') – the statement symbols of the two premises are negative, E(M*,P*)E(M*',S*), (resp. E(M*,P*) E(M*,S*)), and, according to the RofCS #4, the statement symbol of each of the LCs are affirmative: A(S*,P*'); I(S*,P*') if $S^*\neq\emptyset$, I(P*,S*') if $P^*\neq\emptyset$, (resp. I(S*',P*')). In these affirmative LCs the distributions of S* and P* are the same as they were in the PCPs, except in type (1') ei conclusive syllogisms, where ei on S*, (resp. P*) changes S*, (resp. P*), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P*, (resp. S*), remains the same as it was in the PCP – and this verifies RofCS #1 for PCPs of types (1') and (2').) Premises of types (3i') and (3ii'), (one negative and one affirmative statement symbol in the premises), E(M*,P*)I(M*,S*), (resp. I(M*,P*)E(M*,S*)), entail the negative LCs, O(S*,P*), (resp. O(P*,S*)), in which the distributions of S* and P* are again the same as they were in the PCPs. To prove RofCS #4 in a simpler way, it remains to show that if the terms signatures in the LCs change the LC statement's signature, those terms signatures change the premises statement signatures in such a way that the RofCS #4 is still satisfied. Therefore, this slightly simpler proof of RofCS #4 is based on dropping the statement symbol signatures, and considering only the statements' arguments signatures, via introducing the "argumental signature" of each premise, and of the LC. (The formulas (1')-(3ii') were written in such a way that the statement symbol signatures, by themselves, satisfy already the RofCS #4.) For type (1') Barbara, the argumental signature of each negative premise, E(M*,P*), (resp. E(M*',S*)), is [s(M*)+s(P*)] mod 2, (resp. [s(M*)+1+s(S*)] mod 2), the argumental signature of the universal LC, A(S*,P*'), is [1+s(P*)+s(S*)] mod 2 and the particular, ei LCs have the very same signature: (1+s(P*)+s(S*)) mod 2. It follows that when s(P*)=s(S*) the LC is in fact negative, but then the premises are one affirmative and one negative – as they should be according the RofCS #4. When s(P*)≠s(S*), the LC remains affirmative, and the two premises have the same argumental signatures – thus the two premises will both remain negative, or both become affirmative. For the other formulas, (2') - (3ii'), the proof of the RofCS #4 proceeds similarly: For type (2') Darapti, E(M*,P*)E(M*,S*) with the LC I(P*',S*') if M≠Ø, the premises' argumental signatures are (s(M*)+s(P*)) mod 2, (s(M*)+s(S*)) mod 2, and the LC has the argumental signature (2+s(P*)+s(S*)) mod 2. It results that when s(S*)=s(P*), the LC remains an affirmative statement, while the premises have the same signature, i.e., both premises are affirmative or both are negative. When s(S*)≠s(P*), the LC becomes a negative statement, while the premises will have different signatures, i.e., one premise is affirmative and the other is negative. For type (3i') Darii, E(M*,P*)I(M*,S*) with the LC O(S*,P*), the premises' argumental signatures are (s(M*)+s(P*)) mod 2, (s(M*)+s(S*)) mod 2, and the LC has the argumental signature (s(P*)+s(S*)) mod 2. It results that when s(S*)≠s(P*), the LC becomes an affirmative statement, while the premises will have different argumental signatures, i.e., both premises are either affirmative or both are negative. When s(S*)=s(P*), then the LC remains a negative statement, while the premises have the same argumental signatures, i.e., the premises remain one affirmative and one negative. The proof that the type (3ii') conclusive syllogisms also satisfy the RofCS #4 is identical to the one for the type (3i') conclusive syllogisms.

7 The developing of the RofVS "theory"

As an example, one firstly develops the RofVS "teory" – since this "theory" is already well known. See, e.g., Alvarez and Correia [3].
One notes that the RofVS, like the Classical Syllogistics, suppose that the PCPs are formulable using only positive terms, and that, if a PCP entails an LC, then the LC is formulable using only positive terms, too. One lists the RofVS, Copi [7] Hurley [8] Stebbing [10] Keynes [2], in the following order: #1 - "the middle term has to be distributed in at least one premise", #2 - "two negative premises are not allowed", (these two syllogistic rules refer to the PCPs not entailing an LC (of one of the "standard" four formats A(S,P), E(S,P), I(S,P), O(S,P) – the following four rules refer to the PCP and the entailed LC, and will be used to predict the LC of any PCP from the RofVS DofA), #3 - "any term distributed in the LC must be distributed in the PCP", (i.e., the distribution of the end terms, P and S cannot "increase" from undistributed in the premises to distributed in the LC, but can, conceivable, decrease from distributed in the premises to undistributed in the LC), #4 - "if either premise is negative, the LC must be negative", #5 - "from two universal premises, no particular LC may be drawn", #6 - "if one premise is particular, then the LC is particular". The RofVS can be developed into a more rounded "theory", (or even into a set of axioms, see Alvarez and Correia [3]): firstly, one specifies a set of PCPs which is not part of the RofVS DofA: according to RofVS #1, "PCPs in which the middle term is not distributed in any of the two premises, do not entail any LCs", and, according to RofVS #2, "PCPs made of two negative premises do not entail any LCs". Copi [7], Hurley [8], show that such PCPs, indeed, cannot entail LCs formulable via only positive terms, but Carroll [11] and the formulas (1') - (3ii')), show that such PCPs can entail LCs of the format I(S',P'), thus generating conclusive syllogisms. After the "initial set" of PCPs to which the RofVS do not apply is postulated by RofVS #1 and #2, one can deduce that there are other PCP sets for which the RofVS will predict LCs which contradict the RofVS themselves: therefore, those PCP sets will not be part of the RofVS DofA, either. What is left from the set of PCPs formulable via only positive terms, after the above two rounds of

PCP removals, should be exactly those PCPs on which the Classical Syllogistics' and the RofVS results coincide: the PCPs which generate VS, and the PCPs whose entailed LCs are A(P,S) and O(P,S) – the latter PCPs also satisfy all the RofVS, as the VS do. Moreover one generalize the RofVS #5 to become RofCS #2, i.e., to say "From two universal premises follows a universal LC, except when an ei condition on M, S or P is imposed – then a particular LC follows" and one generalize the RofVS #3 to become RofCS #1, i.e., to say "The distribution of the end terms is conserved in all non-ei and ei VS, except in type Barbara ei VS, where ei on S, (resp. P) changes S, (resp. P), from distributed in the PCP to undistributed in the ei LC, while the distribution of the other end term, P, (resp. S), remains the same as it was in the PCP". Then, with the addition of these two generalizations, the RofVS can also predict the ei LCs of Darapti, Felapton/Fesapo, Bramantip, Barbari, Celaront/Cesaro and Camestros/Camenos.

As Stebbing [10] shows, no PCPs made of two particular premises, may produce LCs compatible with the RofVS. Indeed, PCPs made of two particular and affirmative premises – in which, therefore, no term is distributed, (resp. two particular and negative premises – the "not allowed" PCPs), are already excluded from the DofA, by RofVS #1, (resp. RofVS #2), and the PCPs made of one affirmative and one negative particular premises, should have an LC which is particular and negative, (according to RofVS #4 and #6). But this means that an end term will be distributed in the LC, without being distributed in the PCP – since, according to the distribution's definition, only the middle term will be distributed in the negative particular premise: contradiction. Thus, by postulating, via RofVS #1 and #2, two new classes of PCPs which do not entail LCs, one was able to prove that any PCP made of two particular premises does not entail an LC – therefore all PCPs of types (4i) and (4ii) will be excluded from the DofA of the RofVS. One may now prove that PCPs of types (5i) and (5ii) do not entail any LCs, either. Namely, one can check, that out of the four PCPs of type (5i) and four PCPs of type (5ii) which are formulable only via positive terms, two of them contain only negative premises, in four of the PCPs the middle term is not distributed at all, and in another two PCPs one of the premises is particular, one is negative, then in the PCP both end terms are undistributed, since the middle term has to be distributed at least once, and therefore, the LC should be negative and particular, thus one end term would be distributed in the LC, without being distributed in the premises - thus contradicting

RofVS #3. [The complete details are as follows. The formula (5i), E(M*',P*)I(M*,S*), leads, for M*=M, P*=P and S*∈{S, S'} to two PCPs, (containing only positive terms), where M is nowhere distributed – thus, in accordance with RofVS #1 these PCPs do not entail an LC. Similarly, the formula (5ii), I(M*,P*)E(M*',S*), leads, for M*=M, S*=S and P*∈{P, P'} to two PCPs, (containing only positive terms), where M is nowhere distributed. According to RofVS #1, all the above four PCPs will not entail any LC. In A(M,P)O(S,M) – which is formula (5i) for M*=M', S*=S and P*=P', and in O(P,M)A(M,S) – which is formula (5ii) for M*=M', S*=S' and P*=P, the middle term is distributed in both premises, but S and P are nowhere distributed; since the LC should be particular and negative, (due to one premise being negative and particular), the LC would distribute either S or P, thus contradicting RofVS #3. Finally two PCPs, E(M,P)O(S,M) – formula (5i) for M*=M', S*=S and P*=P, and O(P,M)E(M,S) – formula (5ii) for M*=M', S*=S and P*=P, are made only of negative premises, and therefore they are not contained in the RofVS DofA, in accordance with RofVS #2.]

8 The development of the RofCS "theory"

The RofCS "theory" can be presented in a similar way to the RofVS "theory". One defines the Domain of Applicability (DofA) for the RofCS #1 to #4, as the maximal set on which the RofCS will correctly predict the same LCs as the extension of Classical Syllogistics to indefinite terms. As one noticed this RofCS DofA is made of all the 32 conclusive syllogisms. To exclude, (in a "RofCS fashion" – and thus develop a RofCS "theory"), the other 32 PCPs which do not entail any LCs, one postulates, (and this is a true fact - which cannot be said in general about the postulates expressed by the RofVS #1 and #2), that PCPs of type (4i) are not part of DofA for the RofCS. Then one can prove that PCPs of types (4ii), (5i) and (5ii) are not part of DofA for the RofCS, either, since when applied to such PCPs, the RofCS will predict LCs which contradict the RofCS themselves – namely, the LCs predicted by the RofCS #1, #2 and #3, will contradict the RofCS #4. For example, applying the RofCS to (5i) PCPs, E(M*',P*)I(M*,S*), one obtains – since one premise is negative and one is affirmative and particular – that the LC is O(S*,P*). Therefore if s(S*)=s(P*), (resp. s(S*)≠s(P*)), the LC remains a negative statement, (resp. becomes an affirmative statement), and the premises will have opposite, (resp. identical), argumental signatures – this way the premises become either both negative or both affirmative, (resp. the premises remain one negative and one affirmative), contrary to what the RofCS #4 says – while being used to homologate the LCs predicted by the RofCS #1, #2 and #3. (As one already knows from the Introduction, the definition of a syllogism implies that PCPs of types (4i), (4ii), (5i) and (5ii) do not entail any LCs, since they do not pinpoint a unique subset of U.) If one slightly extends the RofCS #3 to say that if at least one premise is particular then the LC is particular, then these "extended" RofCS #1, #2 and #3, when applied to PCPs of type (4i) will predict the I(S*,P*) LCs, and these LCs do not contradict the RofCS #4, but the LCs are provable false, e.g., via the counterexample method, (see below), and when applied to (4ii), the RofCS #1, #2 and #3 would predict LCs which are contradicted by the RofCS #4. The latter proof is identical to the one given above for PCPs of type (5i). The proof that for PCPs of type (5ii), the RofCS #1, #2 and #3 will predict LCs which contradict the RofCS #4 – is also identical to the proof given above that PCPs of type (5i) are not part of DofA for the RofCS. The PCPs left, are those of types (1) - (3ii), on which – as proved in previous Sections - the LC predictions of the RofCS and the LC predictions of the extension of Classical Syllogistics to indefinite terms, coincide.

According to the RofCS #1 to #4, the (4i) type PCPs should entail a particular LC which preserves the distributions that the end terms, (S* and P*), had in the premises. For example, I(M,P)I(M,S), should imply I(S,P) – without contradicting any of the RofCS #1 to #4. But a counterexample shows that the LC does not follow from the premises: I(M,P) is satisfied if $M \cap P \cap S = Ø$ and $M \cap P \cap S' \neq Ø$ and I(M,S) is satisfied if $M \cap P \cap S = Ø$ and $M \cap P' \cap S \neq Ø$. Thus, I(S,P), the LC inferred from RofCS #1 and #3, which also satisfies RofCS #4, cannot be true, since $M \cap P \cap S = Ø$, and the premises do not assert anything about $M' \cap P \cap S$. Therefore, in general, $P^* \cap S^* \neq Ø$, i.e., I(S*,P*), does not follow from the (4i) premises, I(M*,P*)I(M*,S*). The solution, for the Rules of Conclusive Syllogisms (RofCS) "theory" is to postulate that the (4i) type PCPs do not entail any LCs. This postulate has a similar role with the role of the two Rules of Valid Syllogism (RofVS), "the middle term has to be distributed in at least one premise" and "from two negative premises no LC follows": the latter two rules (or postulates) are necessary to transform the RofVS into a (coherent) "theory" – which can then prove that none of the (4i), (4ii) (5i), and (5ii) type PCPs formulable only via positive terms entail any LC. Once the postulate that the (4i) type PCPs do not entail any LCs is accepted, it results as above, that none of the (4i), (4ii) (5i), and (5ii) type PCPs entail any LC.

Consider the PCPs E'E', EE, OE, EO - whose LCs, (or ei LCs), as one already knows, are all the same: I(S',P'). One can see that the RofCS predict the correct LC for these PCPs: In the case of E'E'=E(M',P)E(M',S) =A(P,M)A(S,M), one has two affirmative premises, (as one knows, of type (2) Darapti). According to the RofCS, (which do not depend on the PCP type, provided that the PCP is contained in the RofCS DofA), the LC, (out of which the middle term was eliminated), should be affirmative and should conserve the fact that both S and P are distributed in the premises. But the only statement out of the eight possible LCs remaining after the middle term is eliminated, E(S*,P*), I(S*,P*), where $S^*\in\{S,S'\}$, $P^*\in\{P,P'\}$, which is affirmative and in which both S and P are distributed, is I(S',P'). If, instead one counts only the statements signatures, and one applies the RofCS #4 "directly" to E'E'=E(M',P)E(M',S), counting that the LC of two negative premises has to be an affirmative LC which conserves S and P being distributed in the premises, one again obtains that the unique LC which satisfies all the RofCS #1 to #4, is I(S',P'). Indeed, trying instead A(P,S') as LC of the above premises does not work, since A(P,S') =E(P,S) which contradicts RofCS #4. Thus, for the Darapti's PCP, the RofCS predict even that the only conclusive syllogism from which M is eliminated is an ei conclusive syllogism. One sees once more, that on the set of PCPs of types (1) - (3ii), the RofCS may effectively replace the logic theory by the "prediction rules". In cases, such as OE, EO, the RofCS will similarly choose the same LC, I(S',P') - based on the fact that two negative premises imply an affirmative LC, that both S and P are distributed in the premises, and that if one premise is particular, the LC will be particular. If both premises are universal - as in EE - the RofCS, together, will still predict, as above, that the LC will be particular, (and thus, the ei condition – on M - is necessary). As already noticed the RofVS and the RofCS refer only to conclusive syllogisms out of which the middle term was eliminated. The RofCS cannot, and do not, predict the universal LCs of the type (2) non-ei Darapti conclusive syllogisms, A(M*, S*'P*") or M*=M*S*'P*'. But any universal LC of the type E(S*,P*), (i.e., out of which the middle term was eliminated), that one might try for the type (2) Darapti PCPs, E(M*,P*)E(M*,S*), will not satisfy all the RofCS themselves; the only LCs which satisfy all the RofCS for type (2) PCPs, are the particular LCs, which preserve in the

LCs, the distributions which the end terms already had in the PCPs: I(S*',P*').

9 About empty sets

The four types of conclusive syllogisms may also be used to settle which conclusive syllogisms are compatible with some of the sets S, P, M, S', P', M' being empty. In the modern square of opposition A(M,P), E(M,P) are not contraries anymore - unless one adds the condition M≠Ø. Instead, when both A(M,P), E(M,P) are true, it results that M=MP’+MP=Ø. This empty set constraint (ESC) – which empties all four subsets of M - is compatible with the universal premises of the conclusive syllogisms of type 1 and 2 - but not with the ei on M. Nor is the M=Ø ESC compatible with the type (3i) and (3ii) premises, I(S*, M) and I(P*, M). In fact, since the conclusive syllogisms of the same type follow the same pattern, it results that a complete discussion of the compatibility of various ESCs and conclusive syllogisms may be reduced to examining just three representative cases (since Darii and Disamis are representatives of the same Darii pattern). Moreover, instead of firstly imposing an ESC, and then finding out the PCPs compatible with it, one can do it the other way around, by listing, for each conclusive syllogism type, the ESCs with which that conclusive syllogism type is compatible or incompatible. Darii’s PCP, A(M,P)I(S,M), means MP'=Ø, SM≠Ø, and the LC is SM= SMP+ SMP'= SMP≠Ø. From the LC SMP≠Ø, one may, with some loss of information, eliminate M, and re-express the LC as I(S,P)=”Some S is P”. Thus the PCP is incompatible with the S=Ø, M=Ø, and P=Ø ESCs, but is compatible with the S'=M'=P'=Ø ESCs, (which imply S=M=P=U; thus in this latter, extreme, case Darii’s PCP and LC just assert that U is non-empty).

Darapti’s PCP, A(M,P)A(M,S), means MP'=Ø, MS'=Ø, and the LC is M=MP+MP'=MP=MPS+MPS'=MPS, which may be written as A(M,SP). This time around one may eliminate M only via the ei hypothesis M≠Ø, then re-express the LC as I(S,P). Thus the ei hypothesis is incompatible with the M=Ø, S=Ø and P=Ø ESCs, but is compatible with the S'=M'= P'=Ø ESCs, (which imply S=M=P=U; therefore, in this latter, extreme case, Darapti’s PCP plus the ei on M, assert only that U is non-empty). Note that Darapti’s PCP without the added ei condition is compatible even with U=Ø, in which case the PCP is just “chatter about empty sets”.

Barbara's PCP, A(M,P)A(S,M), means MP'=Ø, SM'=Ø, and the LCs are S=SM+SM'=SM=SMP+SMP'=SMP, and P'=P'M+P'M'=P'M'= P'M'S+P'M'S'=P'M'S'. The first LC may be written as A(S,MP), or, with some loss of information, one may eliminate M, and write A(S,P)=E(S,P'), which now refers to two subsets of U instead of referring to just one of the eight subsets of U. (A "precise" LC always pinpoints just one of the eight subsets of U.) The second LC may be written as A(P', S'M'), or, with some loss of information, one may eliminate M', and write A(P',S') =E(S,P') – the same as the first LC. Since Barbara's PCP contains only universal premises, the PCP is compatible even with U= Ø in which case all the deductions and the LCs – either "precise" or "classically expressed", are just "chatter about empty sets". One may then add an ei hypothesis, S≠Ø, to the 1st LC, and a different ei hypothesis, P'≠Ø, to the 2nd LC, to obtain, after the M, resp. M', elimination the new ei LCs: I(S,P), (Barbari), and resp., (the un-named), I(S',P'). The S≠Ø ei hypothesis means, since S=SPM, that also P≠Ø and M≠Ø, while the compatible ESCs are S'= Ø, or/and, P'= Ø, or/and, M'=Ø. The S'=P'=M'= Ø constraint amounts to Barbari affirming U≠Ø. The P'≠Ø ei condition implies that, also, S'≠Ø and M'≠Ø. If both ei hypotheses are true then all the sets M, M', S, S', P, P' are non-empty, and there are no ESCs compatible with both ei hypotheses.

In conclusion any universal premise is compatible with any ESC. But any ei hypothesis or any LC of a conclusive syllogism of type (3i) or (3ii), (containing one universal and one particular premise - both acting on either M or M'), specifies three sets that are non-empty, and thus pinpoints three ESCs with which the ei hypothesis or the type (3i) or (3ii) LC is incompatible. The above considerations were based on a sort of "temporal commutativity": instead of firstly applying the ESC to obtain a particular universe of discourse, and then searching for the LC in that universe, one writes down the LC in the usual 8-subset universe of discourse U, and one applies an ESC only afterwards, to see if it is compatible with the PCP and its LC.

10 How many sound VS or conclusive syllogisms may one hope to construct out of three given terms, without imposing restrictions on the structure of the universal set U

When three specific terms are given, with one of them already designated as the middle term, one may consider all the 36 or 64 PCPs which can be constructed starting with these three specific terms, (out of which one is the designated middle term), and one can try to see what sound VS or conclusive syllogisms one may construct out of the three terms. As one shows below, given three terms, with one of them already designated as the middle term, then, at most one sound conclusive syllogism of type (1) or (2) may be built out of the three terms without restricting U to particular cases. Since that conclusive syllogism can be formulated either as a Barbara or a Darapti if the terms are appropriately labeled, one may say that given three terms, there exists at most one sound conclusive syllogism of types (1) or (2) – either a Barbara or a Darapti – which can be constructed out of the given three terms, (again, if one of them was already designated as the middle term). If the three given terms generate, (modulo a relabeling), a sound Barbara, then a maximum of two other type (3i) and two other type (3ii) sound conclusive syllogisms may perhaps be constructed with the same given three terms without restricting U to particular cases: these new sound conclusive syllogisms have their universal premises "stolen" from Barbara and their possible particular premises place set elements on the four subsets adjacent to the four subsets emptied by Barbara's two universal premises. One of these other possible four conclusive syllogisms is a Darii/Datisi, and the other three have no names since they assert that subsets – other than the three subsets "preferred" as LCs by Classical Syllogistics (SPM, SP'M, SP'M') – are non-empty. If the three given terms generate, (modulo a relabeling), a sound Darapti, then only two other sound conclusive syllogisms, one of type (3i) and one of type (3ii) may be constructed with the same given three terms without restricting U to particular cases: these two new conclusive syllogisms, a Darii/Datisi and a Disamis/Dimaris will have their universal premises "stolen" from Darapti, and the same LC as the ei Darapti (after the middle term is eliminated): SPM≠Ø. When the middle term is pre-determined, then two distinct PCPs of type (1) Barbara, or two distinct PCPs of type (2) Darapti, or one PCP of type (1) and one PCP of type (2), will necessarily contain either two distinct universal P-premises, or two distinct universal S-premises, (or, equivalently, will contain term inclusions), which will impose a particular structure on the universal set U. For example, if A(M,P) and E(M,P), are both true, as P-premises in two different PCPs, that would imply M being empty, M=Ø. The relationships implied by the other five possible combinations of two universal P-premises being simultaneously true: E&E' imply P=Ø, A'&E' imply M'=Ø, A&A' imply P'=Ø, A&E' imply P=M, A'&E imply P=M', and similar relationships hold for the “top face” of the S-cube, (if one places the four universal premises on the top faces of the P and S cubes, and the particular premises on the bottom faces of the two cubes). (One may represent the eight P-premises as vertices of a cube; similarly for the eight S-premises.) This shows, e.g., that Barbara, AE', and Camestres, E'E, can both be sound, but uninteresting, since in that universal set, P=M and S=Ø. Note that choosing another of the three terms as a middle term, leads to arguments and conclusions similar to the ones above: no two sound and distinct conclusive syllogisms of either type (1), or type (2) may be constructed with the same middle term, unless the universal set has a particular structure; no sound pair of a conclusive syllogism of type (1) and one conclusive syllogism of type (2) may be constructed with the same middle term, unless the universal set has a particular structure.

Thus, the next task is to see if by using a term once as M, and a second time, say, as P, while the term firstly used as P, is afterward used as M, one can produce two distinct PCPs of type (1) without imposing, when all four premises are true, a particular structure on U. By adjoining the “standard” Barbara's PCP, A(M,P)A(S,M)=E(M,P')E(M',S), to each of the eight PCPs of type (1), having P as the middle term and S and M as end terms, E(P*,M*) E(P*',S*), one can show that, given three terms, at most one sound conclusive syllogism of type (1) may be constructed with them, without imposing a particular structure on the universal set U, i.e., in short, one may say, that at most one of the terms, (out of three given terms), may be used as the middle term in a type (1) conclusive syllogism. For example, if all the following four premises, A(M,P)A(S,M) and A(P,M)A(S,P), (where the second Barbara's PCP is obtained by switching the roles which M and P played in the first Barbara's PCP), are true, i.e., MP'=SM'=PM'=SP'=0, then, M=MP'+MP= MP= MP+PM'=P. For a complete proof, one may compare, two at a time, the eight PCPs of type (1) having P as the middle term, with the “standard” Barbara's PCP. From E(M,P')E(M',S) and E(P,M) E(P',S*), it results M=0; from E(M,P')E(M',S) and E(P,M') E(P',S*), it results M=MP=P; from E(M,P')E(M',S) and E(P',M) E(P,S*), it results, if S*=S, that S=PS+P'S=P'S=P'SM+P'SM'=0+0=0, and, if S*=S', that M=MP=MPS= S; from E(M,P')E(M',S) and E(P',M') E(P,S*), it results P'=0.

But one may easily see that the “standard” Barbara's PCP, E(M,P') E(M',S), and these two type (2) Darapti PCPs, one having S as the middle term, E(S,P')E(S,M'), and one having P’ as the middle term, E(S,P')E(M,P'), can be simultaneously sound without imposing a particular structure on U. This reflects the fact that from the chain inclusions, S⊆M⊆P, which characterizes the Barbara PCP, one may deduce exactly the two Darapti type chain inclusions whose two Darapti PCPs were written above: S⊆M, S⊆P, and P'⊆M', P'⊆S'.

Even three type (2) Darapti PCPs can be simultaneously sound without imposing a particular structure on U, under the condition that their middle terms are all different. For example, E(M,P')E(M,S), E(S,P')E(S,M), E(P',M)E(P',S), use M, S, and resp. P' as middle terms, and out of their six premises only three are distinct. These three type (2) Darapti PCPs, empty a total of only four subsets of U – the same number as a single PCP of type (1) Barbara empties. Equivalently, the above three PCPs are described by these three sets of Darapti chain inclusions: M⊆P, M⊆S', corresponds to the first Darapti listed above, S⊆P, S⊆M', corresponds to the second Darapti listed above, and P'⊆M', P'⊆S'.

11 Conclusions

One saw that, based on set inclusions and set intersections, only three distinct patterns of valid syllogisms or conclusive syllogisms do exist – the Barbara, Darapti and Darii patterns.

As it results from the Sections 5 and 6, the Classical Syllogistics and the RofVS seem to work in a more contorted way than necessary – they become easier to grasp when one generalizes them to indefinite terms. The Classical Syllogistics extended to indefinite terms naturally provides the simple formulas (1)-(3ii) and (1')-(3ii') from Section 4, which list all the pairs of categorical premises (PCPs), (conforming to the Barbara, Darapti and Darii patterns), which entail logical consequences (LCs), and list their precise LCs, and their LCs out of which the middle term was eliminated. The Classical Syllogistics extended to indefinite terms also provides the formulas (4i) – (5ii) which completely list all the PCPs which do not entail any LCs – in accord with the intuitive criterion that if the PCP does not pinpoint a unique partitioning subset of the universe of discourse, U – then there is no LC. The Rules of valid syllogism (RofVS) and the Rules of conclusive syllogisms (RofCS) were developed into almost axiomatic "theories", via postulating that some PCP subsets are inconclusive, i.e., do not produce LCs – any LCs at all, or LCs of the required type. Then the rest of the inconclusive PCPs would predict LCs, that contradict the RofCS, (resp. the RofVS), themselves:

therefore, one infers that these PCPs are also inconclusive. This brings the total number of inconclusive PCPs to 32, (resp. 21), out of the 64, (resp. 36), elements of the PCP matrices with indefinite (resp. positive) terms. The rest of the 32, (resp. the 15), possible PCPs from the 64 PCP matrix, (resp. the 36 PCP matrix), are all conclusive and the RofCS, (resp. the RofVS), can "rapidly" predict their LCs out of which the middle term was eliminated. But the formulas (1)-(3ii) and (1')-(3ii') from Section 4, which list all possible conclusive syllogisms and valid syllogisms (VS), including their precise "one partitioning subset of U" LCs, and their LCs out of which the middle term was eliminated, can be used, probably even easier and faster, to decide for any PCP containing three concrete terms if the PCP is conclusive or not and to determine its LC if any, than if one tries to decide, in the RofCS and RofVS fashion, firstly if the PCP contains only positive terms, secondly, if all the RofCS or the RofVS, (as appropriate, considering the positivity or lack thereof of the terms appearing in the given PCP), are satisfied, and, finally, (if one can infer that the proposed PCP is conclusive), to appropriately apply the RofCS or the RofVS to predict the LC out of which the middle term was eliminated. (Probably the safest and fastest strategy for LC finding, is to compare the formulas (1)-(3ii), (and/or (1')-(3ii')), plus the formulas (4i)-(5ii) with a representation of the PCP on a Karnaugh map for n=3.)

Moreover, the PCPs, written as in the formulas (1)-(5ii) and (1')-(3ii'), via only the E and I statements, allow the following general inferences:

(a) Two universal premises always entail at least one universal LC. Namely, if both M and M' appear in the premises, then one deals with a Barbara type PCP which entails two universal LCs: one LC asserts that either the S or S' set is empty except for, possibly, a uniquely determined partitioning subset of U (the universe of discourse), the other LC asserts that either the P or P' set is empty except for, possibly, a uniquely determined partitioning subset of U. The above two LCs have "opposing indexes": if, e.g., one LC is $S=S\cap M'\cap P'$, then the second LC has to be $P=P\cap M\cap S'$. (Note that, e.g., $S=S\cap M'\cap P'$, implies $A(S, S\cap M'\cap P')$ which implies $A(S, S\cap P')$, which implies $A(S, P')$.) If the middle terms are eliminated, i.e., just dropped, from the above, precise, LCs, then the two LCs become less precise, but identical: $A(S,P')= A(P,S')=E(S,P)$. Nevertheless, two independent and separate existential import (ei) conditions can be imposed on S and P: $S\neq\emptyset$ and $P\neq\emptyset$. If in both universal premises only M, (resp. M'), does appear, then one deals with a Darapti type PCP which entails only one universal LC asserting that M, (resp. M'), is empty except for, possibly, a uniquely determined partitioning subset of U. Two examples, out of the possible eight Darapti's type universal LCs are: $M=M\cap S\cap P$ and $M=M\cap S\cap P'$. The existential import (ei) condition has to always be imposed on the smallest set – the one included in all the other sets – in the case of the Darapti type PCPs this set is always M (or M'). Thus if an ei condition is imposed on the precise and universal two LCs above, $M=M\cap S\cap P$ and $M=M\cap S\cap P'$, then a Classical Syllogistics style, less precise, particular LC will result for each of the two examples: I(S,P) (Darapti), and I(S,P')= O(S,P) (Felapton/Fesapo). (For the Darapti type PCPs, there is no universal LC out of which the middle term was eliminated, because an ei condition has to be imposed on the middle term before eliminating it – otherwise, eliminating the "subject" of the LC removes the LC altogether. Note that the precise, one partitioning subset of U, universal LC, uniquely determines the explicit expression of the PCP which entails that precise universal LC. The less precise, Classical Syllogistics style universal LC, obtained for the Barbara type PCPs, determines the explicit expression of the PCP which entails that Classical Syllogistics style LC, up to a replacement $M\leftrightarrow M'$. For example, to the Barbara's Classical Syllogistics style LC, A(S,P), (since only Barbara type PCPs lead to universal LCs out of which the middle term was eliminated, and since cf. RofCS #1 the distributions of the end terms are conserved), correspond the Barbara and Barbara' PCPs: A(M,P)A(S,M)=E(M,P')E(M',S) and A(M',P)A(S,M')= E(M',P')E(M,S). (One defines Barbara' as having the same premises as Barbara up to the substitution $M\rightarrow M'$. Similarly, among all the conclusive syllogisms, there exist a Darapti and a Darapti', a Disamis and a Disamis', etc.)

(b) If the LC is particular, I(S*,P*), in order to recover the PCP entailing the above LC, one needs to know if the LC was obtained via existential import (ei) - and on which term, M*, S*, or P*, the ei condition was imposed, or, one needs to know if the LC is the result of an either Darii type or a Disamis type PCP. In other words, if the given LC is, e.g., I(S,P), then, if the ei was imposed on S, the PCP was the one for either Barbari or Barbari', if the ei was imposed on P, the PCP was the one for either Bramantip or Bramantip', if the ei was imposed on the middle term, the PCP was the one for either Darapti or Darapti', and if one knew that the PCP contained one universal and one particular premises then the possible PCPs are either the ones for Darii or Darii', or, the ones for Disamis or Disamis'.

(c) According to the formulas (3i) and (3ii), (or according to the RofCS #1 to #4), the Darii and Disamis type PCPs entail a particular LC which asserts that a uniquely determined partitioning subset of U (out of the eight partitioning subsets of the universe of discourse), is non empty.

(d)The formulas (4i), (4ii), (5i) and (5ii) characterize all the PCPs which do not entail any LCs.

The "one partitioning subset of U" paradigm, i.e., the realization that if the premises pinpoint a unique partitioning subset of U then the premises entail an LC, and otherwise there is no LC, characterizes what logical consequence (LC) means, in a way that differs from the characterization of an LC in the Classical Syllogistics which only asserts that the LC cannot be false if the premises are true. The difference between these two LC characterizations also exposes the difference between the role which the middle term plays in the Classical Syllogistics, (as a facilitator of a direct connection between the end terms S* and P*, the only terms which appear in the LC), and the role the middle term plays in the universe of discourse set model where the terms are "interpreted in extension" only, and where the middle term remains an essential part of the LC – since one cannot uniquely label a partitioning subset of U using only two terms out of the three syllogistic terms.

Note that Aristotle's definition (Striker [12]) "A syllogism is an argument in which, certain things being posited, something other than what was laid down results by necessity because these things are so", provides not only a characterization of a syllogism – both premises are necessary to obtain the LC and the LC has to validly follow from the premises - but also a justification, (or a pretext – embodied by the expression "something other than what was laid down"), for the elimination of the middle term from the LC. Nevertheless, this elimination always weakens the LC, which instead of asserting something about a unique subset of U, will now assert the same thing, less precisely, about two subsets of U – namely that two subsets might be non-empty, (e.g., Barbara's LC, A(S,P), means $S \cap P'=\emptyset$; therefore the LC asserts that $S=S \cap P:= SP=SPM+SPM'$, although the premises already assured that $SPM'=\emptyset$), or that at least one of the two is definitely non-empty. Moreover, the contradictory statement of the weakened LC is stronger, (since it negates something about a larger number of sets), than the contradictory statement of the initial, stronger LC – which referred to a unique subset of U. (Example: compare "John lives in Miami" with "John lives in Florida": the negation of the less precise information places John out of Florida, while the negation of the stronger info about John, places him only out of Miami. It reflects the fact that negating a multiple "Or" statement produces a multiple "And" statement. Analogously, a negative statement such as "John does not live in Florida" is more powerful than the negative statement "John does not live in Miami"; by negating them, one obtains the affirmative statements "John does live in Florida" and, respectively, "John does live in Miami" – whereby out of the two latter affirmative statements, the last one is the strongest.) Thus, in Classical Syllogistics, when performing an indirect reduction, i.e., a reductio ad absurdum proof of validity, one proves a weakened LC by using stronger than necessary premises. For example, Darii's validity may be proved, by impossibility, from Camestres. But this is unnecessary: suppose, by impossibility, that Darii's precise LC, E(M,P') I(M,S):SPM≠Ø, is false, i.e., SPM=Ø (by the law of excluded middle). Then, from Darii's general premise, A(M,P), i.e., MP'=Ø, it results SM=SMP+SMP'=Ø, which already contradicts Darii's particular premise, I(S,M) or SM≠Ø – no Camestres had to be invoked, and there is no need to suppose, (the stronger), SP=Ø, (the contradictory of Darii's weakened LC, I(S,P), i.e., SP≠Ø), since supposing, by impossibility, that SPM=Ø, suffices. In Classical Syllogistics, one can obtain from Camestres' PCP, A(P,M) E(M,S), via reductio ad absurdum, its Classical Syllogistics' conclusion, E(S,P) or $S \cap P$:=SP=Ø, which is a weaker LC than each of the precise LCs provided by the "one subset of U" paradigm LCs: $S=S \cap M' \cap P'$:=SP'M' and $P=P \cap M \cap S'$:=PMS' (cf. formula (1)). After eliminating, (i.e., dropping), the middle terms from each of these LCs, the weaker LCs out of which the middle term was eliminated, become identical: A(S,P')=A(P,S')=E(S,P). In Classical Syllogistics the reductio ad absurdum method will prove Camestres' LC by showing that the supposition $S \cap P$≠Ø, (i.e., I(S,P), which negates E(S,P)), when paired up with any of the two of Camestres' premises, A(P,M)E(M,S), will entail an LC which directly contradicts the other of the Camestres' premises. But all this is unnecessary: Camestres premises assert that PM'=SPM'+S'P'M' =Ø, and that SM=SPM+PSP'M=Ø, out of which the two precise LCs, $S=S \cap M' \cap P'$:=SP'M' and $P=P \cap M \cap S'$:=PMS', and their Classical Syllogistics style (and weaker) LC, A(S,P')=A(P,S')=E(S,P), easily follow.

Author's Declaration:

The submitted paper, "An extension of valid syllogisms to indefinite

terms and a reduction of all the conclusive syllogisms to only Barbara, Darapti and Darii", complies with the Ethical standards - it was not submitted anywhere else and any other ethical considerations are not applicable.
The author has no conflicts of interest to declare that are relevant to the content of this article.
The research for this article did not involve asking for an Informed consent
from anybody - only the author did the research.
The research for this article did not involve any Ethical approval from anybody; no Ethical approval is applicable for this article.
Thank you.

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