# A Closer look at the Sub-harmonic Test for Watthour Meters

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## 1. Introduction

The Sub-harmonic Test (also called Burst Control Test or Integral Cycle Control Test) has been in several meter specifications for decades. This test is intended to ensure that watthour meters can cope with a load control method that turns the load current on and off for a few 60 Hz (or 50 Hz) complete cycles at a time. The test itself is defined as having a continuous sinusoidal voltage and a sinusoidal test current that is on for two cycles and then off for two cycles, repeatedly. These test waveforms are illustrated in Figure 1. AC voltage and current definitions are based on the root-mean-square or rms value of the AC waveform, which provide traceability to DC voltage and current values. Both the rms measurements and the presence of sub-harmonics depend on one's view of the current waveform in Figure 1. Is it four cycles of a 60 Hz waveform or one 'cycle' of a 15 Hz waveform?

The rms calculation can be done on a cycle-by-cycle basis, or once over the four cycles of the repetitive test waveform. These two methods do not produce the same measurements, resulting in some confusion about this test and its expected outcome. Which is correct?

Similarly, a FFT done over four cycles will indicate that the (current) waveform has a fundamental frequency of 15 Hz, with significant components at the 3rd, 4th, and 5th harmonics (45 Hz, 60 Hz, and 75 Hz) components, and smaller components at the higher odd harmonics. If one assumes that the fundamental frequency is actually 60 Hz, then the FFT components at 15 Hz and 45 Hz must be sub-harmonics of 60 Hz. Are these sub-harmonics real, or mathematical side-effects of the FFT?

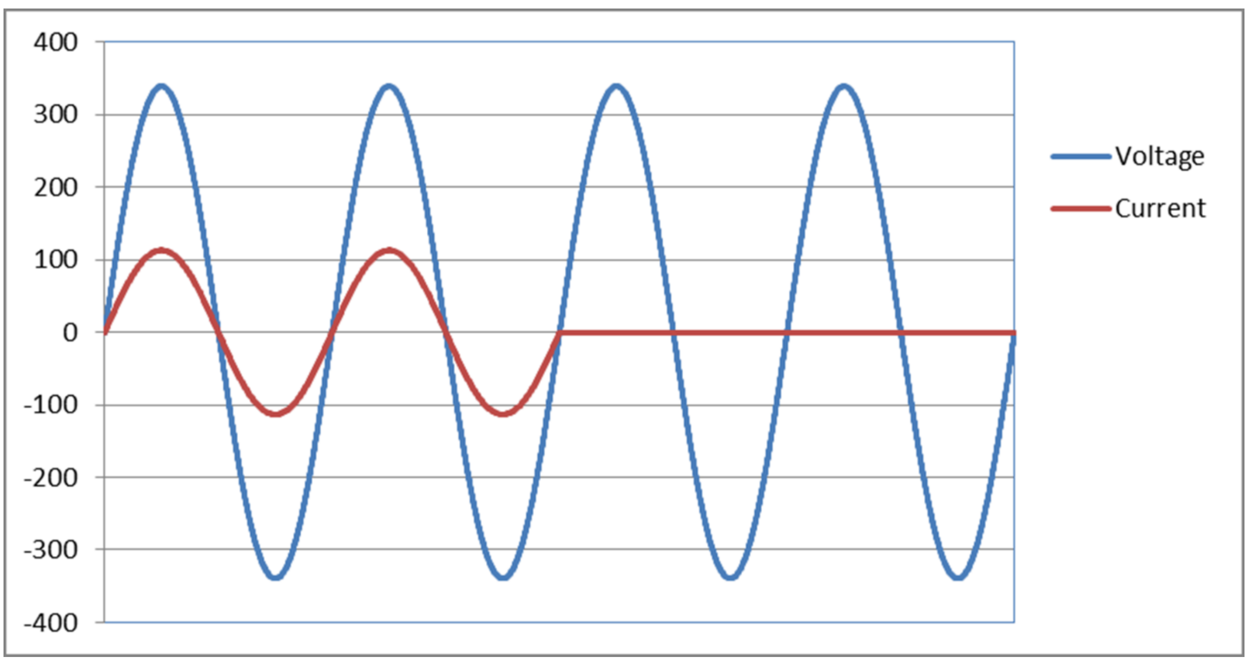



**Figure 1 – The Sub-harmonic Test Waveform**

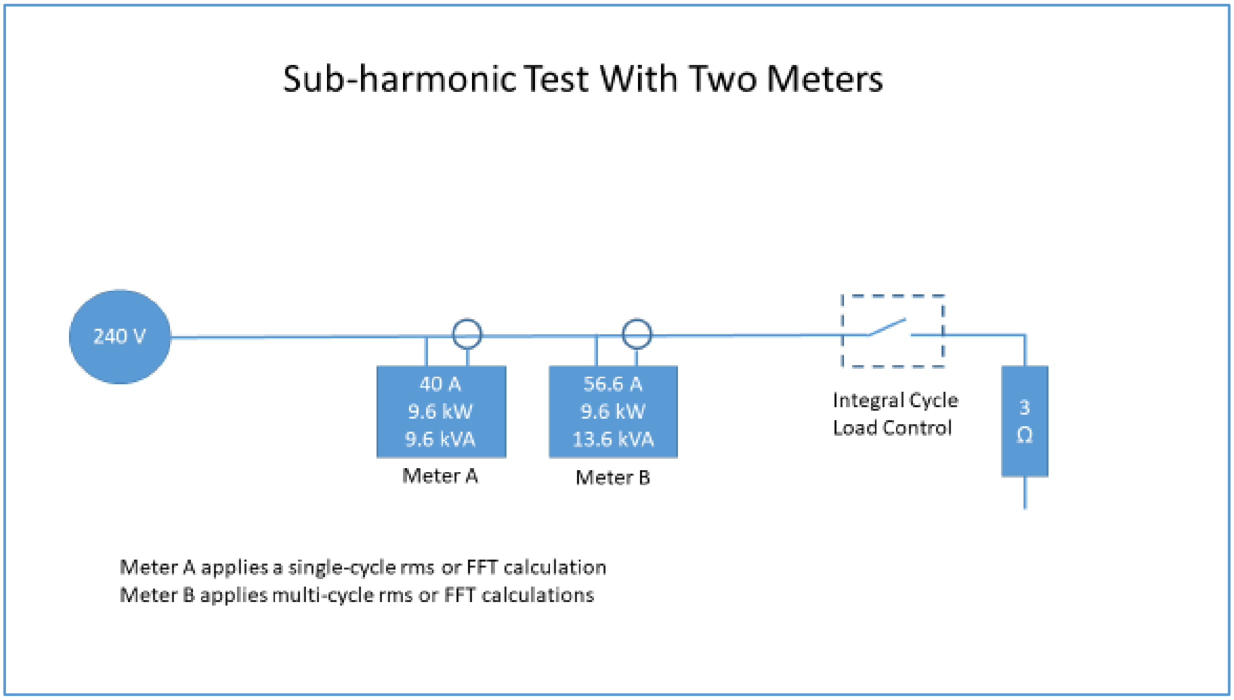



**Figure 2 – Sub-harmonic Test Diagram**

## 2. The RMS Calculation Issue

The rms calculation, as used in electrical measurements, must meet two conditions:

a) The calculation must be made over one or more complete cycles
b) For multi-cycle measurements, each cycle (values) must be identical in shape and magnitude

The first of these conditions is fairly easy to meet, but the second is violated by the current waveform in the Sub-harmonic test. The waveforms in Figure 1 correspond to a 240V rms sinusoidal source applied to a 3 Ω resistive load for two consecutive cycles out of four, connected as in Figure 2.

If one views this cycle by cycle, one finds an 80 A rms current flow for two cycles and no current for the following two. This in turn equates to a load of 19.2 kW for two cycles followed by no load for the following two, or an average load of 9.6 kW over the four cycles. Over an hour, a revenue meter would be expected to register 9.6 kWh for this load (Meter A).

If one calculates the rms value over four cycles, one gets a current of 56.6 A (80 A * 1.414 / 2), and the rms voltage over four cycles is the same as before, at 240 V. This means that the rms apparent power must be 13.6 kVA (Meter B).

Since there is no reactive component to the load, there is no fundamental reactive load, and fundamental reactive power $Q_1$ must be zero. A meter that derives (Fryze's) reactive power from active and apparent power would find 9.6 kvar, and a meter that extracted the non-fundamental apparent power from the four-cycle rms figures would report $S_N$ as 9.6 kVA. Table 1 summarises the two sets of measured quantities one would get for the above example. Note that each set of figures on its own looks reasonable at first glance. There is no glaring indication of a flawed measurement.

| RMS Calculation Period: | Single Cycle (Meter A) | Four Cycles (Meter B) |
|---|---|---|
| **Voltage rms** | 240 V | 240 V |
| **Current rms** | 80 A (40A average) | 56.6 A |
| **Apparent Power S = Vrms*Irms** | 19.2 kVA (9.6 kVA average) | 13.6 kVA |
| **Active Power P** | 19.2 kW (9.6 kW average) | 9.6 kW |
| **Fundamental Reactive Power $Q_1$** | 0 kvar | 0 kvar |
| **Fryze Reactive Power $Q_F$** | 0 kvar | 9.6 kvar |
| **Non-Fundamental Apparent Power $S_N$** | 0 kVA | 9.6 kVA |

**Table 1 – RMS Quantities for the Sub-harmonic Test Waveforms**

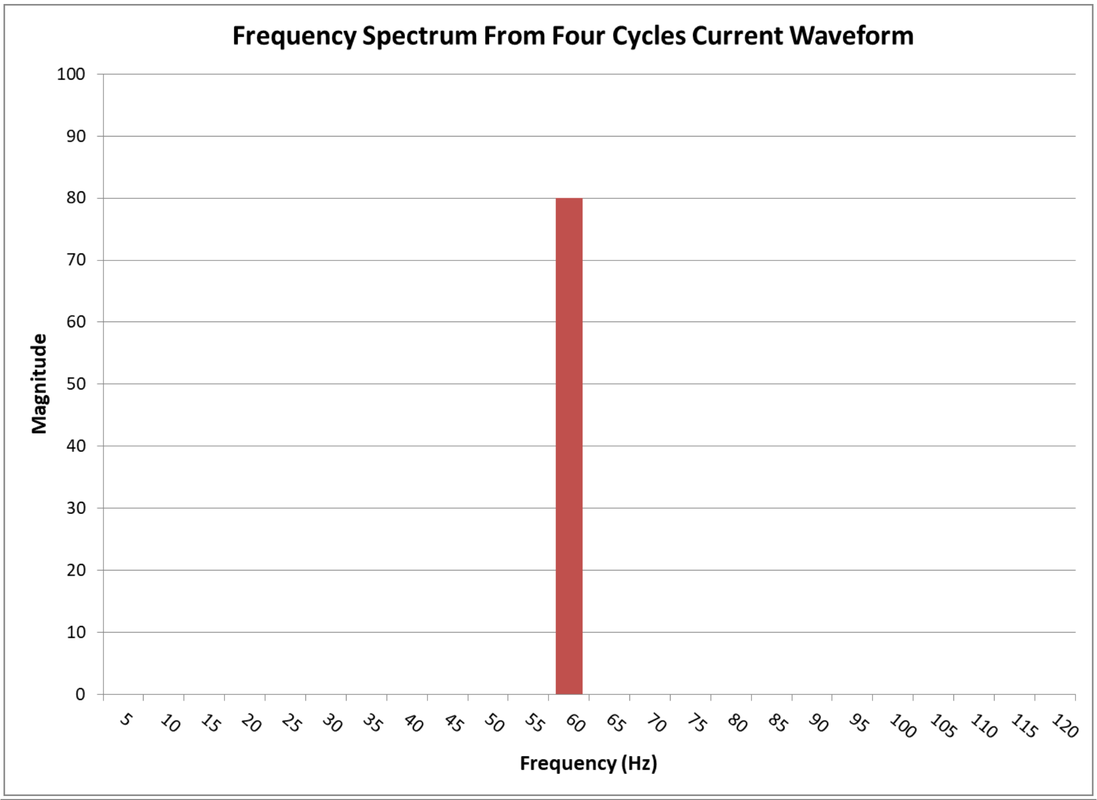
The most important differences are the rms current and rms apparent power measurements, where the four-cycle measurements are 41% higher than the single cycle values.

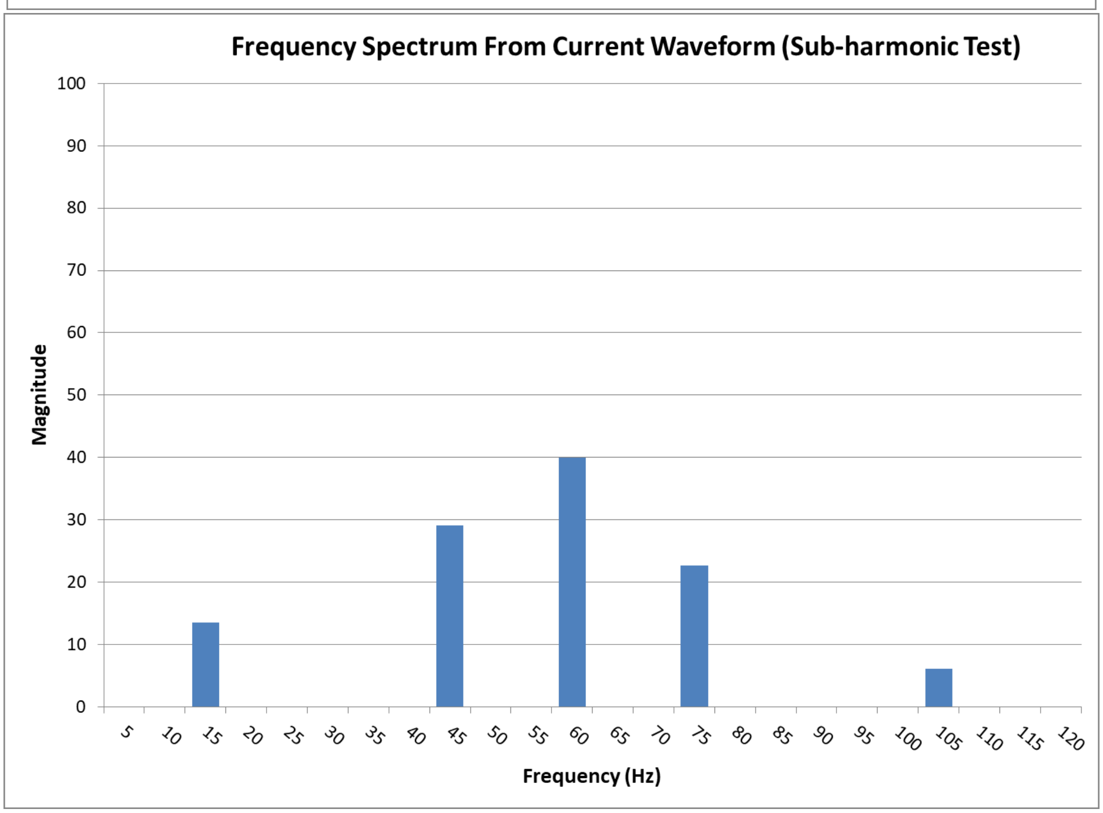
## 3. The Sub-harmonics Issue

Much like the rms calculation, the FFT calculation assumes that, if the measurement window spans more than one (complete) cycle, then all the cycles will be identical. Again, the current waveform violates this assumption. Figure 3 shows the results of a FFT carried out on four cycles of continuous sinusoidal 80A rms current. There is only one frequency component at 60 Hz and it contains all the content of the waveform – 80A rms. Figure 4 is the FFT of the Sub-harmonic test waveform, with two cycles at 80A rms and the next two at zero. Note the components at 15 Hz, 45 Hz, and 75 Hz, and the 60 Hz component containing only 50% of the total current magnitude (40 A rms).

Figure 3 implies that:

a) The Sub-harmonic test waveform contains actual sub-harmonics – current components at 15 Hz and 45 Hz, as well as inter-harmonics at 75 Hz and higher odd multiples of 15 Hz.
b) The magnitude of the 60 Hz component comprises only 50% of the current waveform




**Figures 3 and 4 – FFT Output for four cycle Sinusoidal and Sub-harmonic Waveforms**

While the results are mathematically correct, they are also artifacts of the 4-cycle window width and the zero current values for half the window. The 50% magnitude for the 60 Hz component corresponds to the 40 A rms equivalent. However, the other sub-harmonic and inter-harmonic components are valid only in the context of a four-cycle FFT window. If you applied the FFT over each cycle separately the result would be an average of 40 A rms at 60 Hz (80A+80A+0A+0A)/4, with no subharmonics or inter-harmonics in sight. Table 2 lists the measurements obtained from FFTs applied to the sub-harmonic test waveforms.

| FFT Window Period: | Single Cycle (Meter A) | Four Cycles (Meter B) |
|---|---|---|
| **Voltage rms at 60 Hz** | 240 V | 240 V |
| **Current rms** | 80 A (40 A average) | 56.6 A |
| **Current rms at 60 Hz** | 80 A (40 A average) | 40 A |
| **Sub-harmonic Content** | 0 A | 32 A |
| **Inter-harmonic Content** | 0 A | 24 A |

**Table 2 – FFT Quantities for the Sub-harmonic Test Waveforms**

The primary issue here is that the four cycle window implies that there are significant current components at sub-harmonic and inter-harmonic frequencies. As with the rms measurements, both sets of figures look reasonable. In fact, if you checked the four cycle rms measurements against the four cycle FFT readings, you could conclude happily that both are correct. The FFT total current appears to agree with the four cycle rms current measurement at 56.6 A, and the FFT 60Hz current component appears to be identical to the single cycle average rms current of 40 A. Alas, this would be a false sense of security. It is also possible that both methods are equally incorrect.

## 4. Revenue Metering and the Sub-harmonic Test

The Sub-harmonic Test is a Type Test, typically done by a regulatory body to ensure that a specific meter model meets the specified technical requirements for its jurisdiction. The meter readings would be compared to those of a reference standard, with traceability to national standards. If your reference standard calculates rms current over one cycle, it would define the above load as 40 A and 9.6 kVA. Alternatively, if it used four cycles (or multiples of 4), it would characterise the load as 56.6 A and 13.6 kVA. We do not define the rms measurement window in any meter specification (and thus define our requirement for the reference standard), so either set of measurements is possible, and traceable under steady state sinusoidal conditions.

Most meters today do the rms calculation over 10 to 60 cycles, and would measure 56.6 A and 13.6 kVA for the Sub-harmonic Test. However, few people are aware of these intricacies, and as long as your reference standard and test meter use the same approach, no-one is any the wiser. Nevertheless, careful review of the numbers will reveal the problem.

## 5. The Correct Measurements for Revenue Metering

Is the Meter A load of 40 A and 9.6 kVA correct, or Meter B with 56.6 A and 13.6 kVA?

Do the ‘sub-harmonics’ apparently revealed by the FFT explain the differences?

It is easy to show that Meter A gives the correct readings (40 A and 9.6 kVA) while the measurements from Meter B (56.6 A and 13.6 kVA) are incorrect. We simply add a second load, identical to the first except that it is on for cycles 3 and 4, when the first load is off (Figure 5). The combination of these loads presents a continuous sinusoidal load to the meters, which both report the load correctly as 80 A, 19.2 kW, and 19.2 kVA. This is double the Meter A reported load, to be expected for double the load.

There are no harmonics, sub-harmonics, or inter-harmonics.

There is no reactive load, real or fictitious. There is no non-fundamental load.

Only sinusoidal load as expected. Meter A clearly reported the measurements for both single and dual loads correctly, while Meter B produced incorrect current and VA readings for the single load.

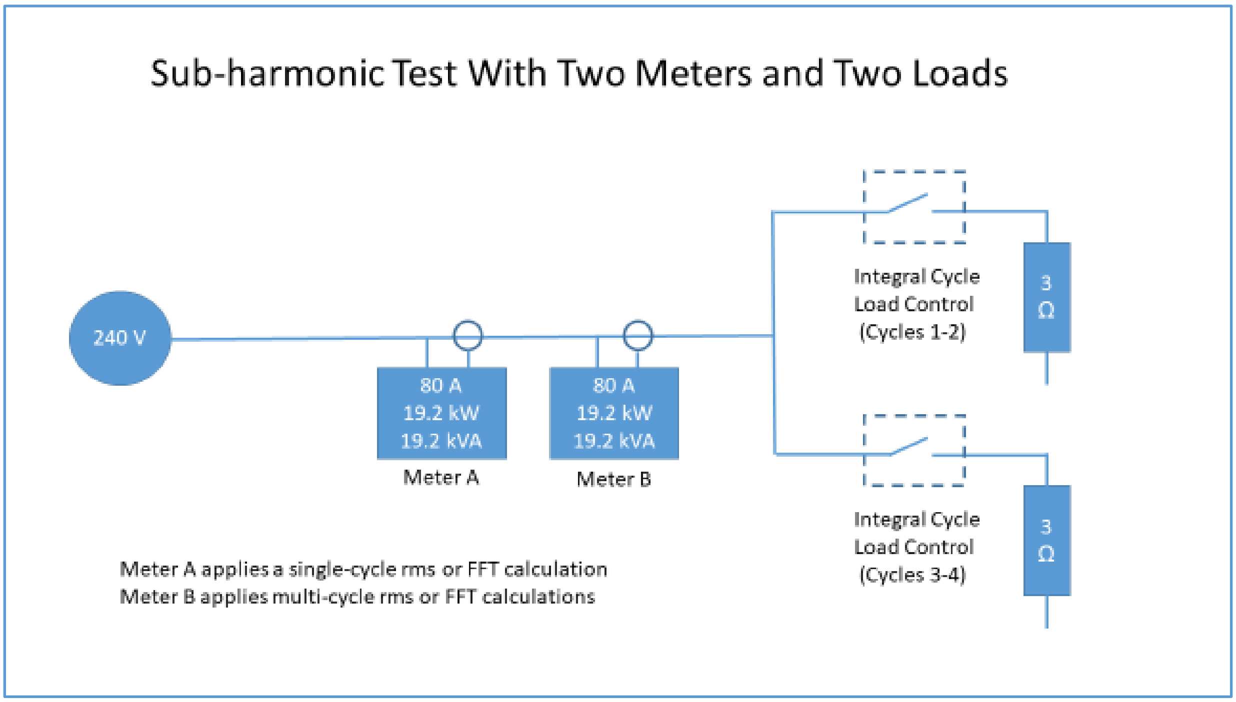



**Figure 5 – Sub-harmonic Test with Two Loads**

## 6. A Practical Example with a Modern Meter Test Set

The deeper problem, as with many issues affecting electrical measurements, is that it is difficult to determine if an electrical measurement is accurate or not at face value. One cannot see electricity, or estimate its magnitude with any of our senses. This problem is compounded by the fact that metering professionals are aware that different definitions or algorithms applied by different meters can result in different measurements in unusual circumstances.

The author recently acquired a new Calmet TS33 portable three phase test system. The TS33 can generate and measure harmonics, and includes the sub-harmonic test waveform as one of six special predefined waveforms. This was applied to a three phase four wire Vision meter, reprogrammed for the HIP project carried out by BC Hydro. The HIP meter, like any modern digital meter, takes thousands of samples per second of each voltage and current and uses a digital signal processor (DSP) to do the rms and power calculations. The DSP program in the HIP meter has been modified to produces traditional harmonic-inclusive power quantities (P, $Q_F$, $Q_B$, and S), fundamental only power quantities ($P_1$, $Q_1$, and

$S_1$), and some power quality related quantities. The TS33 was configured to deliver 240V rms and 10A rms to the meter, using the sub-harmonic current waveform.

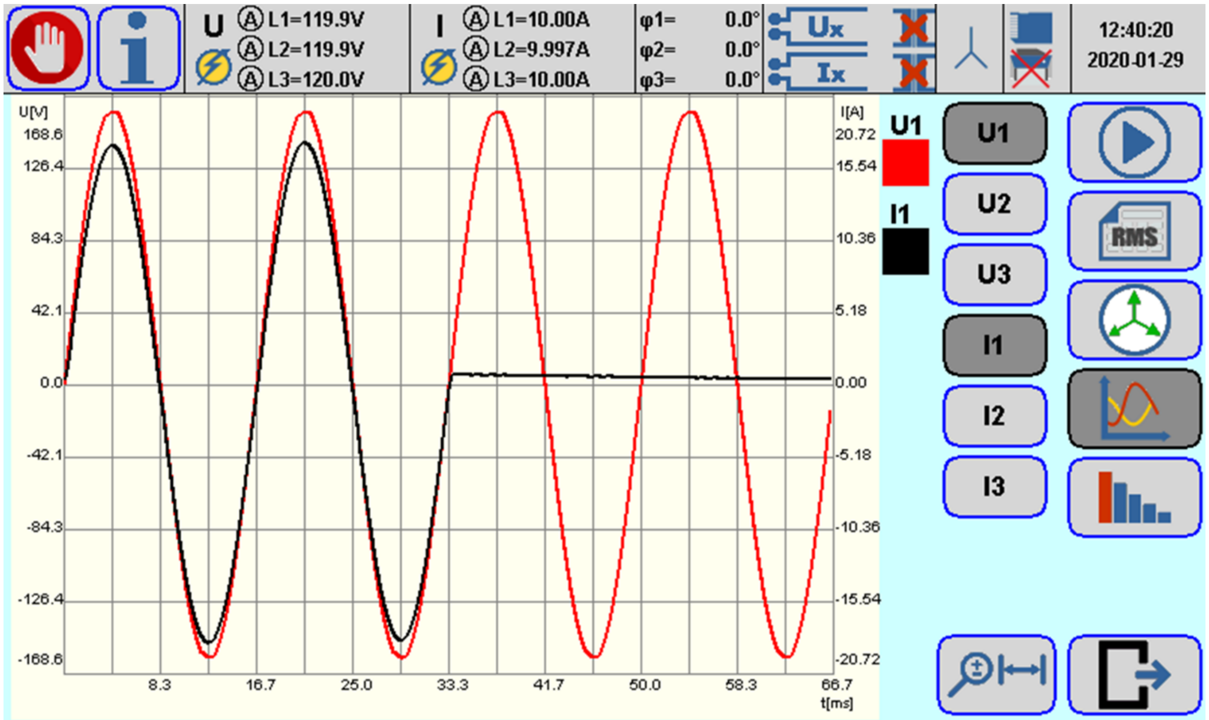
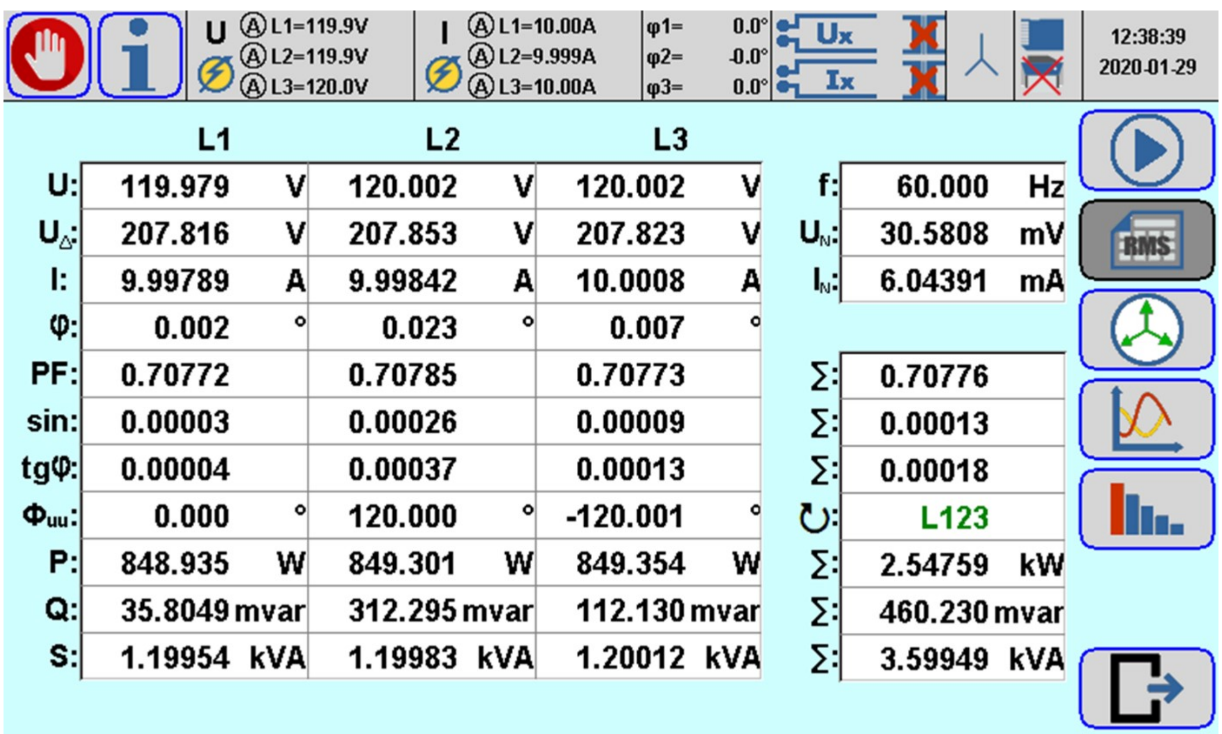



**Figure 5 – TS33 Screenshot of Sub-harmonic Test Waveforms**




| | L1 | | L2 | | L3 | | | | |
|---|---|---|---|---|---|---|---|---|---|
| U: | 119.979 | V | 120.002 | V | 120.002 | V | f: | 60.000 | Hz |
| $U_\Delta$: | 207.816 | V | 207.853 | V | 207.823 | V | $U_N$: | 30.5808 | mV |
| I: | 9.99789 | A | 9.99842 | A | 10.0008 | A | $I_N$: | 6.04391 | mA |
| φ: | 0.002 | ° | 0.023 | ° | 0.007 | ° | | | |
| PF: | 0.70772 | | 0.70785 | | 0.70773 | | Σ: | 0.70776 | |
| sin: | 0.00003 | | 0.00026 | | 0.00009 | | Σ: | 0.00013 | |
| tgφ: | 0.00004 | | 0.00037 | | 0.00013 | | Σ: | 0.00018 | |
| $\Phi_{uu}$: | 0.000 | ° | 120.000 | ° | -120.001 | ° | ↻: | L123 | |
| P: | 848.935 | W | 849.301 | W | 849.354 | W | Σ: | 2.54759 | kW |
| Q: | 35.8049 | mvar | 312.295 | mvar | 112.130 | mvar | Σ: | 460.230 | mvar |
| S: | 1.19954 | kVA | 1.19983 | kVA | 1.20012 | kVA | Σ: | 3.59949 | kVA |

**Figure 6 – Screenshot of TS33 Measurements for Sub-harmonic Test**

Figure 5 is a screenshot from the TS33, showing the test voltage and current waveforms. Note the rms voltages and currents as measured in the bar above the waveforms, and the peak voltage (168.6 V) and current readings (20.72 A) from the graph. These correspond to single cycle rms values of 119.2 V and 14.64 A. It is immediately clear that the TS33 applies a multi-cycle rms calculation for the current, as the average of 14.64 A over four cycles is 7.32 A, not 10 A.

The measurement screenshot contains a lot more information (Figure 6). These confirm that the TS33 is using a multi-cycle rms calculation, as the apparent power S is correctly calculated from the rms voltage and current, and is 41% higher than the active power. The reactive power is almost zero, yet the power factor is 71%.

A screenshot of the measured values as seen by the HIP meter with the sub-harmonic test applied is displayed in Figure 7. Note the harmonic-inclusive quantities in the left column tie up almost perfectly with those from the TS33. The difference between the 3600 VA and the 2548 Watts is explained by the Fryze reactive power of 2543 vars, or the non-fundamental power $S_N$. This of course implies that there are non-fundamental power components (such as sub-harmonics and inter-harmonics).

File Edit View Call Transfer Help

```
   Time entry              = 2020 JAN 30 THU 15:51:08
P  Active Power Watts      =  2548.000    P1 Fundamental Watts       =  2547.000
QB Reactive Power VARs     = -    1.000
QF Reactive Power VARs     = -2543.959    Q1 Fundamental VARs        = - 240.091
S  Apparent Power VA       =  3600.560,   S1 Fundamental VA          =  2558.291
Va RMS Voltage             =   120.062,   V1a Fundamental Volts      =   120.043
Vb RMS Voltage             =   119.972,   V1b Fundamental Volts      =   119.953
Vc RMS Voltage             =   119.974,   V1c Fundamental Volts      =   119.955
Ia RMS Current             =    10.003,   I1a Fundamental Current    =     7.108
Ib RMS Current             =    10.009,   I1b Fundamental Current    =     7.114
Ic RMS Current             =     9.992,   I1c Fundamental Current    =     7.100
Sn Nonfundamental VA       =  2533.610,   Dv Voltage Distortion VAR  =    30.368
Di Current Distortion VAR  =  1689.437,   THDv Voltage Distortion %  =     1.779
Ph Harmonic Active Power   =     1.000,   THDi Current Distortion %  =    99.018
PF  Power Factor           =      .707,   PF1 Fundamental P.F.       =      .995
P  kWh Consumed            =      .429,   P1 kWh Consumed            =      .429
P  kWh Generated           =      .000,   P1 kWh Generated           =      .000
QF kVARh Lagging           =      .031,   Q1 kVARh Lagging           =      .004
QF kVARh Leading           =      .330,   Q1 kVARh Leading           =      .031
QB kVARh Lagging           =      .000
QB kVARh Leading           =      .000
S  kVAh (Arithmetic)       =      .501,   S1 kVAh (A or V option)    =      .374
EOF
>_
```

Connected 0:01:11 | Auto detect | 19200 8-N-1 | SCROLL | CAPS | NUM | Capture | Print echo

**Figure 7 – Screenshot of HIP Meter Measured Values**

The right column lists the fundamental only quantities measured by the HIP meter. (The reason for 240 vars is unknown at this point, but not significant here). The fundamental-only digital filter effectively ‘smooths’ the current waveform, so that it appears continuous to the rms calculation which produces the correct average load current of 7.1 A rather than the multi-cycle rms value of 10 A. There are fundamental active power. The fundamental rms current and apparent power readings are correct, while the traditional measurements, based on an incorrect application of the rms concept, are incorrect both in the meter and in the TS33 test set. Nevertheless, most meters tested by the TS33 would pass the Sub-harmonic Test, as both would apply the rms calculation incorrectly. However, this is hidden from view, and there is no obvious reason to doubt either the TS33 or the meter. The TS33 in this respect is no different to any other meter test set used by the author.

## 7. Conclusion

This study can draw several conclusions:

1. The Sub-harmonic Test has nothing to do with sub-harmonics – there are none. The sub-harmonics reported by the four-cycle FFT are a mathematical side-effect of the process, not a measurement of a physical property of the load. In fact, the FFT does not report sub-harmonics of 60 Hz – it reports integer harmonics of 15 Hz, which is the fundamental frequency of its four cycle window. They only appear to be sub-harmonics if one incorrectly assumes that the FFT views 60 Hz as the fundamental.

2. An rms calculation that violates the consistency assumption can be proven to produce incorrect results. Coincidentally this error is identical to that introduced by the four-cycle FFT as fictitious sub-harmonics. This reinforces the belief that both methods are producing correct measurements, while in fact both are incorrect.

3. The digital filter used in the Vision HIP meter effectively smooths or averages the current waveform so that it appears to be continuous, and this meets the criteria for accurate multi-cycle rms measurements. This means that fundamental-only filter-based digital meters not only eliminate metering errors caused by harmonics, but inherently eliminate the multi-cycle rms measurement error discussed in this paper.

4. The multi-cycle rms calculation error affects both meters and meter test equipment. This issue needs to be exposed to organizations responsible for meter specifications and reference standard manufacturers.

The sub-harmonic test is appropriate as a Type Test, as it covers the meter’s ability to cater for rapidly switched loads, which are a growing load type in an era of inverters. The term ‘sub-harmonic test’ should be replaced by ‘integral cycle control test’ or ‘burst control test’ and its requirements spelled out in more detail in meter specifications. This will ensure that both meter and reference standard manufacturers are aware of the problems discussed here, and that we no longer install revenue meters that can produce VA measurement errors of 41% for customers with integral cycle load control.