Math II Independent and Dependent Conditional Events

Scrabble has 100 tiles, 98 of which are letters and two of which are blank. The numbers of tiles of each letter are shown in the diagram

Find the probability of the following events:

1. Choosing an "S" tile and keeping it, then choosing another "S" tile.

dependent

$P(S \cap S) = P(S) \cdot P(S|S)$

$P(S \cap S) = \frac{4}{100} \cdot \frac{3}{99} = 0.0012 \rightarrow 0.12\%$

2. Choosing a vowel tile and keeping it, then choosing a blank tile.

dependent

$P(V \cap B) = P(V) \cdot P(B|V)$

$P(V \cap B) = \frac{42}{100} \cdot \frac{2}{99} = 0.0085 \rightarrow 0.85\%$

3. Choosing a vowel tile and keeping it, then choosing another vowel tile and keeping then choosing another vowel.

dependent

$P(V \cap V) = \frac{42}{100} \cdot \frac{41}{99} \rightarrow 0.1739 \rightarrow 17.39\%$

A	9	H	2	O	8	V	2
B	2	I	9	P	2	W	2
C	2	J	1	Q	1	X	1
D	4	K	1	R	6	Y	2
E	12	L	4	S	4	Z	1
F	2	M	2	T	6	Blank	2
G	3	N	6	U	4		

A number is selected, at random, from the set $\{1,2,3,4,5,6,7,8\}$. Find:

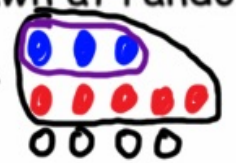
a) $P(odd) = \frac{4}{8} = 0.50 \rightarrow 50\%$

b) $P(prime \mid odd) = \frac{4}{4} = 1 \rightarrow 100\%$

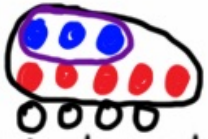
A box contains three blue marbles, five red marbles, and four white marbles. If one marble is drawn at random, find:

a) $P(blue \mid not\ white)$

b) $P(not\ red \mid not\ white)$



$\rightarrow \frac{3}{8} \rightarrow 37.5\%$



$\rightarrow \frac{3}{8} \rightarrow 37.5\%$

A number is selected randomly from a container containing all the integers from 10 to 50. Find:

a) $P(even \mid greater\ than\ 40)$

b) $P(greater\ than\ 40 \mid even)$

c) $P(prime \mid between\ 20\ and\ 40)$

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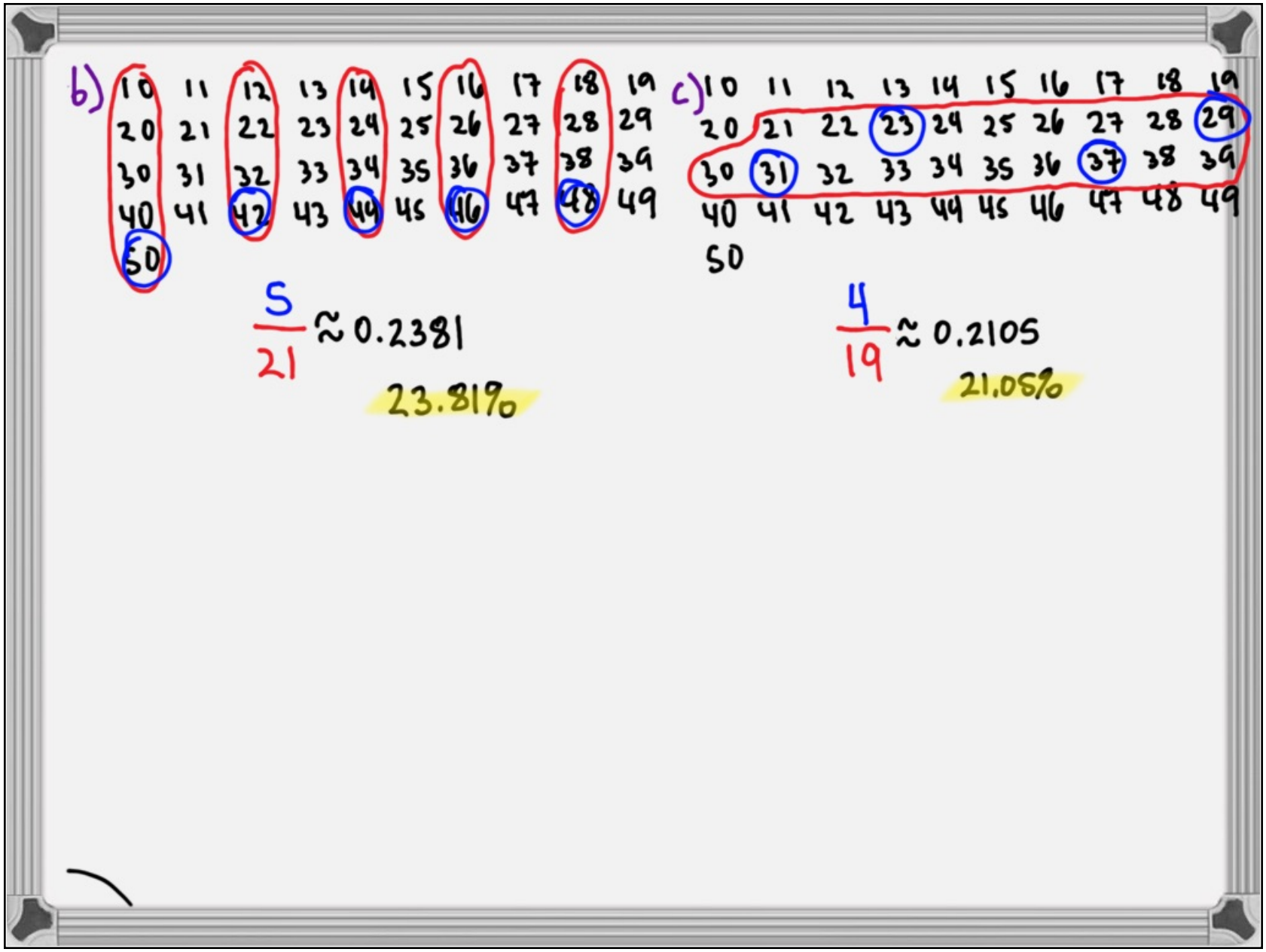
a)

10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50									

$\frac{5}{10} = 0.5 = 50\%$ $\frac{1}{10} \cdot \frac{5}{10}$


b)
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29
30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49
50
5/21 ≈ 0.2381
23.81%
c)
10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29
30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49
50
4/19 ≈ 0.2105
21.05%

Events *A* and *B* are independent. Find the indicated probability.

. $P(A) = 0.3$
$P(B) = 0.9$
$P(A \text{ and } B) = \underline{?}$

$P(A \text{ and } B) = P(A) \cdot P(B)$
$P(A \text{ and } B) = (0.3)(0.9)$
$P(A \text{ and } B) = 0.27$

$P(A) = \underline{?}$
$P(B) = 0.3$
$P(A \text{ and } B) = 0.06$

$P(A \text{ and } B) = P(A) \cdot P(B)$
$\frac{0.06}{0.3} = \frac{P(A) \cdot 0.3}{0.3}$
$P(A) = 0.2$

$P(A) = 0.75$
$P(B) = \underline{?}$
$P(A \text{ and } B) = 0.15$

$P(A \text{ and } B) = P(A) \cdot P(B)$
$\frac{0.15}{0.75} = \frac{0.75 \cdot P(B)}{0.75}$
$P(B) = 0.2$

Events *A* and *B* are dependent. Find the indicated probability.

$P(A) = 0.1$
$P(B \mid A) = 0.8$
$P(A \text{ and } B) = \underline{?}$

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
$P(A \text{ and } B) = (0.1)(0.8)$
$P(A \text{ and } B) = 0.08$

$P(A) = \underline{?}$
$P(B \mid A) = 0.5$
$P(A \text{ and } B) = 0.25$

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
$\frac{0.25}{0.5} = \frac{P(A) \cdot 0.5}{0.5}$
$P(A) = 0.5$

$P(A) = 0.9$
$P(B \mid A) = \underline{?}$
$P(A \text{ and } B) = 0.54$

$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$
$\frac{0.54}{0.9} = \frac{0.9 \cdot P(B \mid A)}{0.9}$
$P(B \mid A) = 0.6$

Three friends are taking an English class that has a summer reading list. Each student is required to read one book from the list, which contains 3 biographies, 10 classics, and 5 historical novels.

- Find the probability that the first friend chooses a biography, the second friend chooses a classic, and the third friend chooses a historical novel.
- Find the probability that the three friends each choose a different classic.

P(Biography and Classic and Historical) → $P(B) \cdot P(C) \cdot P(H) \rightarrow \frac{3}{18} \cdot \frac{10}{18} \cdot \frac{5}{18} =$ 0.0257 → 2.57%

$\frac{10}{18} \cdot \frac{9}{17} \cdot \frac{8}{16} = 0.1471 \rightarrow 14.71\%$

COSTUMES You and four of your friends go to the same store at different times to buy costumes for a costume party. There are 20 different costumes at the store, and the store has at least five duplicates of each costume. Find the probability that all five of you choose different costumes.

20 different costumes 5 of each kind

you	friend one	friend two	friend three	friend four
$\frac{100}{100}$	$\cdot \frac{95}{100}$	$\cdot \frac{90}{100}$	$\cdot \frac{85}{100}$	$\cdot \frac{80}{100} =$

0.5814 → 58.14%