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Methods For Reducing Vibration (Intro to Vibration Control)

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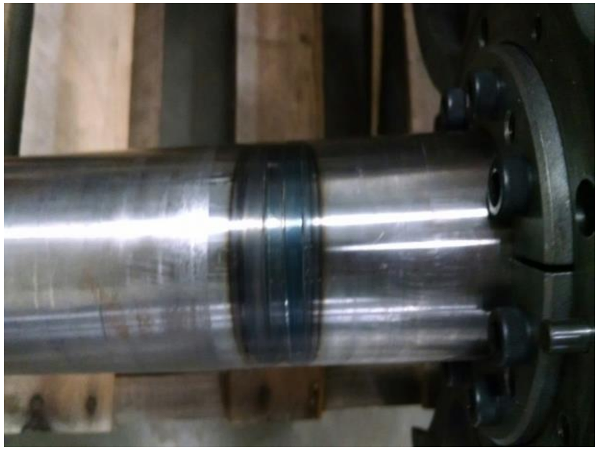
Vibration & Machine Reliability

Excessive Vibration leads to Higher Cyclic Stress that shortens Fatigue Life & adversely affects Reliability.



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Basic Vibration Response

$$x = F_d/k[\{1-(f_d/f_n)^2\}^2 + \{2\zeta(f_d/f_n)\}^2]^{1/2}$$

Where: F_d = magnitude of dynamic force (lbs)

k = stiffness (lb/in)

f_d = frequency of dynamic force (Hz)

f_n = natural frequency (Hz)

ζ = damping (percentage of critical)

$f_n = (1/2\pi)(k/m)^{1/2}(1- \zeta^2)^{1/2}$

m = mass (lb-sec^2/in)

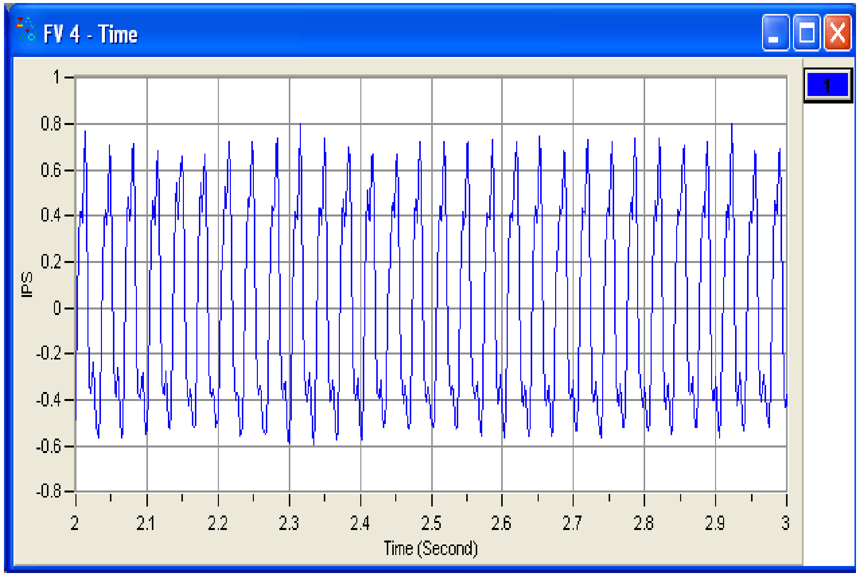
Stiffness & Mass are prominent in the Basic Vibration Response equation.

Basic Vibration Response

$$x = F(\theta)/k[\{1-(f_d/f_n)^2\}^2 + \{2\zeta(f_d/f_n)\}^2]^{1/2}$$

Vibration can be controlled by:

1) Changing magnitude and/or frequency of Force
2) Isolation of Dynamic Force
3) Increasing/Decreasing Stiffness
4) Increasing/Decreasing Mass
5) Adding Damping
6) DVA (Special use of Stiffness/Mass/Damping)



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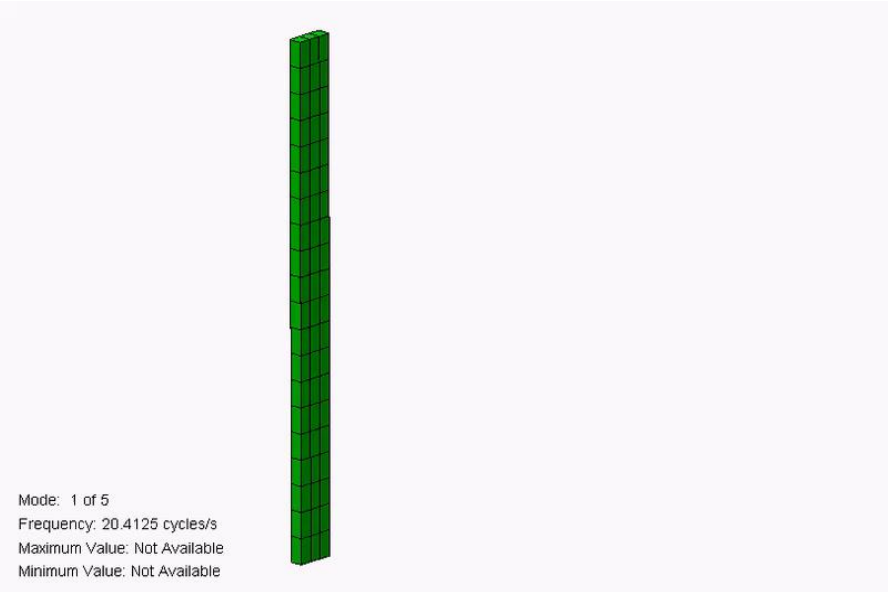
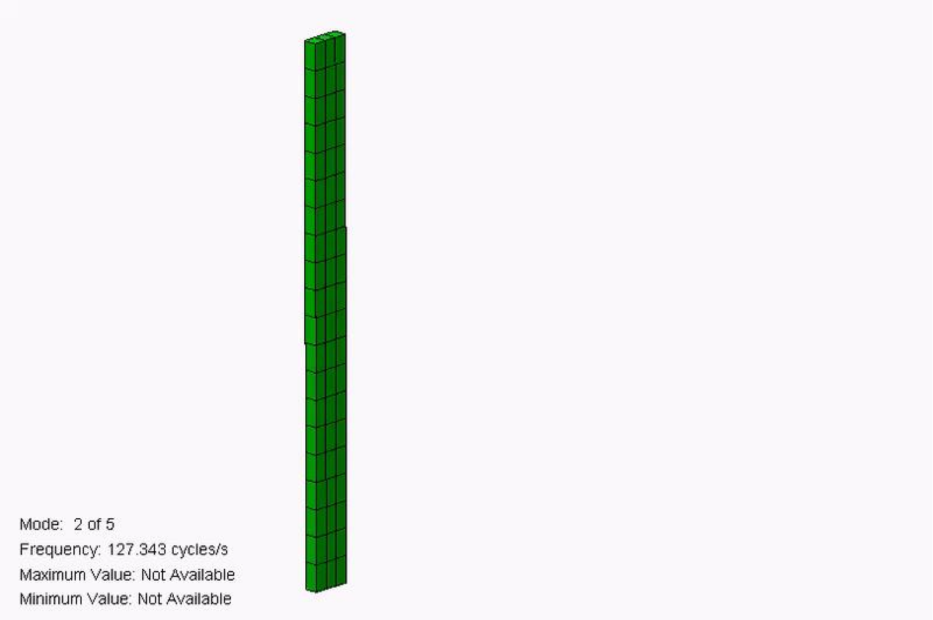
Vibration Control – Modal Participation (mdof)





- Typical Structural Mechanical Systems have more than 1 natural frequency to be considered.
- Each Natural Frequency is distinguished by a unique (orthogonal) Mode Shape.

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Vibration Control – Modal Participation (mdof)

Define $AF = 1/ [\{1-(f_d/f_n)^2\}^2 + \{2\zeta(f_d/f_n)\}^2]^{1/2}$

Where AF = amplification factor

MPF = modal participation factor

$$x = \sum_i \sum_n F_i(\theta)\ (AF_n)\ (MPF_n)/k$$

A detailed Discussion of mdof response is beyond this Presentation. However, stiffness and mass still dominates this equation.

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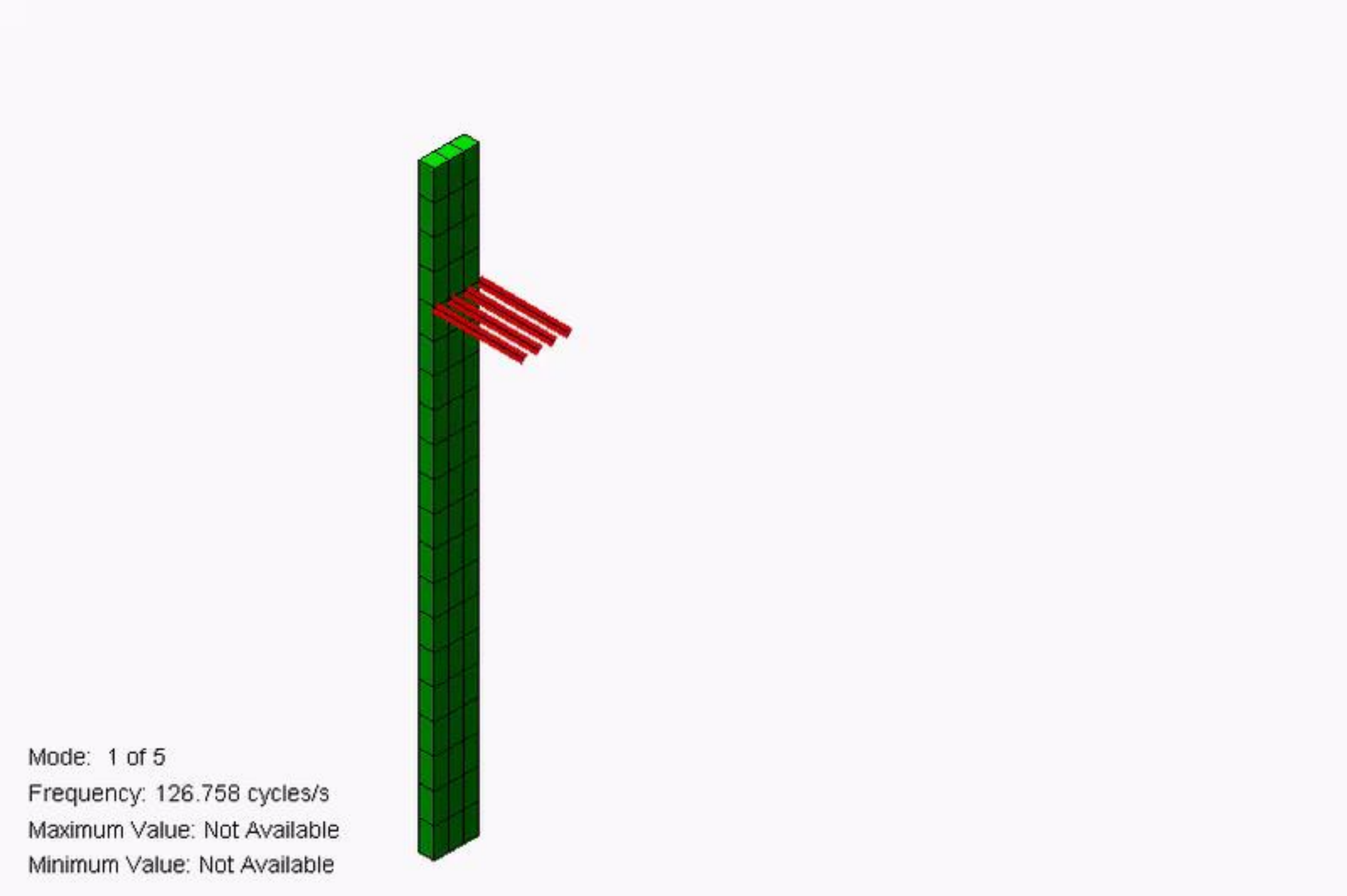

Vibration Control – Modal Participation

- Stiffness affects Mode 1; No effect on Mode 2

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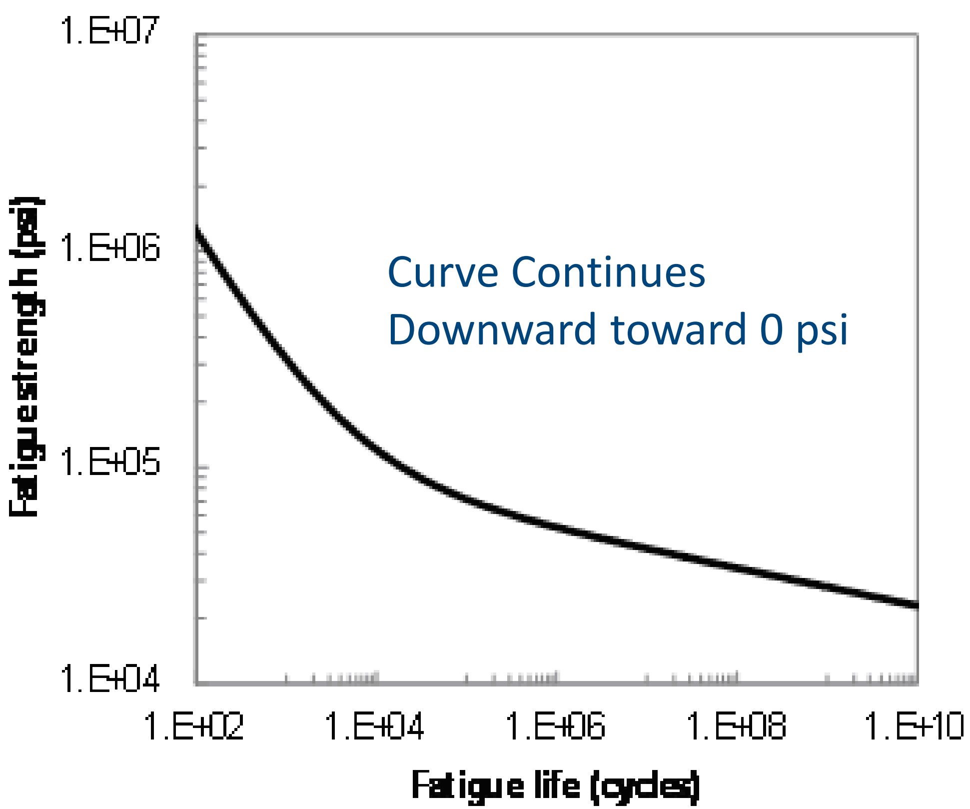
Why is it Important to Minimize Vibration?

Fatigue Life Curve
Notch-Strain Method
By SAE



$$(\Delta\sigma/2) = (\sigma_f`)(2N)^b + (\varepsilon_f`)E(2N)^c$$

Where σ_f` is the fatigue strength coefficient, b is the fatigue strength exponent, $\varepsilon_f$`is the fatigue ductility coefficient, and c is the fatigue ductility exponent.

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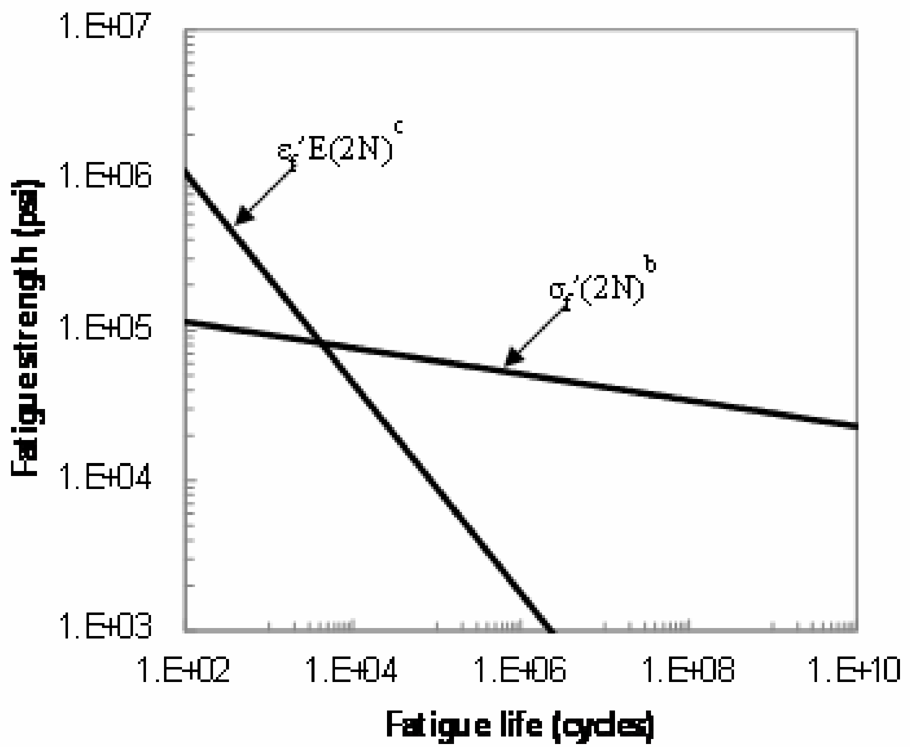
Fatigue Life Curve Components



Fatigue life curves, developed from constant amplitude fatigue life tests, can be described by a low-cycle fatigue component and a high-cycle fatigue component. For high cycle fatigue,>1E06 cycles, the equation simplifies to

$$(\Delta\sigma/2) = (\sigma_f` - \sigma_m)(2N)^b$$

Effect of Vibration on Fatigue Life

Consider a Fan that at one vibration level is subjected to 20,000 psi cyclic stress, with constant mean stress of 50,000psi, operating @ 1,200 rpm. (cycles = 1.73E06 per day)

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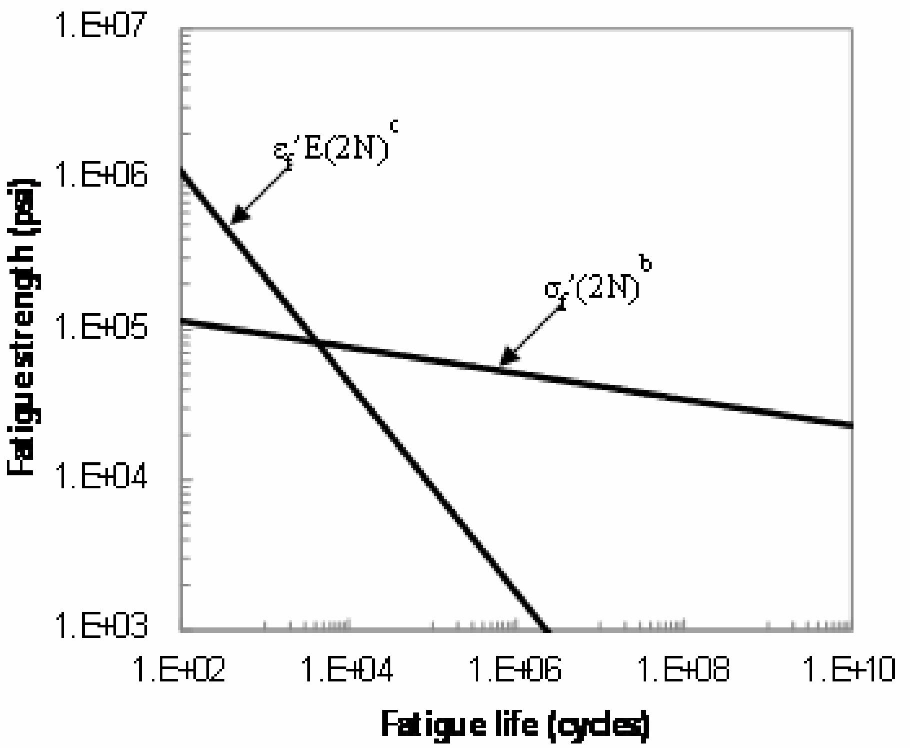
Effect of Vibration on Fatigue Life



Note: Stress is measured or calculated @ point of max stress concentration. It is not the nominal stress

High Cycle Fatigue life for 20,000 psi; 2N ~ 1E09 cycles = 1.5 years

Reduce Vibration & Stress by 33% to 15,000 psi; 2N ~ 3E10 cycles = 45 years

A 33% reduction in Vibration results in an 30x increase in machine life!!!!!!

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Damped Amplification Factor
Why is Natural Frequency Important?

$AF = 1/[\{1-(f_d/f_n)^2\}^2 + (2\zeta f_d/f_n)^2]^{1/2}$

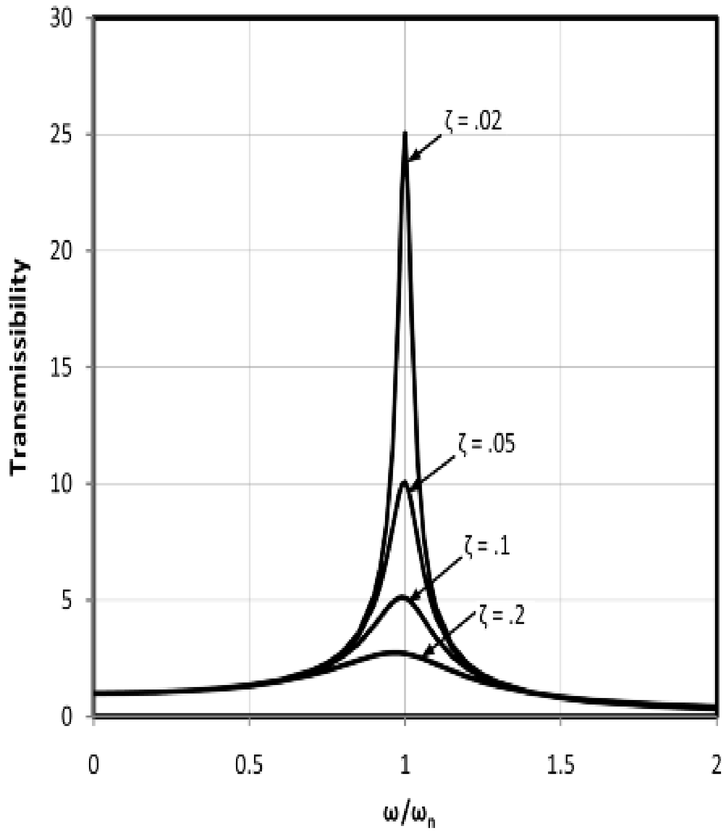
When the frequency of dynamic force (f_d) approaches the natural frequency (resonance), damping controls the amount of vibration.

For $\zeta = 0.02$; AF = 25

Vibration & Stress amplified by 25:1;

Normal Stress = 1,000 psi becomes 25,000 psi

Fatigue Severity 25ksi >>> 1 ksi



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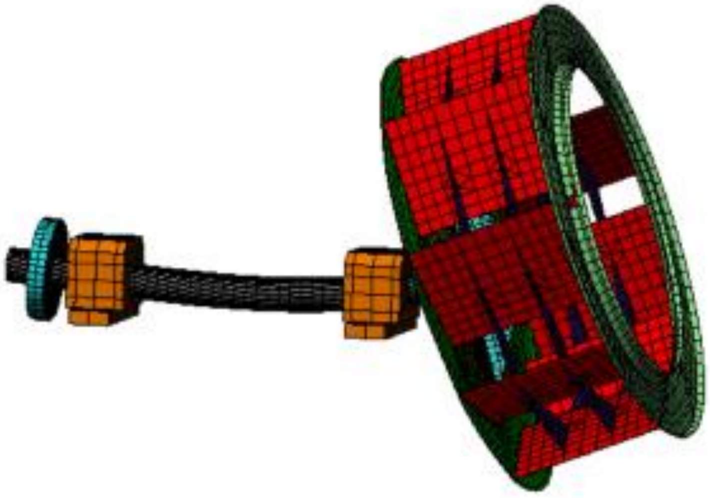
Resonance Amplification Effects on Machine Reliability

- The natural frequencies of two identical mechanical systems can differ slightly due to variations in material properties, manufacturing process (tightness of bolts, consistency of weldments).
- If 2 pieces of equipment operate at the same frequency, but have slightly different natural frequencies, the resonant amplification of vibration (and stress) will differ.

Example of Fatigue Sensitivity



Mode: 3 of 20
Frequency: 24.0175 cycles/s
Maximum Value: Not Available
Minimum Value: Not Available

The natural frequency of the fan wheel will increase during operation due to centrifugal stress stiffening. For the SWSI fan of this example, stress stiffening resulted in the natural frequency increasing from 24.0 Hz at rest to 29.85 Hz during operation. The operating frequency ratio (fo/fn) for this fan is:

$$r = 29.67/29.85 = 0.994$$

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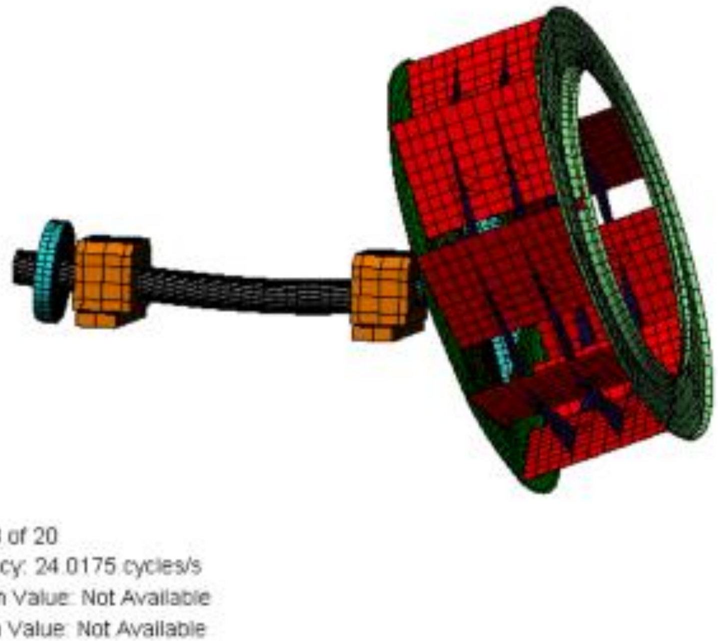
Example of Fatigue Sensitivity



The damping available in fan rotors is typically very low. A damping constant of 0.01 is not unusual for the wheel wobble -shaft flexural mode. This damping factor and frequency ratio, yields an amplification factor of 49.4. Consider the fact that the alternating stresses in the fan wheel would have been only +/-530 psi if the resonant condition were not present. The resonant amplification results in a cyclic stress increasing to +/-26,200 psi.

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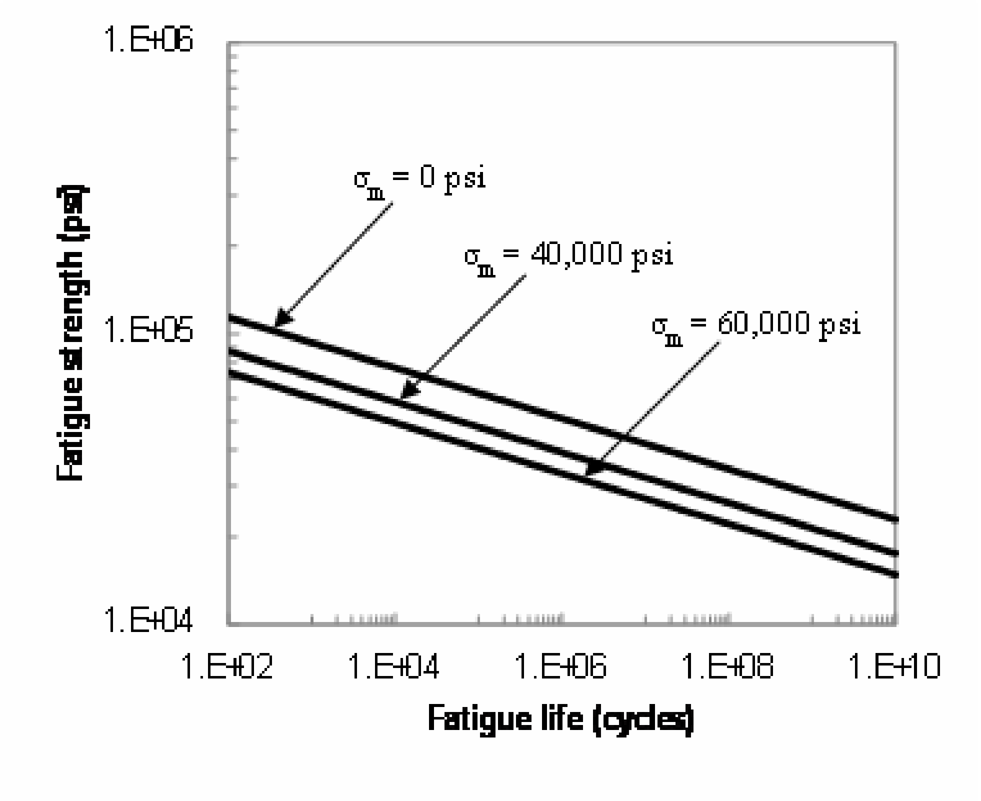
Fatigue Life Estimation



Both fan wheels are constructed of same material and operate under a continual mean stress of 40,000 psi. The high-cycle fatigue life equation for this material is defined by:

$$(\Delta\sigma/2) = (130 - \sigma_m)(2N)^{-0.087}$$

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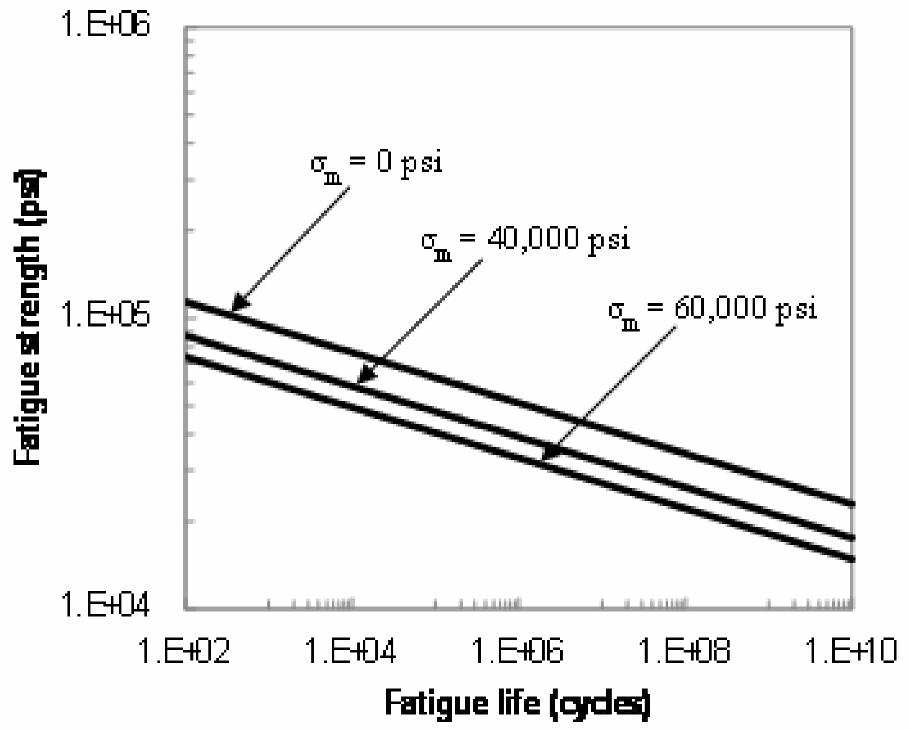
Fatigue Life Estimation

Substituting the amplified cyclic stress of 26.2 ksi for the first fan into the fatigue life equation provides an estimated life of 100 million cycles to failure. Since the fan operates at 1,780 rpm, the stress is applied 29.67 cycles per second. If the fan were operating continuously, which is not unusual for most industrial applications, failure would occur in 39 days.



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Fatigue Life Estimation

Now consider the fact that, although theoretically identical, the stiffness of the second fan differs slightly from the first. The small difference in stiffness results in the at-rest natural frequency being 24.4 Hz. This is only 2 percent different from the natural frequency of the first fan. The stress-stiffened operating natural frequency of the second wheel is 30.43 Hz. The frequency ratio for the second fan is 0.975, whereas it was 0.994 for the first fan. The frequency ratio for the second fan provides an amplification factor of 18.8, which is quite a reduction from the amplification factor of 49.4 for the first fan.

Fatigue Life Estimation

The operating cyclic stress in the second fan would be around +/- 10,000 psi or 10 ksi, compared to 26.2 ksi for Wheel 1. The predicted fatigue life of Wheel 2 would be over 1 trillion cycles. This fan could operate for 1000 years before fatigue failure caused by normal unbalance would occur.

This surely could lead to an erroneous deduction that the failures are not the result of a design deficiency, but are caused by something else in the system.

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Bearing L10 Life

The American Bearing Manufacturers Association (ABMA),defines the Basic Rating Life, L10 as the bearing life associated with a 90% reliability when operating under conventional conditions.
i.e. after a stated amount of time 90% of a group of identical bearings will not yet have developed metal fatigue.

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Basic Vibration Response

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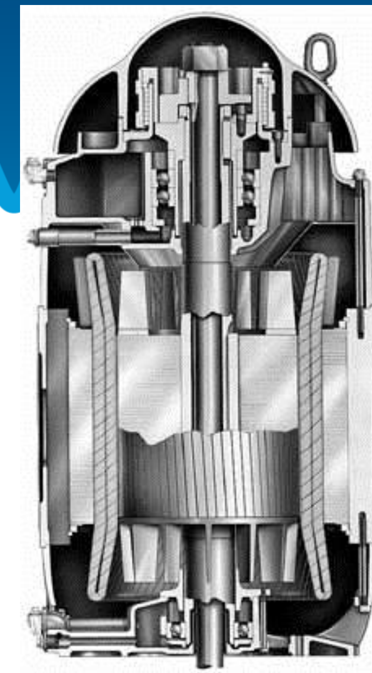
Vibration can be controlled by:

1) Changing magnitude and/or Frequency of Force
2) Isolation of Dynamic Force
3) Increasing/Decreasing Stiffness
4) Increasing/Decreasing Mass
5) Adding Damping
6) DVA

Case History: Vertical Pump



Dynamic Force is not always Unbalance or Misalignment.

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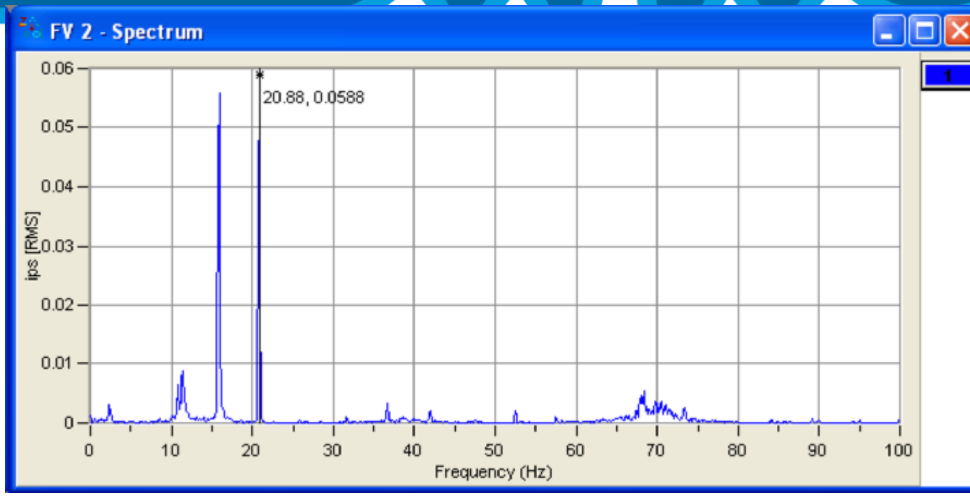
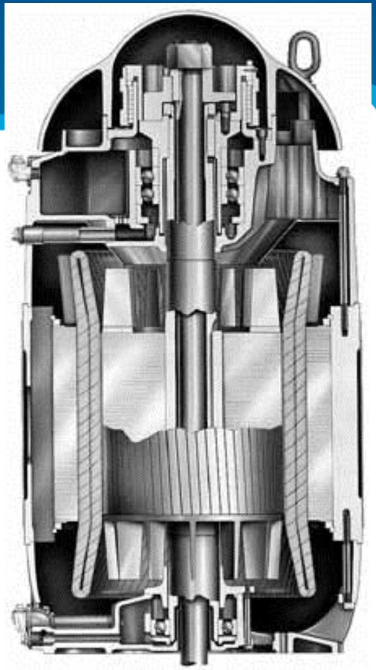
Case History: Vertical Pump



1800 rpm Pump Operates with VFD Control. At low flow (speed) settings, check valve not completely open. A rotating stall condition initiates. Vibration @ sub-harmonic frequency (~16 Hz) as well as operating speed (1250 rpm ~ 20.88 Hz).

Case History: Vertical Pump



Problem: Sub-harmonic excites internal shaft within hollow motor shaft, causing impacting between both shafts.

Solution: Program VFD to prohibit operating in the range that produces rotating stall. (i.e. Controlling the Frequency of Dynamic Force)

Sayer's Theory on Variable Frequency Drives

"Variable Frequency Drives (VFD) were invented so that the speed of a Machine could be tuned to operate at it's Principal and most sensitive Natural Frequency."

However, a VFD can be used to tune a system away from Resonance. In some cases, VFD can be your Friend.

Mining Tower with Log Washer

A Log Washer is a piece of Process Equipment used in the mining industry. It is used to remove clay and other foreign material from stone.

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Case History: Mining Log Washer

Problem: The rotating speed of the paddle rotor was 65 rpm (1.08 Hz). Each row of paddles contained four paddles and all paddles were in line from front to back of the machine, the log washer produced pulsations 4 times per revolution of the rotor which was 4 x 1.08 ~ 4.3 Hz. This matched natural frequency of bldg and led to plant shutdown.

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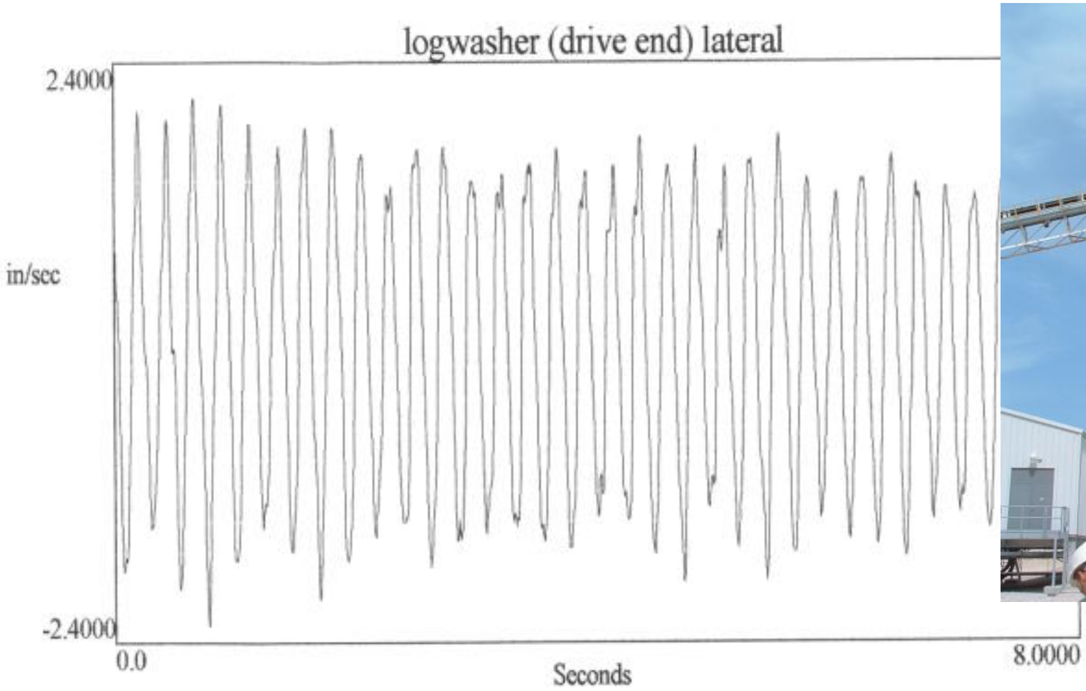
Case Study – Log Washer Tower



Structural vibration was dominated by response at 4.3 Hz, reaching magnitudes of 2.4 ips with the machine only lightly loaded.

Peak-Peak Displace = 2x2.4/2π(4.3) ~ 180 mils @ Machine

Peak-Peak Displace > 0.6 inches @ top of bldg

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Case History: Mining Log Washer

Solution: Stagger Paddle Arrangement (i.e. Controlling the Frequency & Magnitude of Dynamic Force)

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Case History: Mining Log Washer

The Cost required to modify the structure to increase natural frequency outside of resonant excitation range was prohibitive.

Alternate solution was to replace rotors. A new design was installed whereby each row of paddles was indexed by approx 3 degrees, thus repeating the pattern once every 30 rows. This resulted in an increase in the pulsation frequency from 4.3 Hz to 30x4.3 Hz = 129 Hz. Also, since the pulsation was produced by a single blade instead of 30 blades, its magnitude was greatly reduced.

The vibration was almost completely eliminated.

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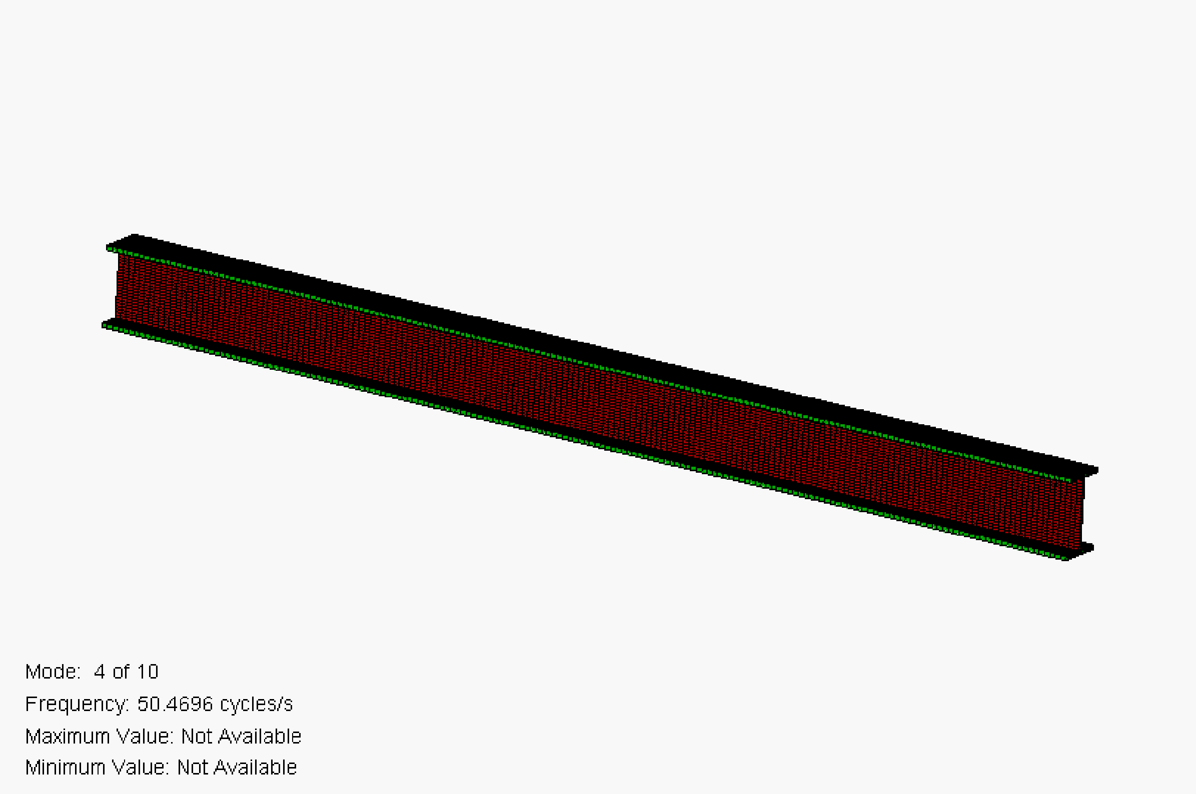
Reduce Vibration by Changing Stiffness



Stiffness depends on the 1) direction of motion (deformation/vibration), 2) material, and 3) geometry.

Given a simple supported beam, allowed to grow axially at one end.

Flex Stiffness = k = $48EI/L^3$

E = Modulus of Elasticity is a Material Property

I = Moment of Inertia is a Geometric Property

L = Length between Bearings is a Geometric Property

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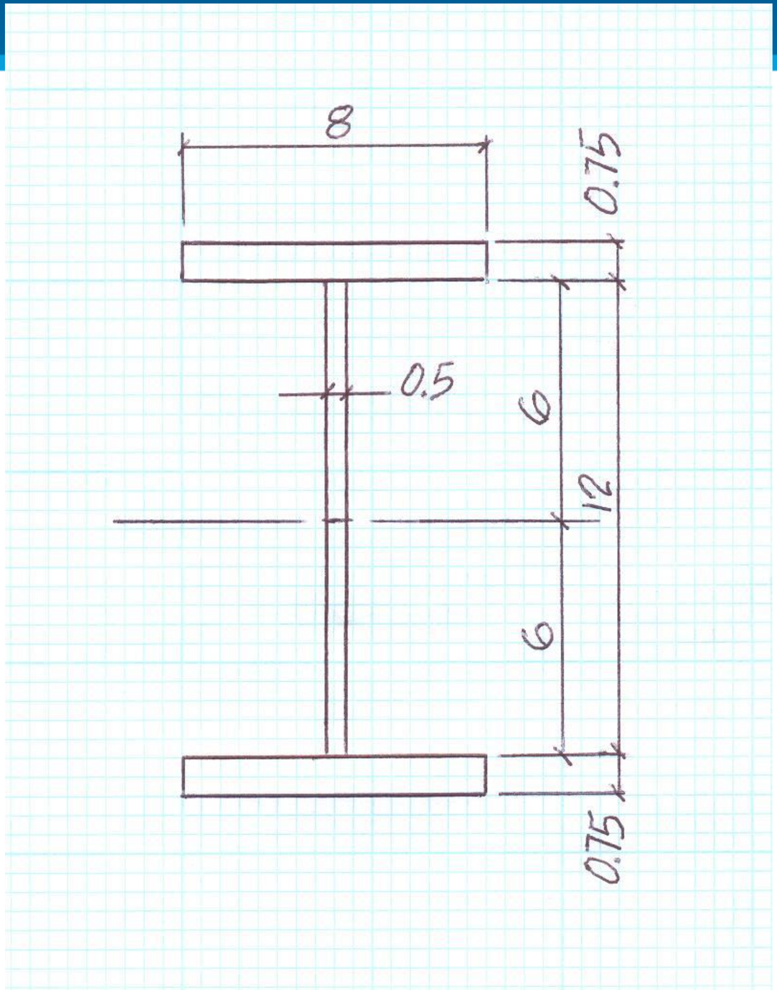
Flexural Mode (Example)



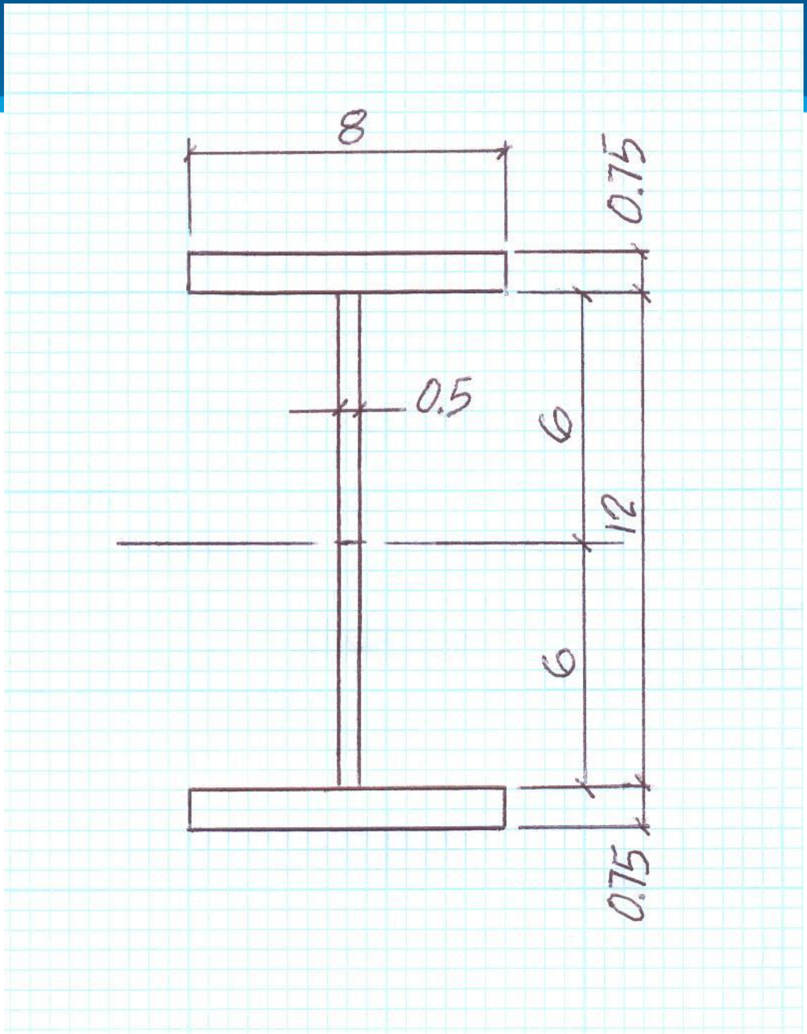
Flexural Stiffness = k = $48EI/L^3$

E (steel) = 29E06 psi

L = 180”

What is I?

Geometric Property: Moment of Inertia



$I = \sum(I + Ad^2)$

$I = bh^3/12$

Top & Bottom Flanges:

$I = 2x(8)(.75)^3/12 = 0.56\ in^4$

Negligible

$Ad^2 = 2x(8)(.75)(6.375)^2 = 488\ in^4$

Web:

$I = (0.5)(12)^3/12 = 72\ in^4$

$Ad^2 = 0.0$

Total Beam:

$I = \sum(I + Ad^2) = 560\ in^4$

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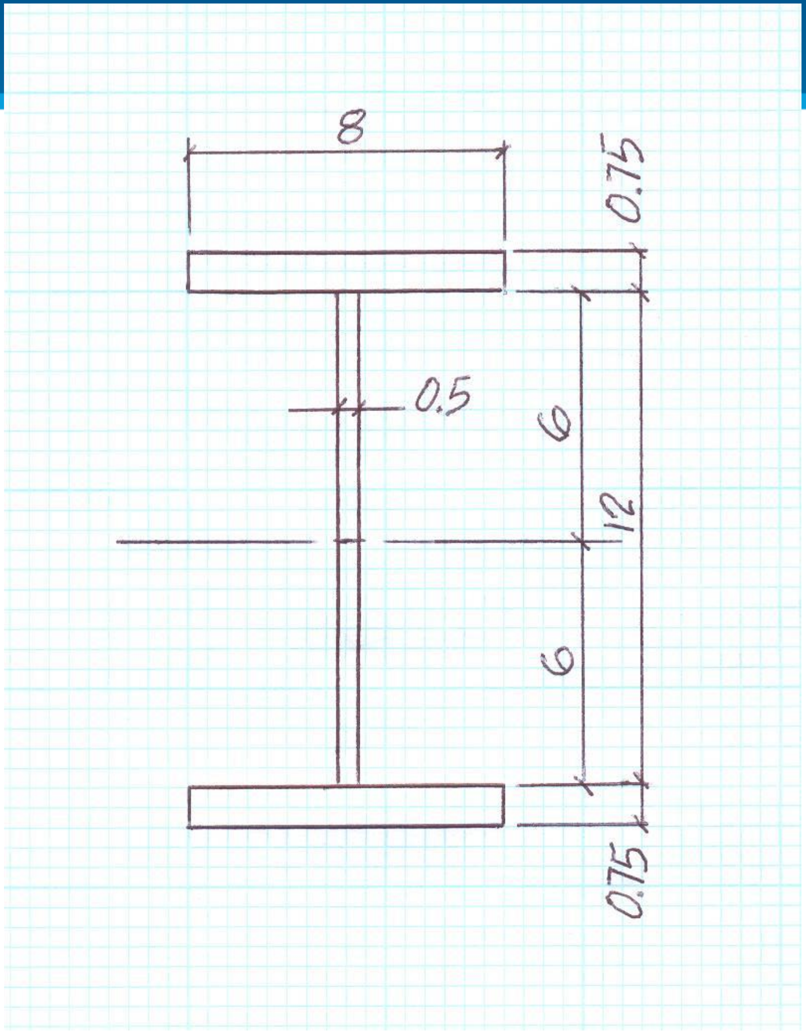
Stiffness, Mass, & Natural Frequency



$$k = 48EI/L^3 =$$

$$k = 48(29{,}000{,}000)(560)/(180)^3 =$$

133,660 lb/in

$$A = 2(.75)(8) + (.50)(12) = 18 \text{ in}^2$$

$$m = 18(180)(0.283)/386.4 =$$

2.37 lb-sec^2/in

m/2 = 1.185 lb-sec^2/in

$$f_n = (1/2\pi)[133{,}660/1.185]^{1/2} =$$

53.5 Hz

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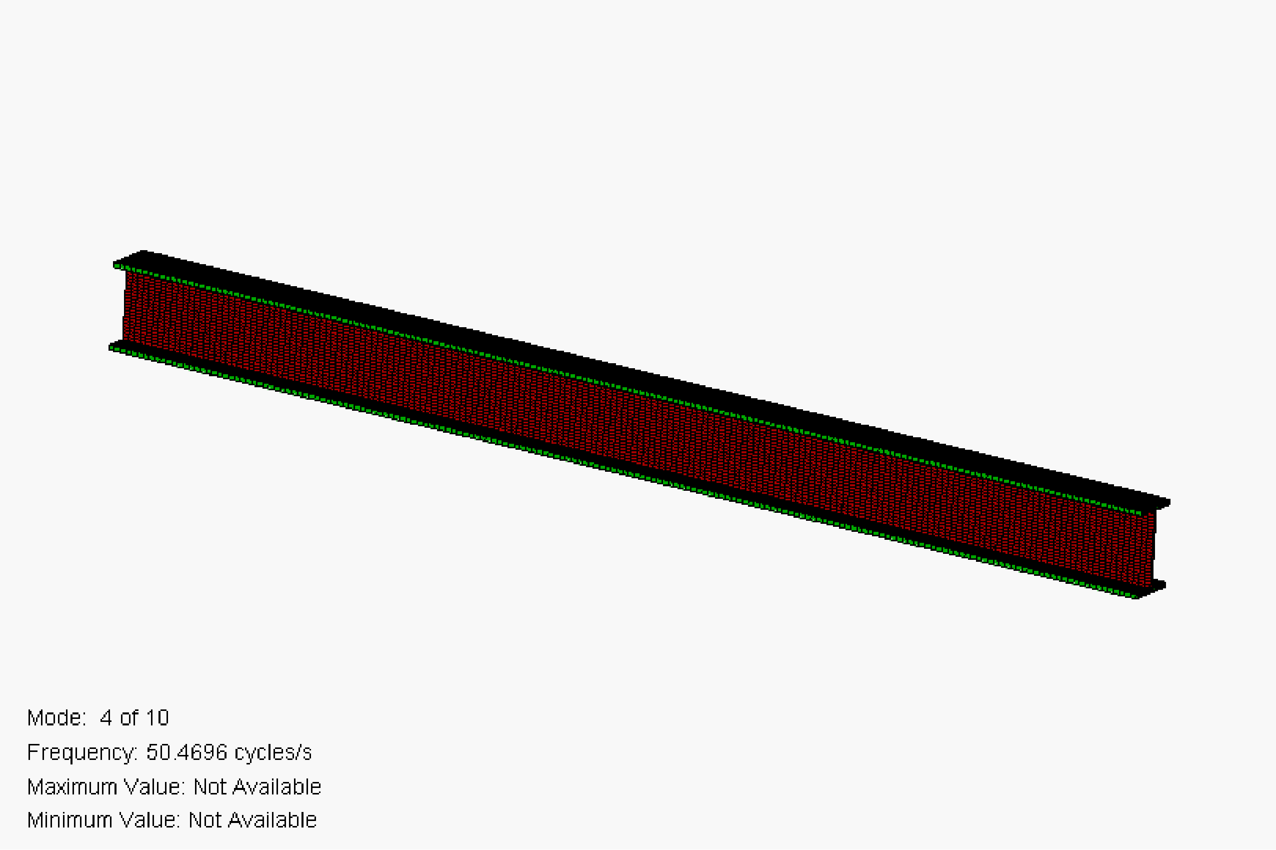
Natural Frequency (Formula vs. FEA)



Formula assumes pinned support @ CL of beam cross section.

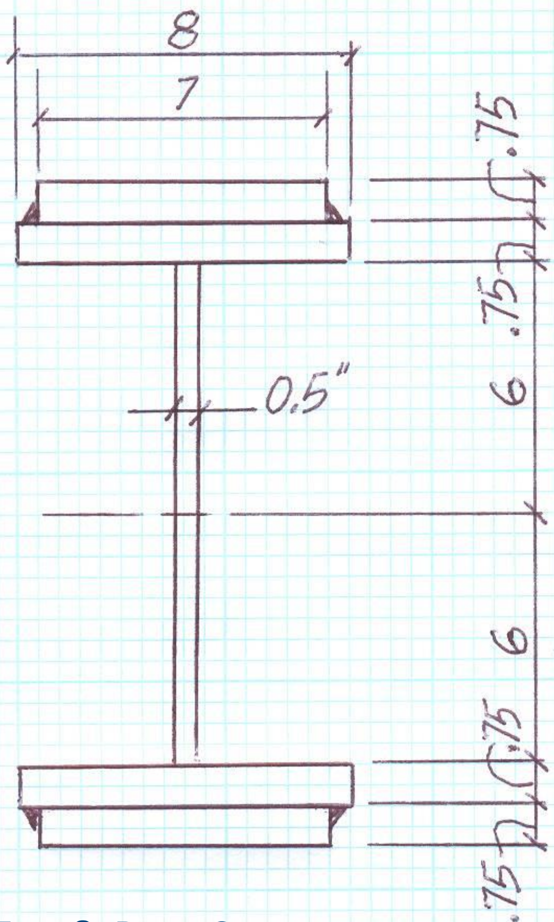
FEA assumes ledge support along bottom flange of beam.

$$f_n = (1/2\pi)[133{,}660/1.185]^{1/2} = 53.5 \text{ Hz vs.}$$

$$f_n \text{ (FEA)} = 50.5 \text{ Hz}$$

Is m/2 effective mass of beam absolutely correct? No, it is an approximation!

Geometric Property: Moment of Inertia



Add Top & Bott Cover Plates (3/4")

$$I = \sum(I + Ad^2)$$

$$I = bh^3/12$$

Top & Bottom Flanges:

$$Ad^2 = 2x(8)(.75)(6.375)^2$$

$$+2x(7)(.75)(7.125)^{2=}\ 1{,}021\ in^4$$

Web:

$$I = (0.5)(12)^3/12 = 72\ in^4$$

$$Ad^2 = 0.0$$

$$I = \sum(I + Ad^2) = 1{,}093\ in^4$$

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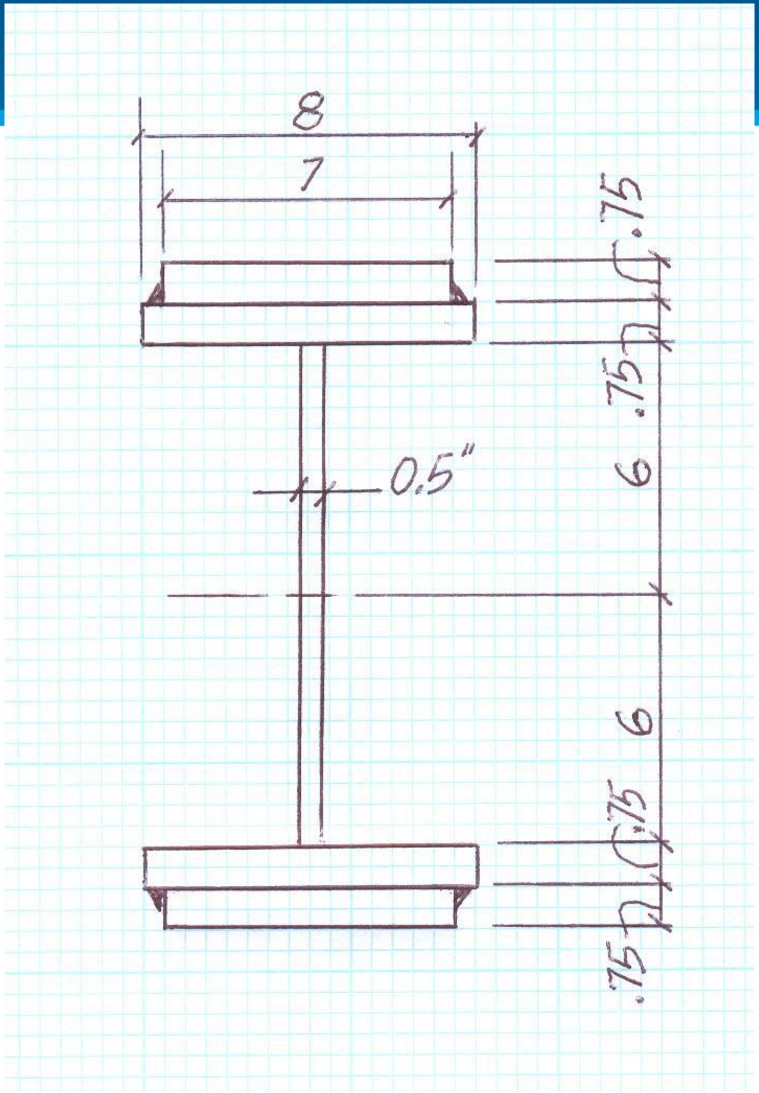
Natural Frequency (w/Cover Plates)



$k = 48EI/L^3 =$

$k = 48(29{,}000{,}000)(1093)/(180)^3 =$

260,880 lb/in

$A = 2(.75)(7) + 18 = 28.5\ in^2$

$m = 28.5(180)(0.283)/386.4 =$

3.75 lb-sec^2/in

m/2 = 1.875 lb-sec^2/in

$f_n = (1/2\pi)[260{,}880/1.875]^{1/2} =$

59.4 Hz vs 53.5 Hz

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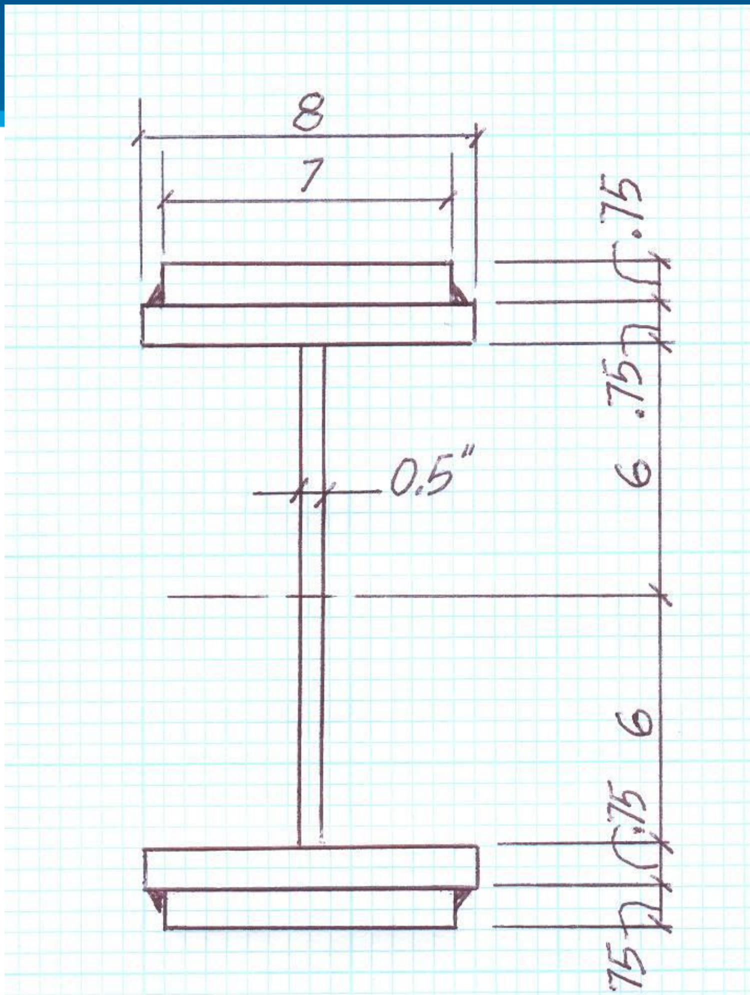
Comparison of Results



Mod adds Stiffness & Mass

The Flange Stiffener Plates

Resulted in a 1.95X increase in Stiffness

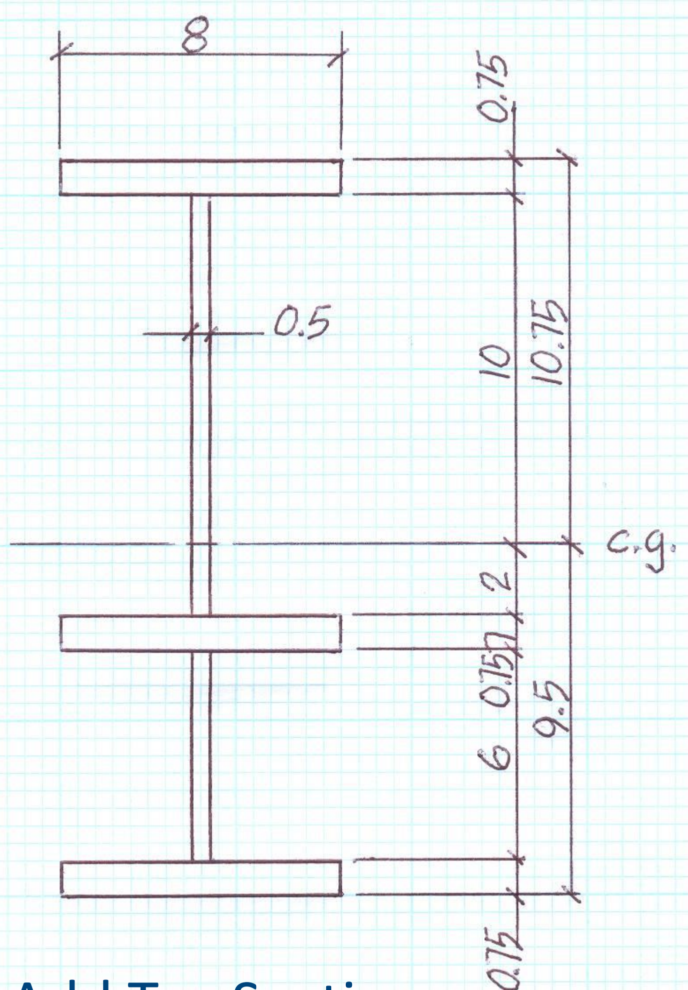
k = 260,880/133,360 = 1.95

Natural frequency only increased by 59.4/53.5 = 1.11x.

Mass increased by 3.75/2.37 = 1.58

Nat Freq increase $[1.95/1.58]^{1/2} = 1.11x$

Geometric Property: Moment of Inertia



Add Tee Section

$$I = \sum(I + Ad^2) \text{ where } I = bh^3/12$$

Top & Bottom Flanges:

$$\sum Ad^2 = (8)(.75)(10.375)^2$$
$$+(8)(.75)(2.375)^2 + (8)(.75)(9.125)^2$$
$$= 1{,}179.3 \text{ in}^4$$

Webs:

$$\sum Ad^2 = (12)(.50)(4)^2$$
$$+(6)(.50)(5.75)^2 = 195.2 \text{ in}^4$$
$$\sum I = [(0.5)(12)^3 + (0.5)(6)^3]/12 =$$
$$81 \text{ in}^4$$
$$I = \sum(I + Ad^2) = 1{,}455.5 \text{ in}^4$$

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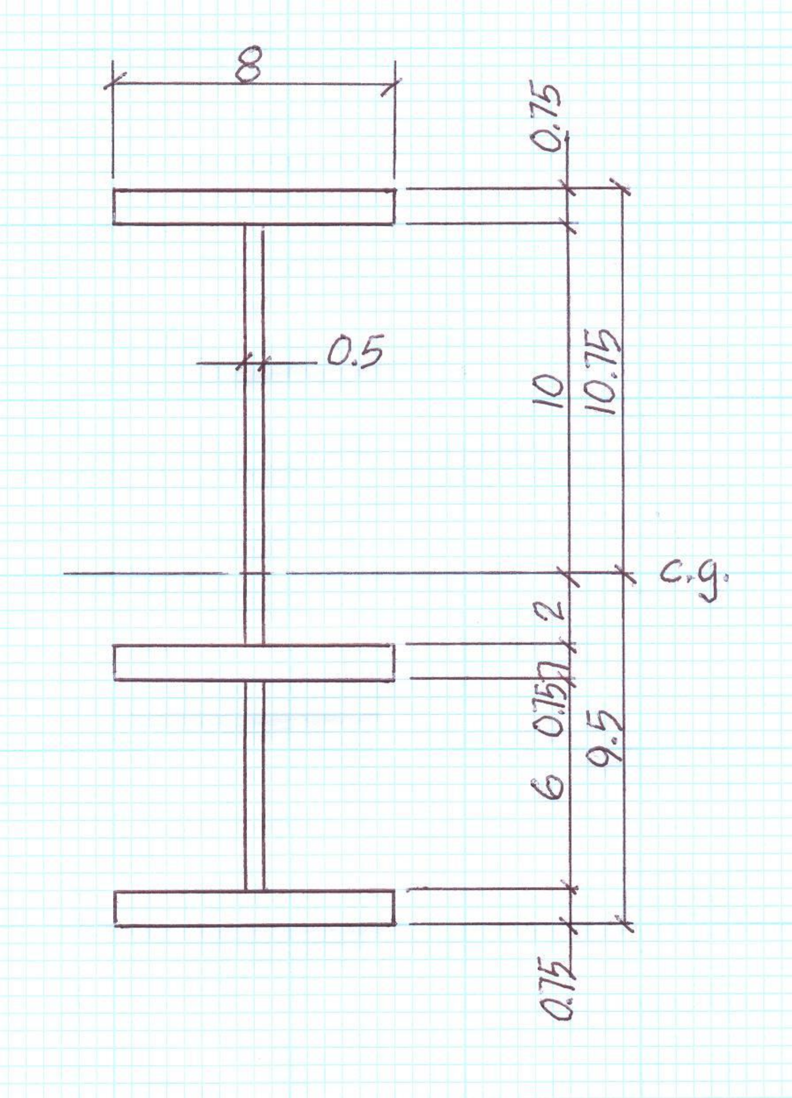
Natural Frequency (T-Section Mod)



$k = 48EI/L^3 =$

$48(29{,}000{,}000)(1455.5)/(180)^3 =$

347,400 lb/in

$A = (.75)(8) + (.50)(6) + 18 =$

$27.0\ in^2$

$m = 27.0(180)(0.283)/386.4 =$

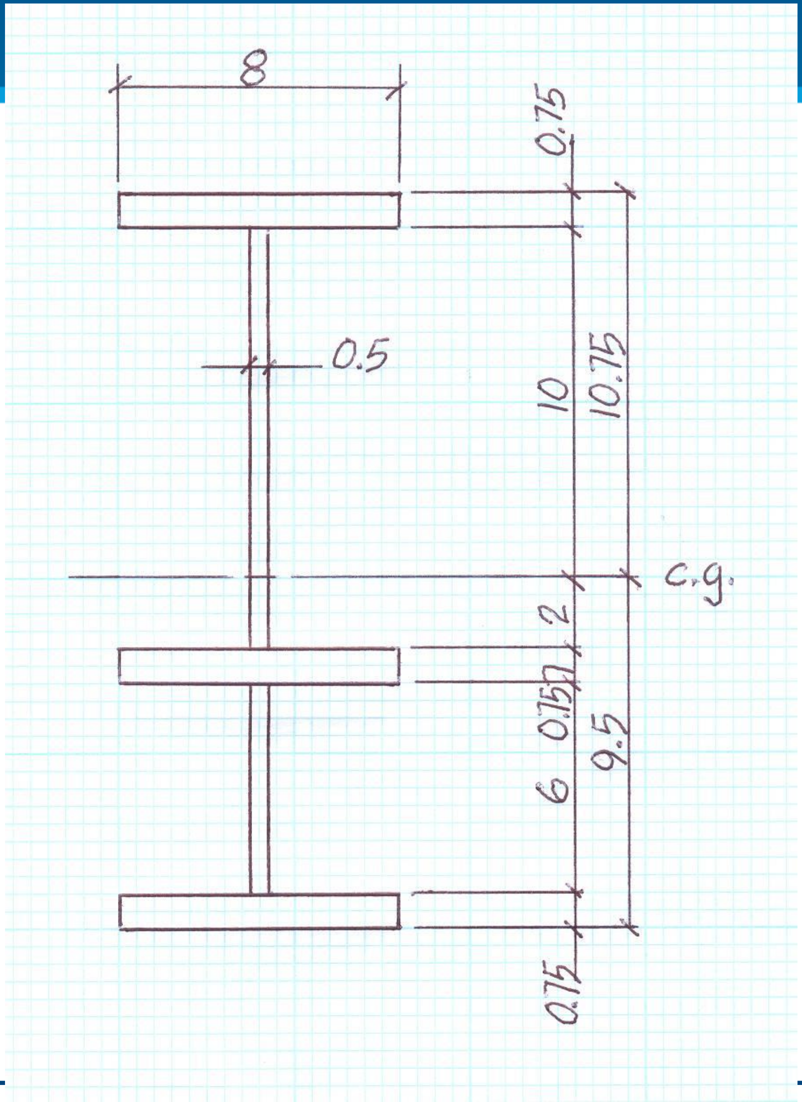
$3.56\ lb\text{-}sec^2/in$

$m/2 = 1.78\ lb\text{-}sec^2/in$

$f_n = (1/2\pi)[347{,}400/1.78]^{1/2} =$

70.3Hz vs 53.5 Hz

Comparison of Results (T-Section Mod)



The Flange Stiffener Plates

Resulted in a 2.60X increase in Stiffness

K = 347,400/133,360 = 2.60

Natural frequency only increased by 70.3/53.5 = 1.31x.

Mass increased by 3.56/2.37 = 1.50

Nat Freq increase $[2.60/1.50]^{1/2}$ = 1.31x

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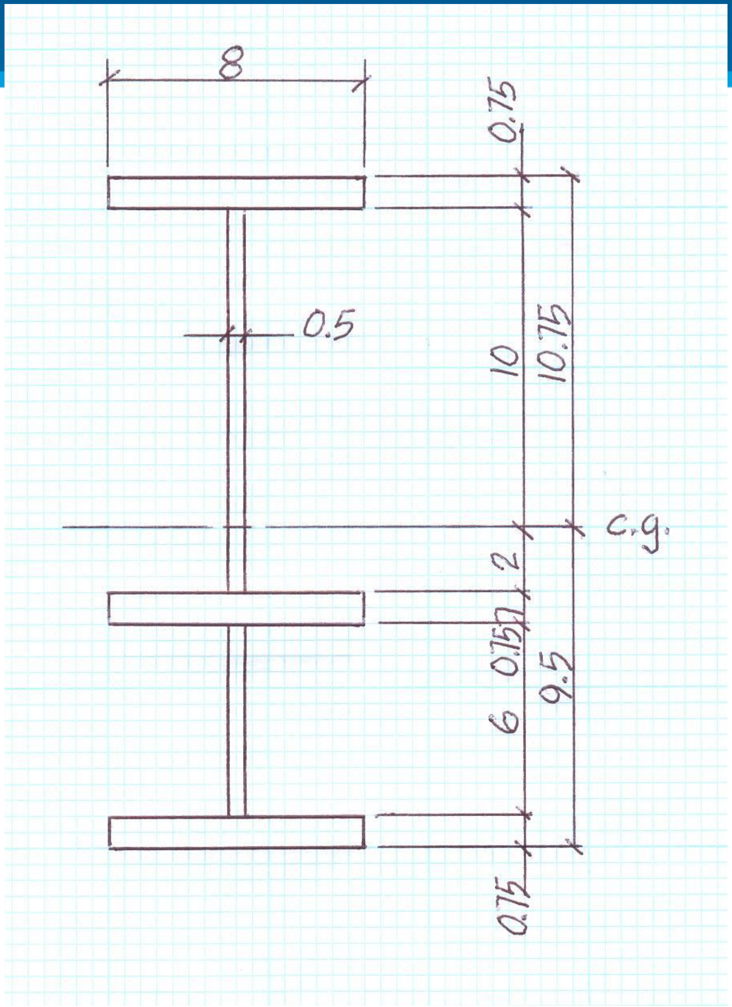
Damping

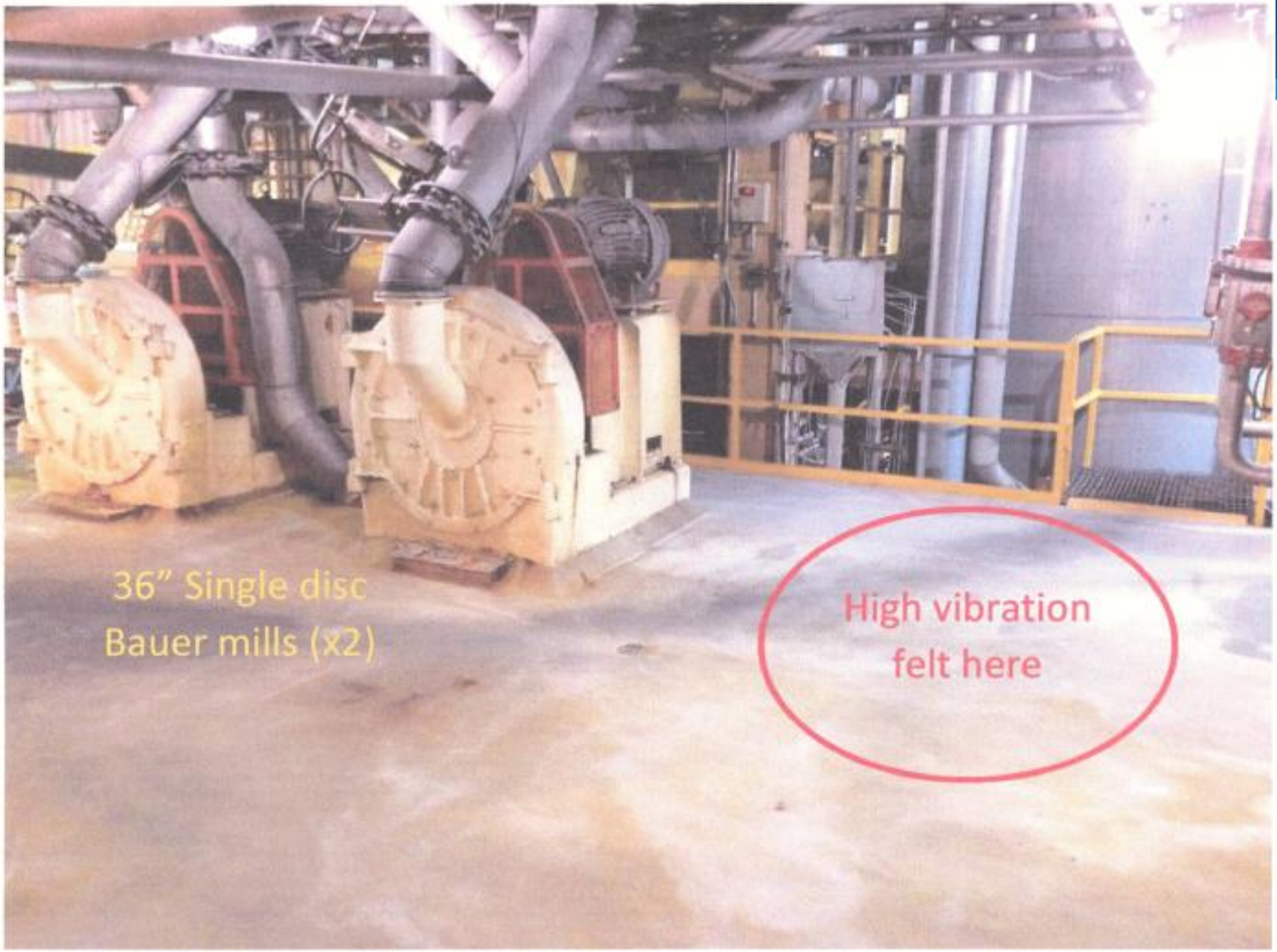


As Beam is stiffened, hysteretic damping (due to strain) is lowered.

The damping factor (ζ) may be 0.03 for the original beam and 0.026 for the modified beam.

$$x = F(\theta)/k[\{1-(f_d/f_n)^2\}^2 + \{2\zeta(f_d/f_n)\}^2]^{1/2}$$

Case History - Corn Milling Floor



Milling Machine on Concrete Slab supported by Structural Steel Framing.

Machine Speed =

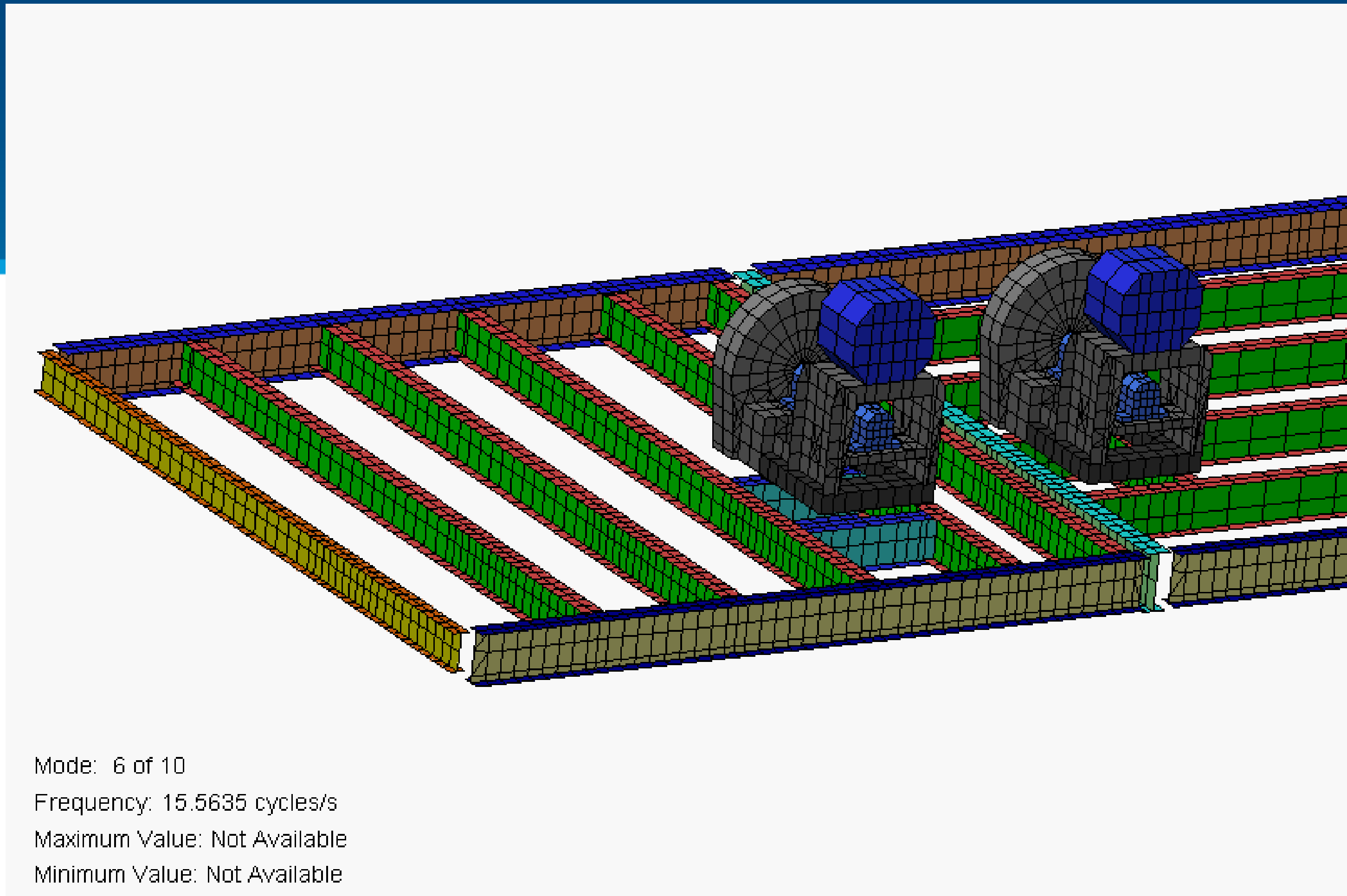
890 rpm = 14.83 Hz

Floor Vibration Nuisance

Milling Machine Rotor

False Brinneling of Bearings when Idle

Case History - Corn Milling Floor

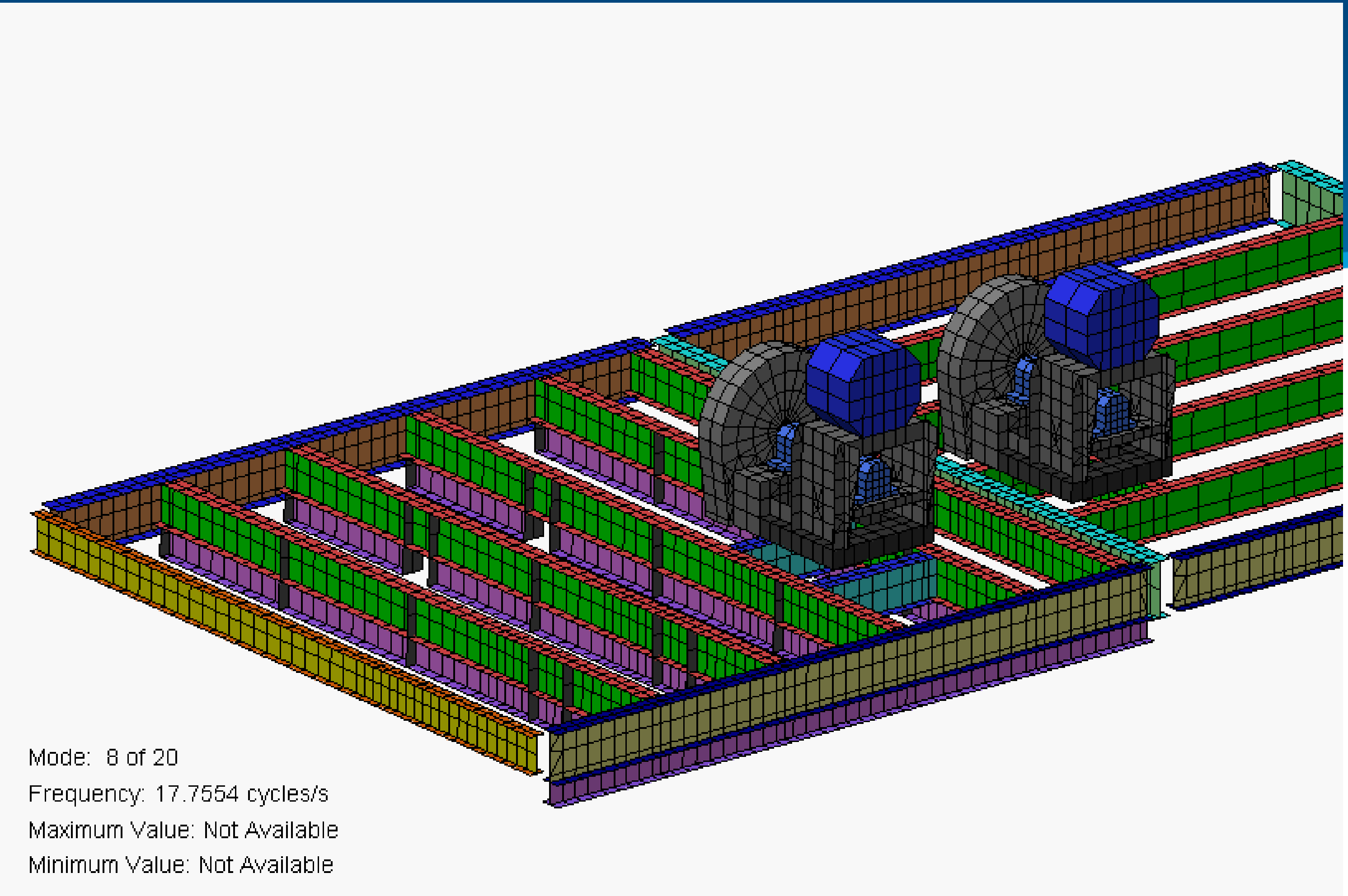


Natural Frequency of Original Floor @ 15.56 Hz

Concrete Floor not shown. Freq Ratio = 14.83/15.56 = 0.953

Case History Corn Milling Floor

Effect of Concrete Floor On Flexural Stiffness?



Natural Freq of Modified Floor = 17.75 Hz

Freq Ratio =14.83/17.75 = 0.835

Not all beams could be modified over entire span. Notches for Piping.

Case History Forced – Response Ratio

$$x = F(\theta)/k[\{1-(f_d/f_n)^2\}^2 + \{2\zeta(f_d/f_n)\}^2]^{1/2}$$

Where:

$F(\theta)$ = dynamic force = unchanged

$r = f_d/f_n = 0.953$ before modif

$r = f_d/f_n = 0.835$ after modif

$\zeta = 0.03$ before & 0.028 after

AF (before) = 9.25

AF (after) = 3.26

Is New Vibration Level = (3.26/9.25) = 0.35x Old Level?

Case History Forced – Response Ratio

$$x = F(\theta)/k[\{1-(f_d/f_n)^2\}^2 + \{2\zeta(f_d/f_n)\}^2]^{1/2}$$

Is New Vibration Level = (3.26/9.25) = 0.35x Old Level?

No! The stiffness changes from original configuration to modified configuration.

$$k_{new} = 1.30k_{original}$$

$$x_{new} = 0.35/1.30 = 0.27x_{original}$$

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Banbury Mixer

Motor Bearing Race

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Banbury Mixer

DC Motor & Gear Box

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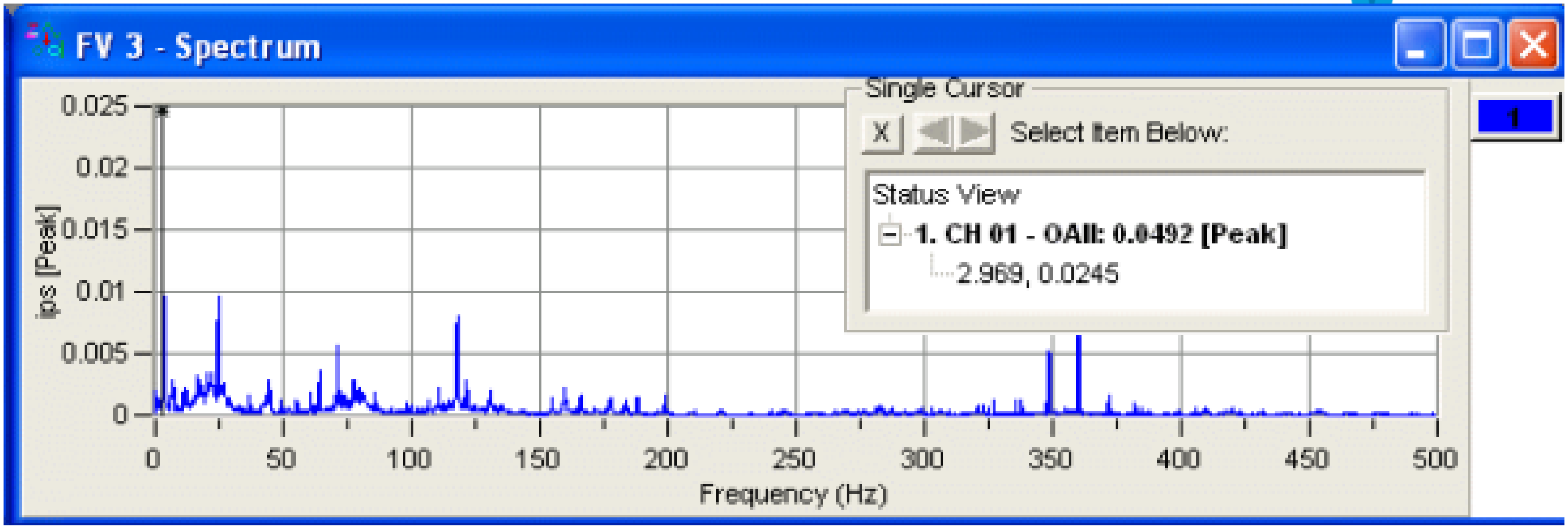
Banbury Mixer Case Study



FFT of Motor Vibration (Horizontal) during normal mixing indicates low vibration level (0.025 ips) dominated by 4x Mixer Frequency (2.96 Hz).

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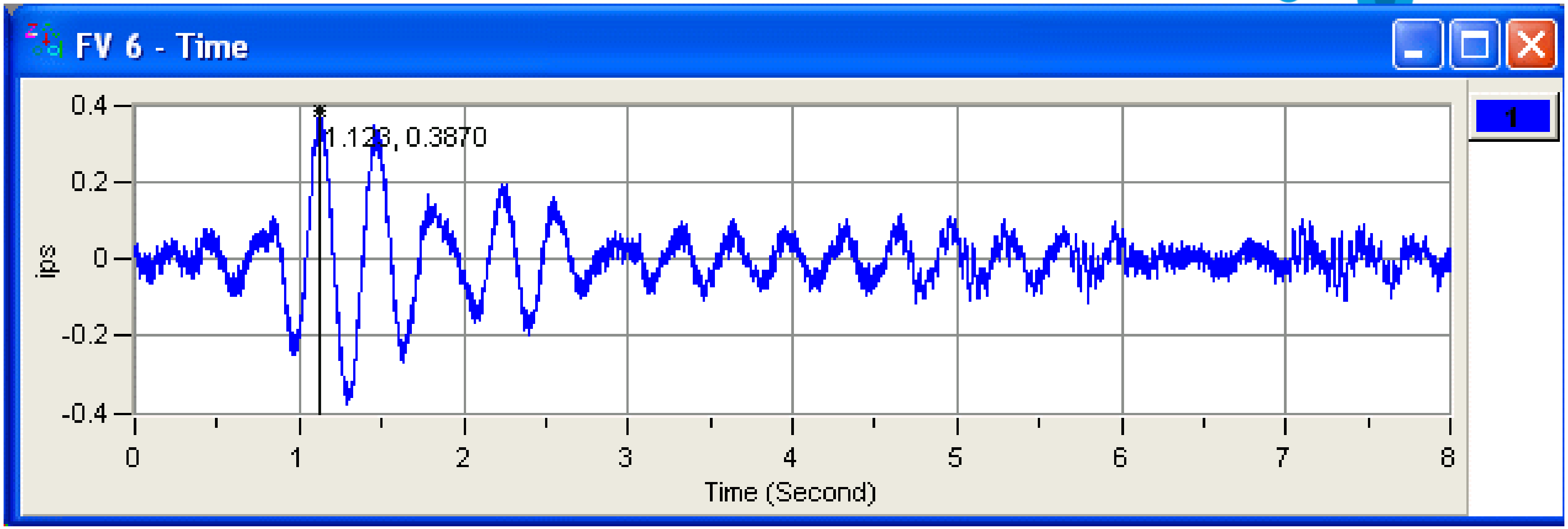
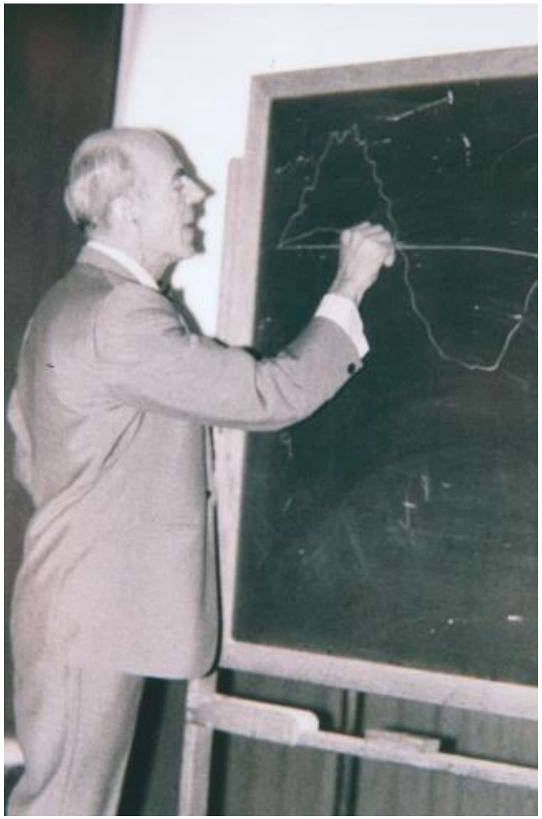
Banbury Mixer Case Study



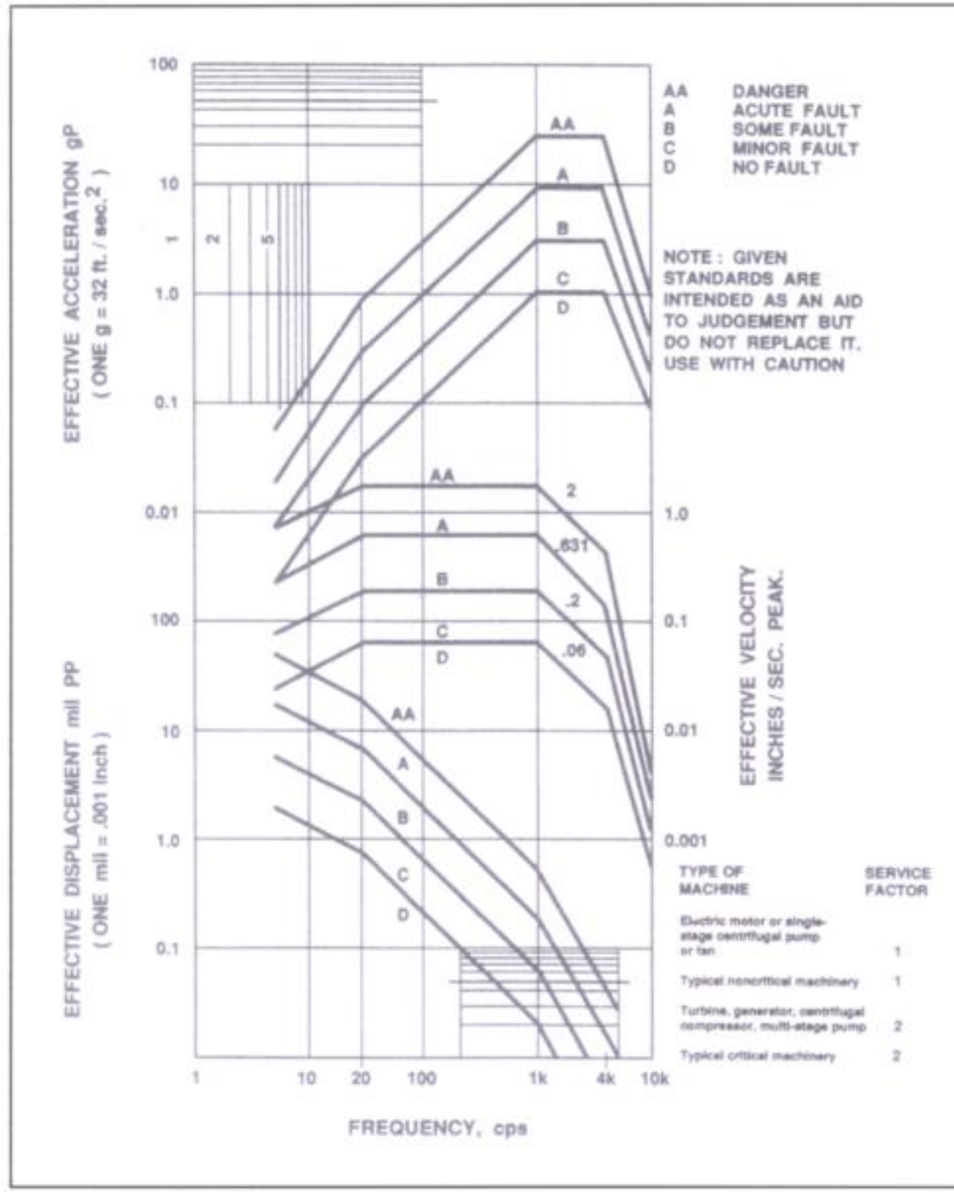
Waveform during Gate Activation approaches 0.40 ips, sometimes reaching 0.60 ips.

Transient Response @ 2.7 Hz (not 4X).

Low Frequency Vibration Severity Criteria – Blake Chart 1972



Michael Blake (Original Founder of VI)



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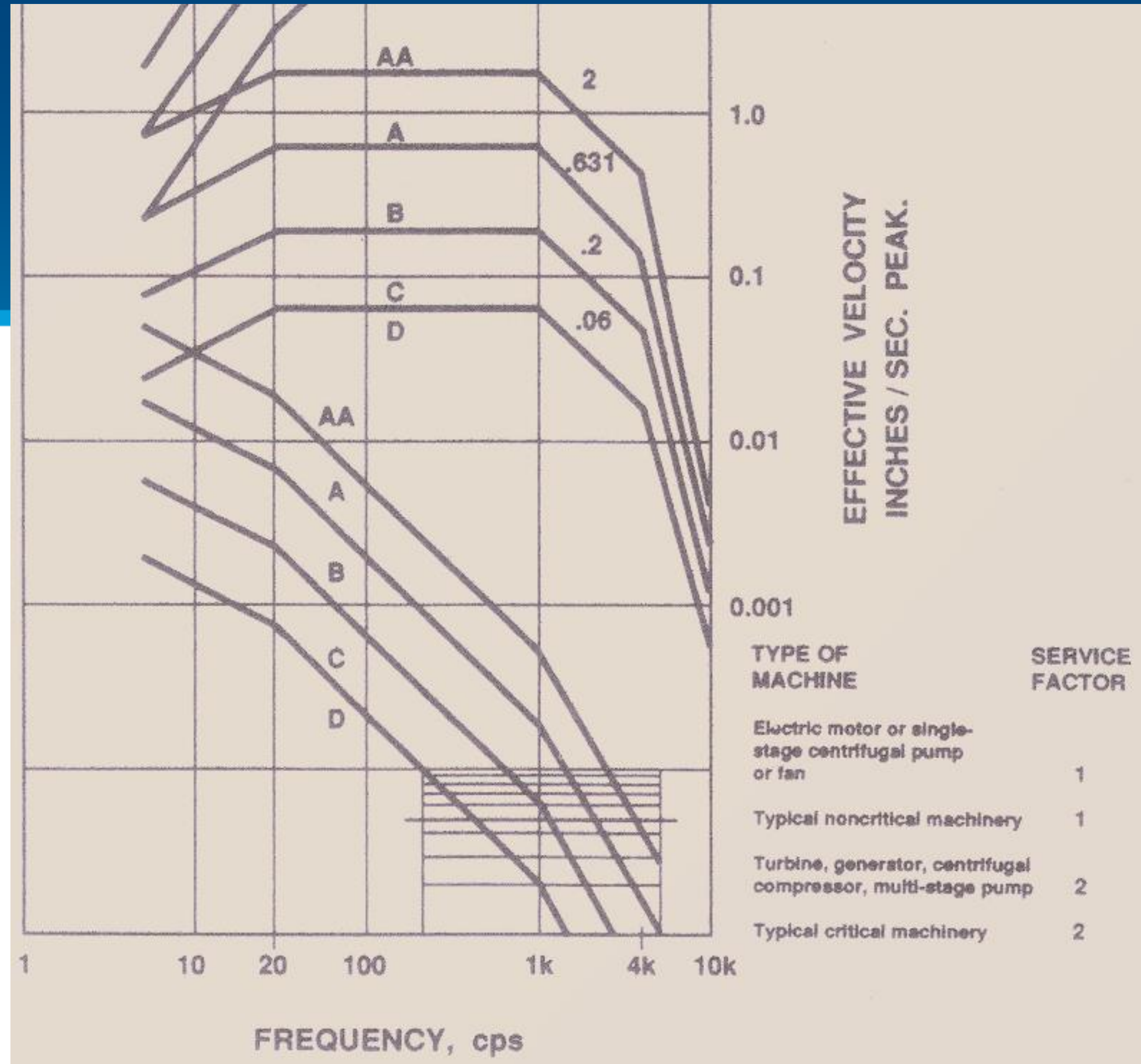
Low Frequency Vibration Severity Criteria – Blake Chart 1972

AA DANGER
A ACUTE FAULT
B SOME FAULT
C MINOR FAULT
D NO FAULT

NOTE : GIVEN STANDARDS ARE INTENDED AS AN AID TO JUDGEMENT BUT DO NOT REPLACE IT. USE WITH CAUTION



A Line @ 5 Hz; V < 0.30 ips; Critical Equipment has Service Factor = 2

A Line @ 20 Hz – 1 kHz; V < 0.63 ips

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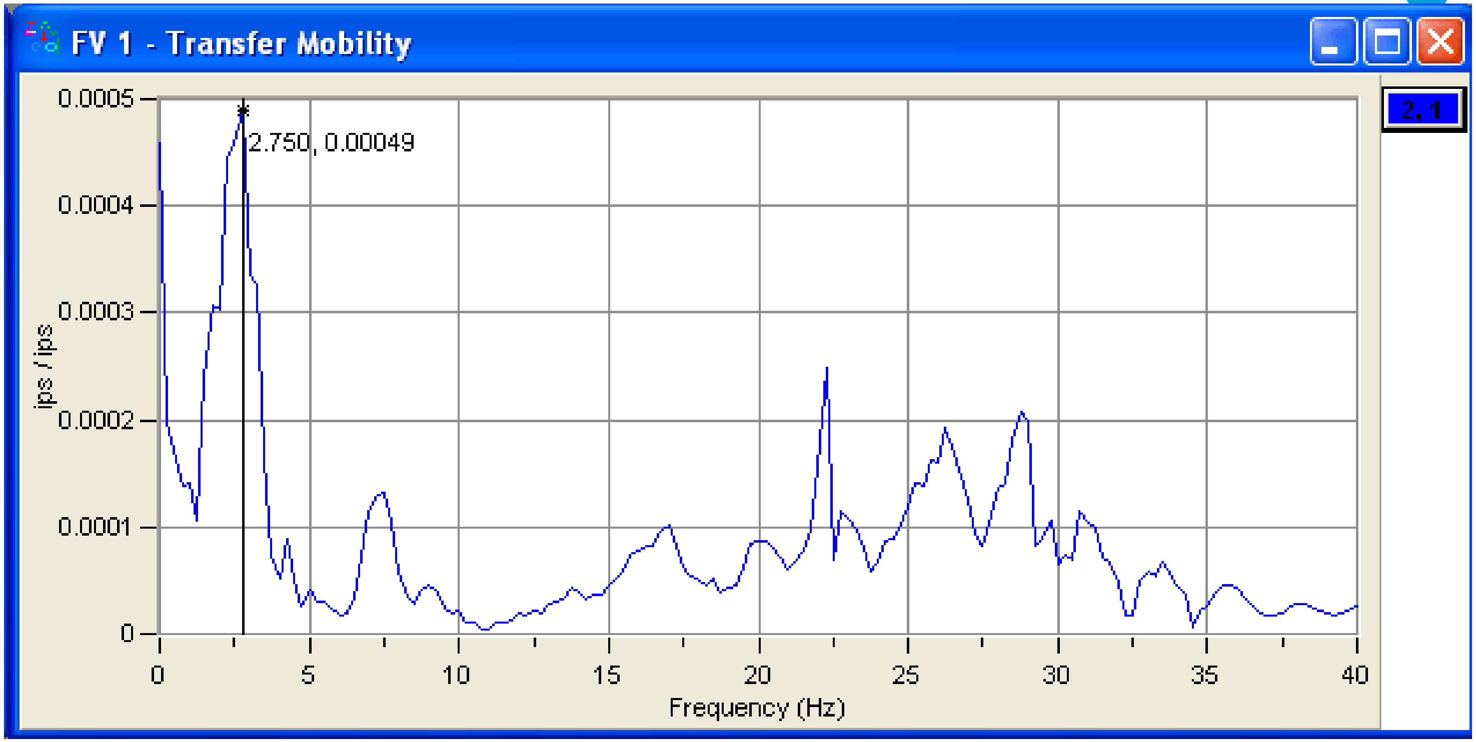
Banbury Mixer Case Study



Natural Frequency Test of Motor Support Structure identifies fn ~ 2.7 Hz.

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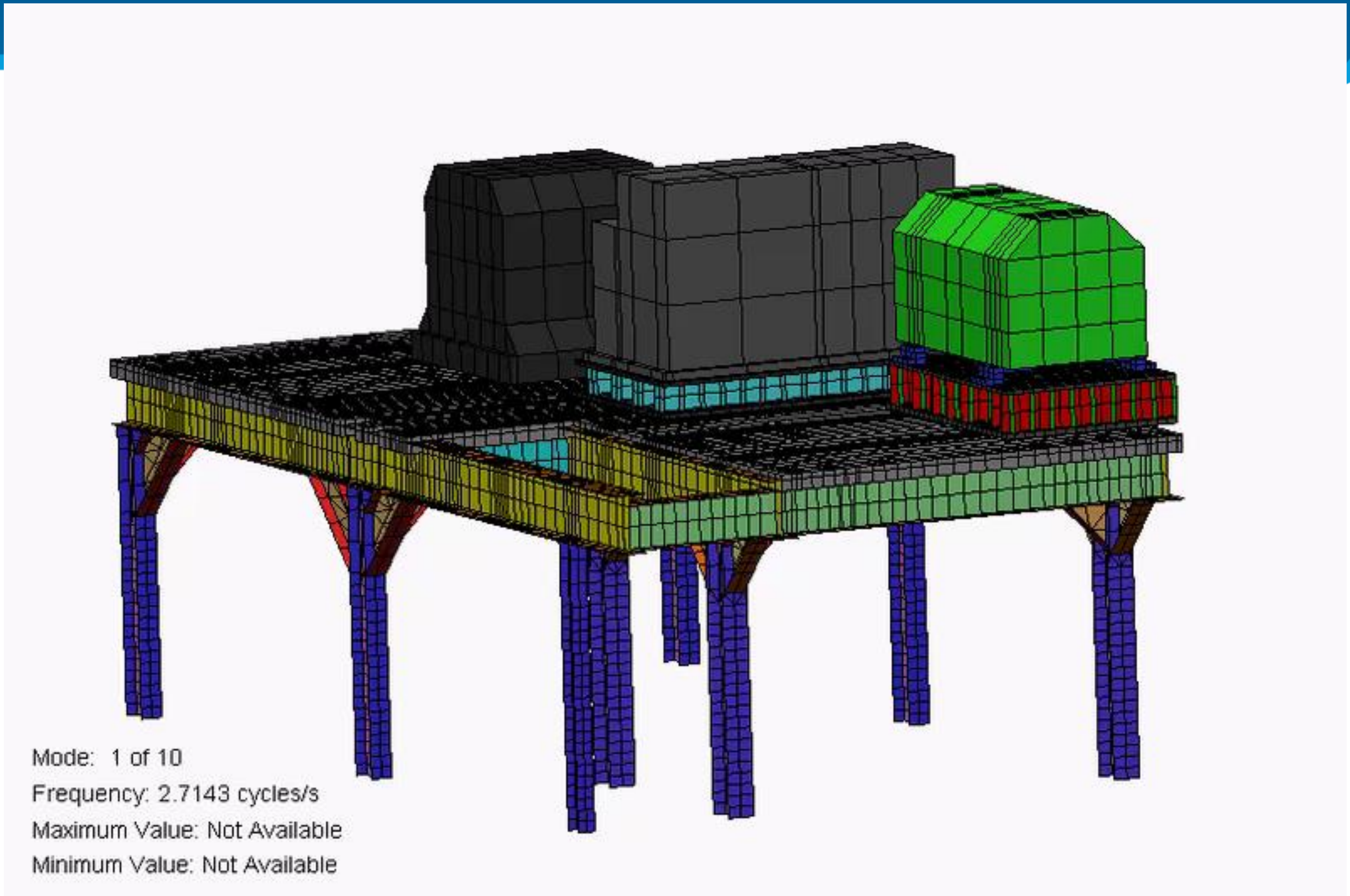
Banbury Mixer Case Study



FEA shows mode shape of Very Flexible Support System.

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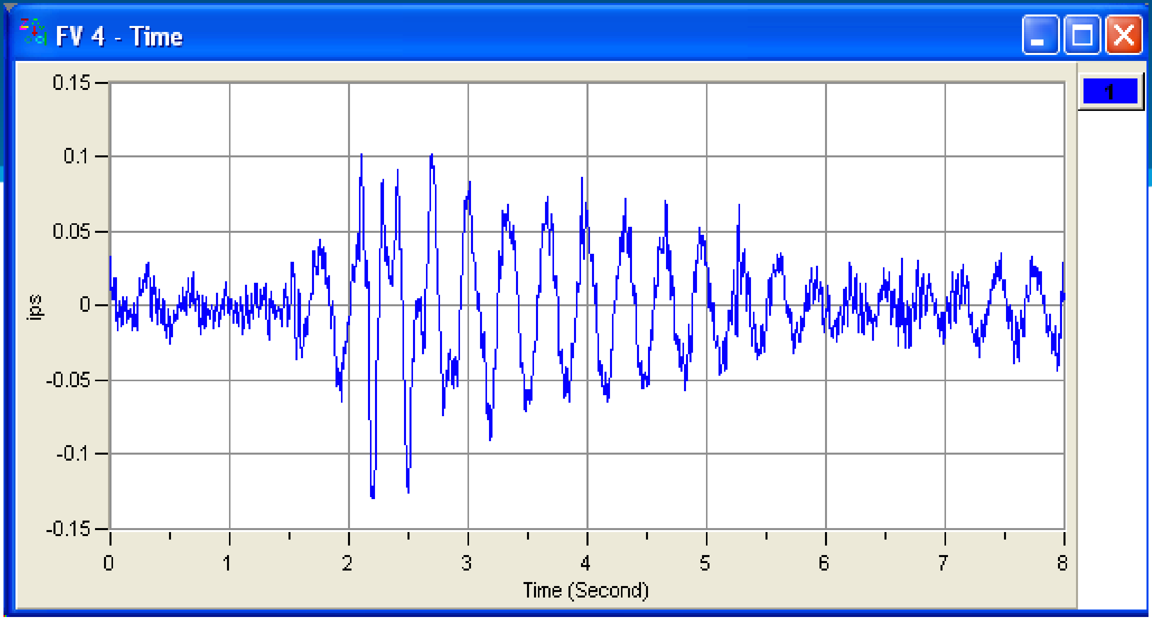
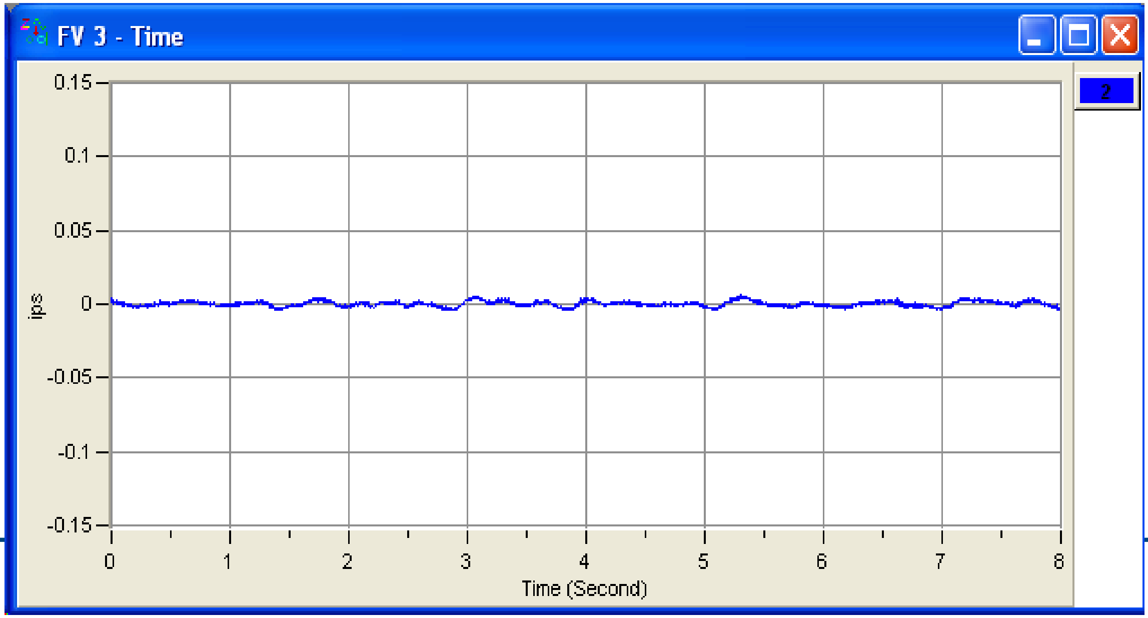
Vibration Waveforms





Waveform @ Top of Column

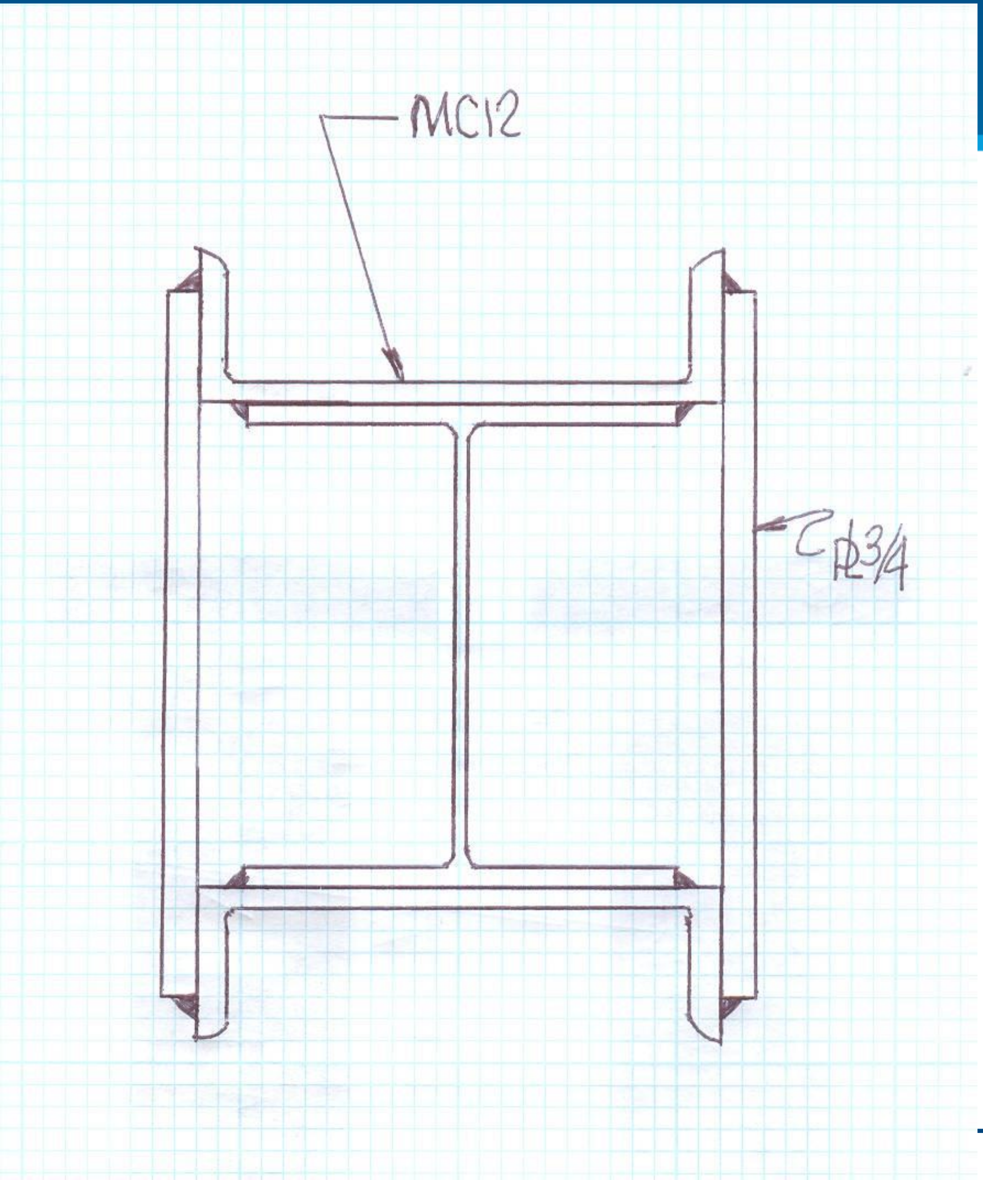
versus

Waveform @ Bottom of Column

Modification Objective

- Current Natural Freq ~ 2.7 Hz
- Increase as much as possible without getting close to Motor Speed (12 – 13 Hz; 720 – 780 rpm)
- Target Natural Freq ~ 7.5 Hz; ratio = 7.5/12 = 0.63

Modification Try #1



- MC12 Channels welded to Exist Column Flanges
- Plate welded to Flanges of MC12
- Cover Plate(s) @ Top of MC12 to Prevent Buildup of Material between Exist Col & Plate

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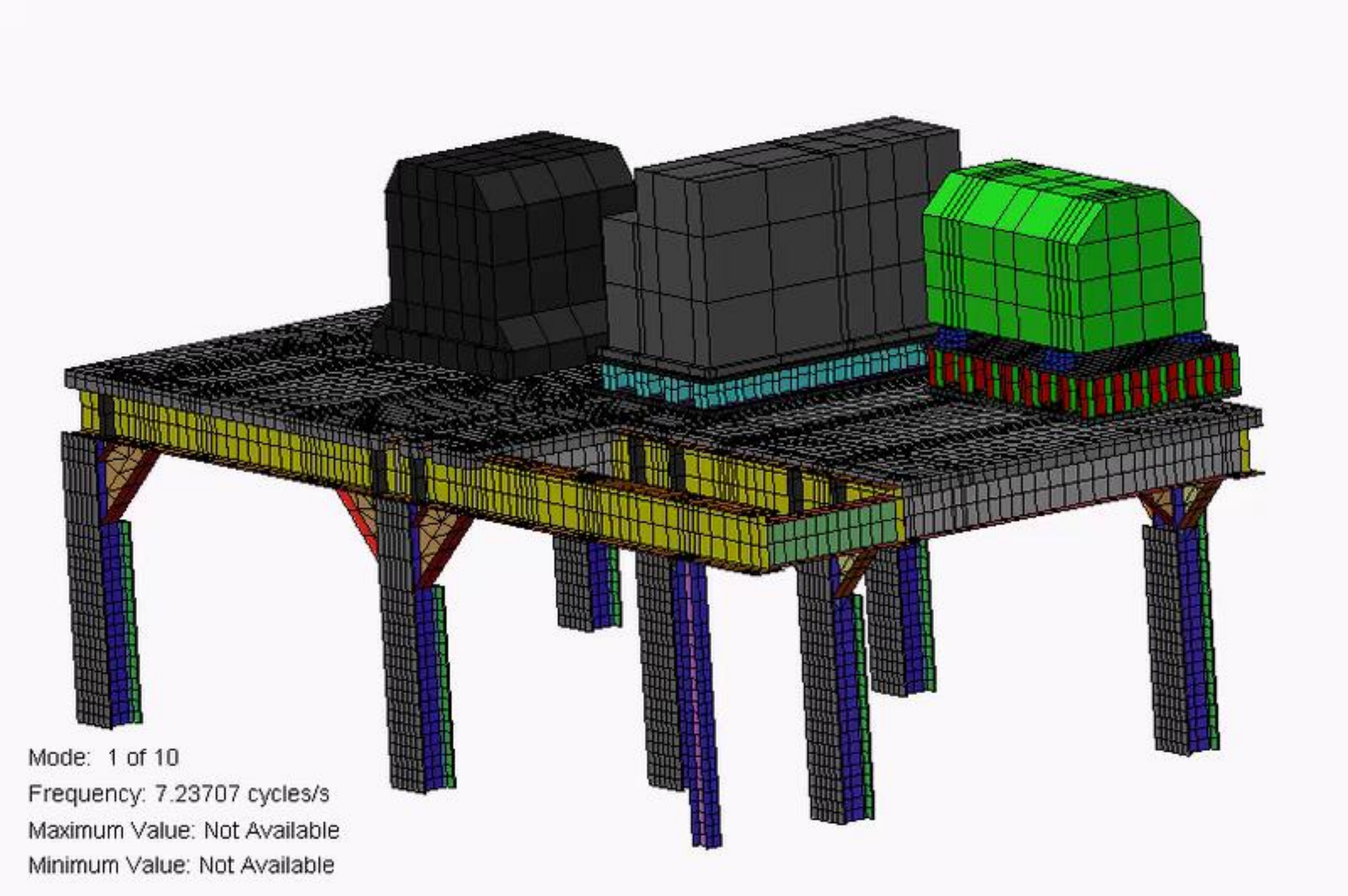
Modified Column

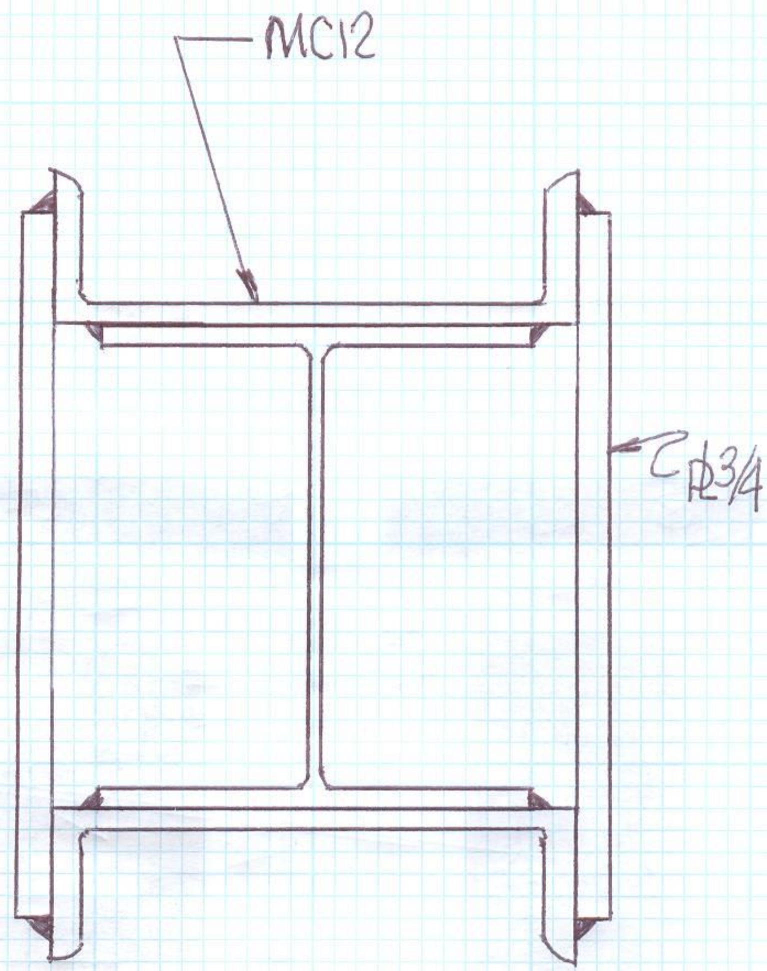


Natural Frequency increases to 7.3 Hz

Close to Objective

Modification Effectiveness

FEA says OK
Formula Says No Way!



FEA Freq Ratio:

7.3/2.7 = 2.7x Increase

$I_{original} = 107\ in^4$

$I_{modified} = 1435\ in^4$

Stiffness Increase = 13.6x

Projected Nat Freq Increase:

$(13.6)^{1/2}(2.7) = 9.96$ Hz >

FEA Calculation of 7.3 Hz

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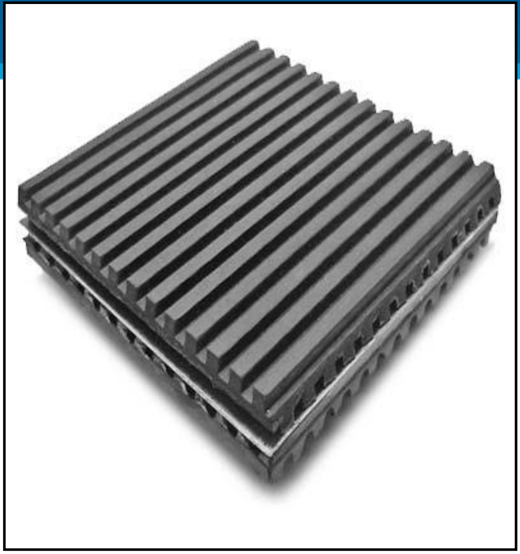
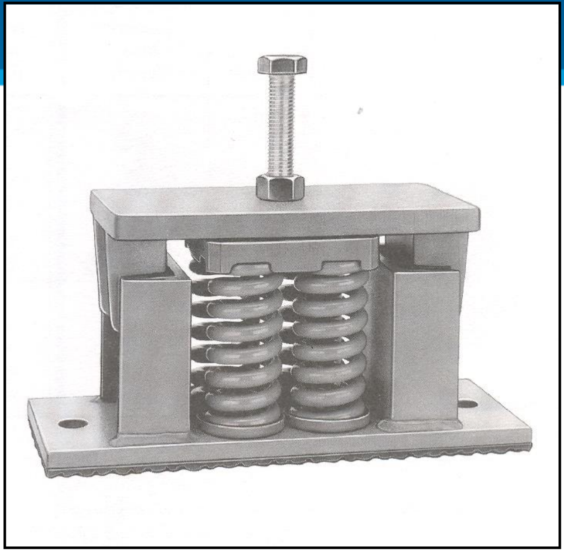
Vibration Response – Force Isolation



Elastomeric Pad



Mechanical Coil Spring

Air Spring

Isolators can be placed between "*Foundation*" & Equipment to either:

1) Reduce (not eliminate) transmission of Dynamic Force to Surroundings

2) Reduce transmission of Surrounding Vibration to Equipment

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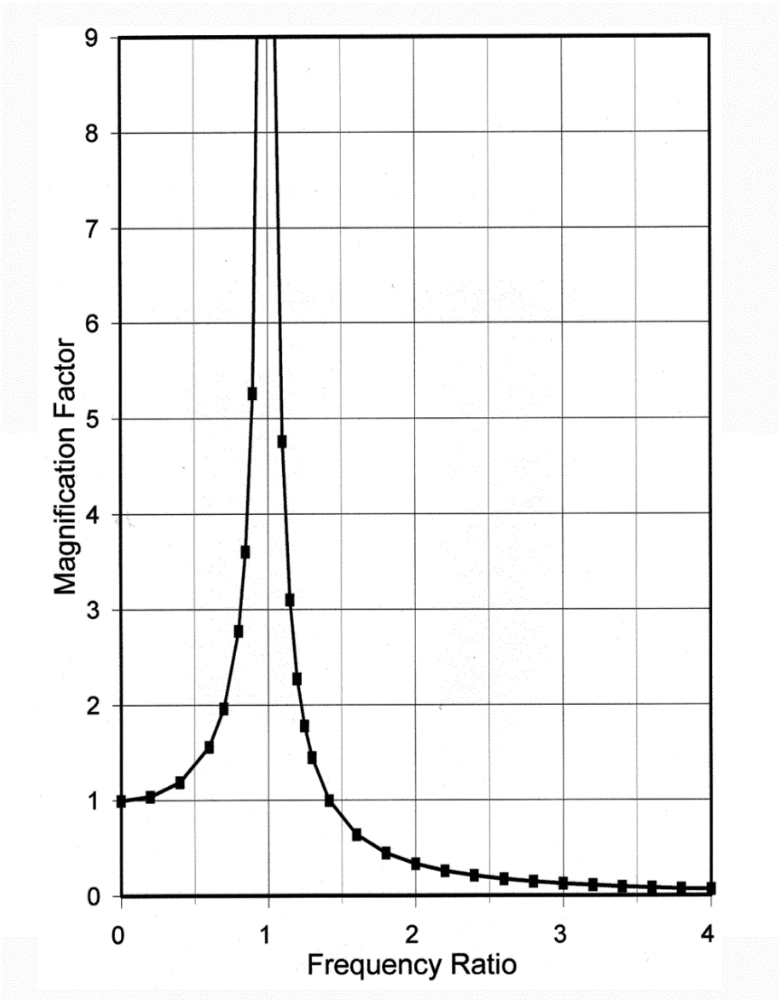
Vibration Response – Force Isolation



$$AF = 1/[\{1-(f_d/f_n)^2\}^2 + (2\zeta f_d/f_n)^2]^{1/2}$$

Neglecting Damping:

$$AF = 1/\{1-(f_d/f_n)^2\}^{1/2}$$

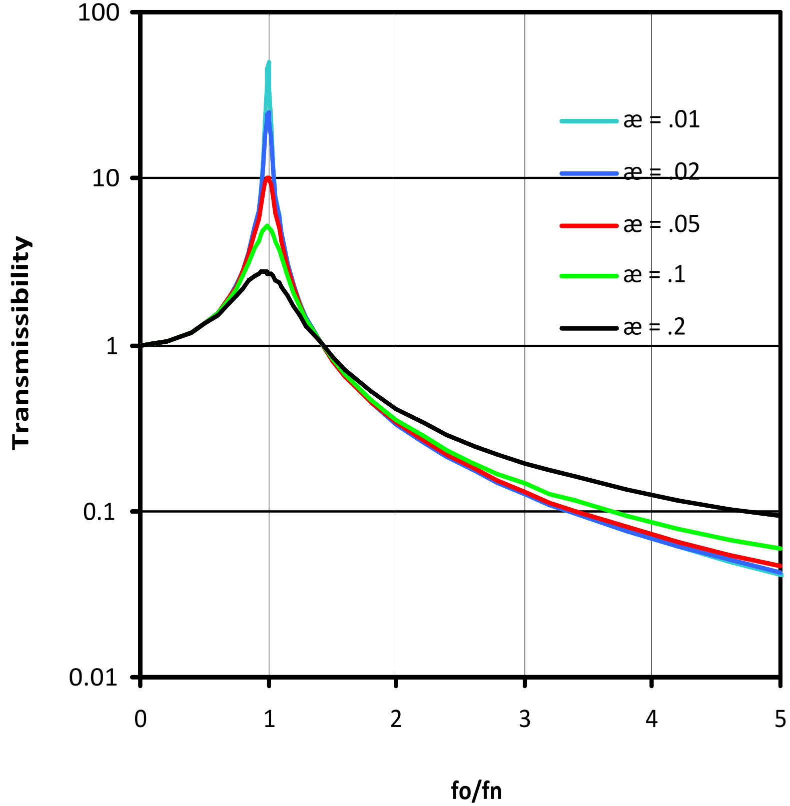
@ $(f_d/f_n) > 1.414$,

Amplification Factor < 1.0

Dynamic Force is attenuated instead of amplified.

Objective: $f_d >> f_n$

Vibration Response – Force Isolation



Frequency Ratio	Transmission
1.414	1.00
2.0	0.333
4.0	0.067
6.0	0.029

Efficiency of Isolation

The amount of dynamic force that is transmitted across an isolator reduces as the frequency ratio (f_d/f_n) increases.

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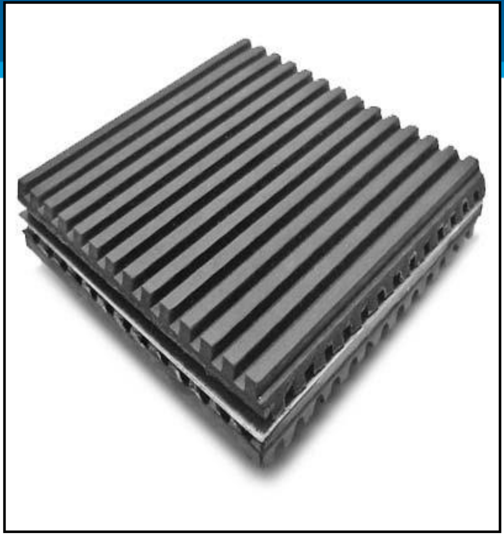
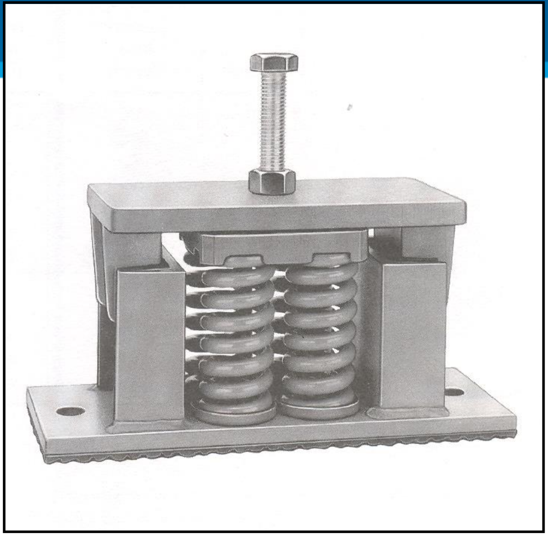
Vibration Response – Force Isolation



Elastomeric Pad



Mechanical Coil Spring

Air Spring

Generally, air springs are more efficient than metal coil springs, which are more efficient than elastomeric springs and pads. However, air springs are generally more expensive than metal coil springs, which are generally more expensive than elastomeric springs and pads.

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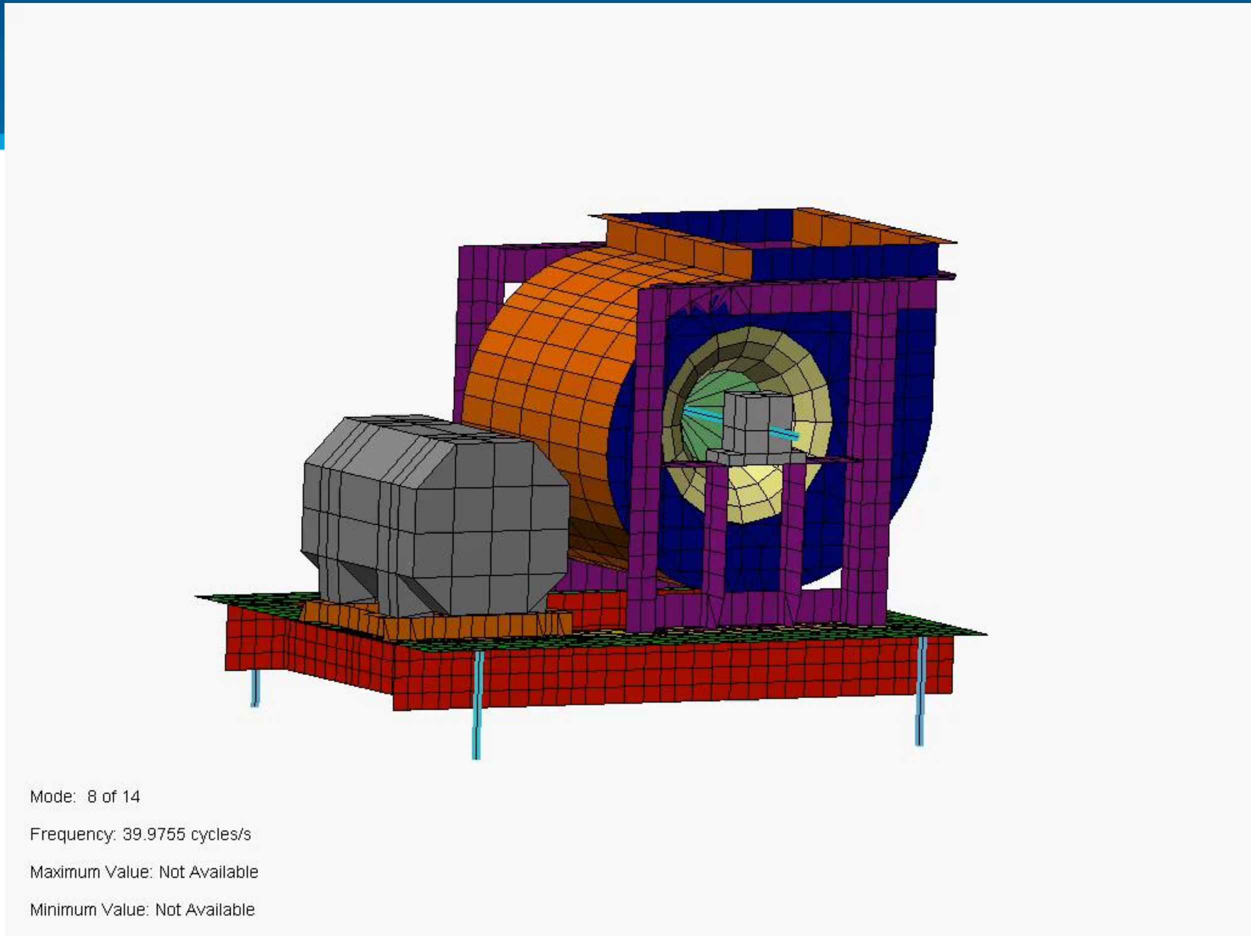
Fan on Isolator Base



Torsional Mode Shape that, if excited, results in stress in belts and bearings

Case Study – Impulse Force Isolation

Vertical pumps servicing a cooling tower at a co-generation facility were experiencing premature bearing and seal failures. The pump array consisted of six 1,000 hp units, supported on a common flexible steel framing system that spanned the sump pit. Provisions (pipe stubs) were provided for future expansion to include three additional pumps, for an eventual total of nine pumps. The operating speed of the pumps was 900 rpm (15.0 Hz).

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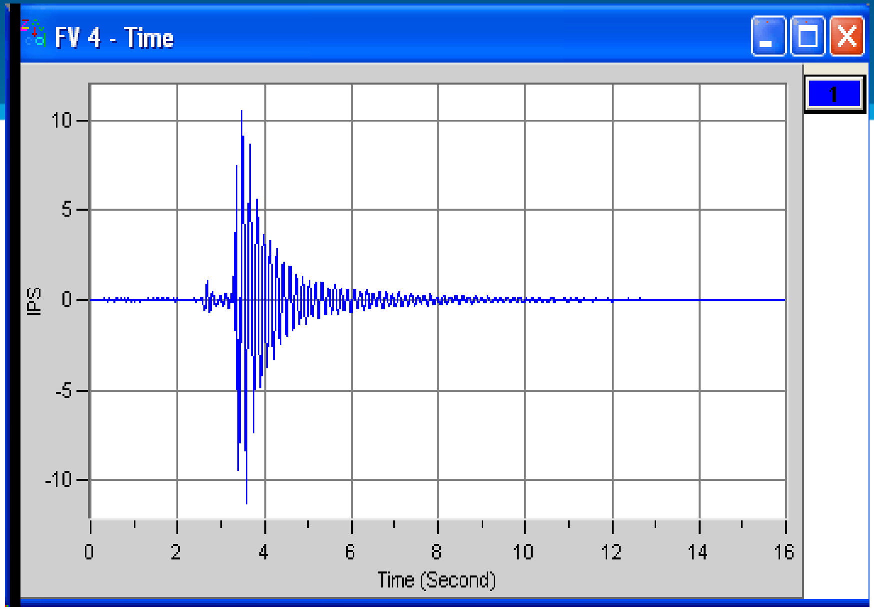
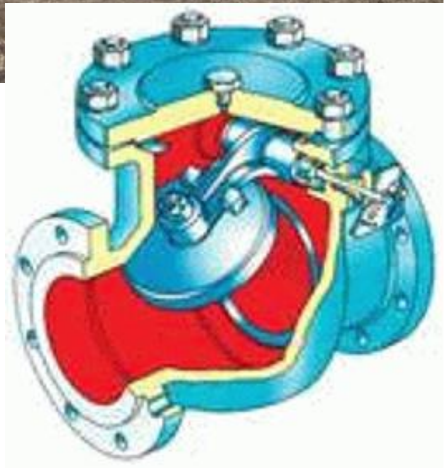
Case Study – Impulse Force Isolation





A check valve was provided in the discharge piping, close to the outlet of each pump, to isolate the pump from the header when it was not operating. At pump shutdown, the check-valve would slam shut when the flow reversed direction due to the positive pressure developed in the common header by the other operating pumps.

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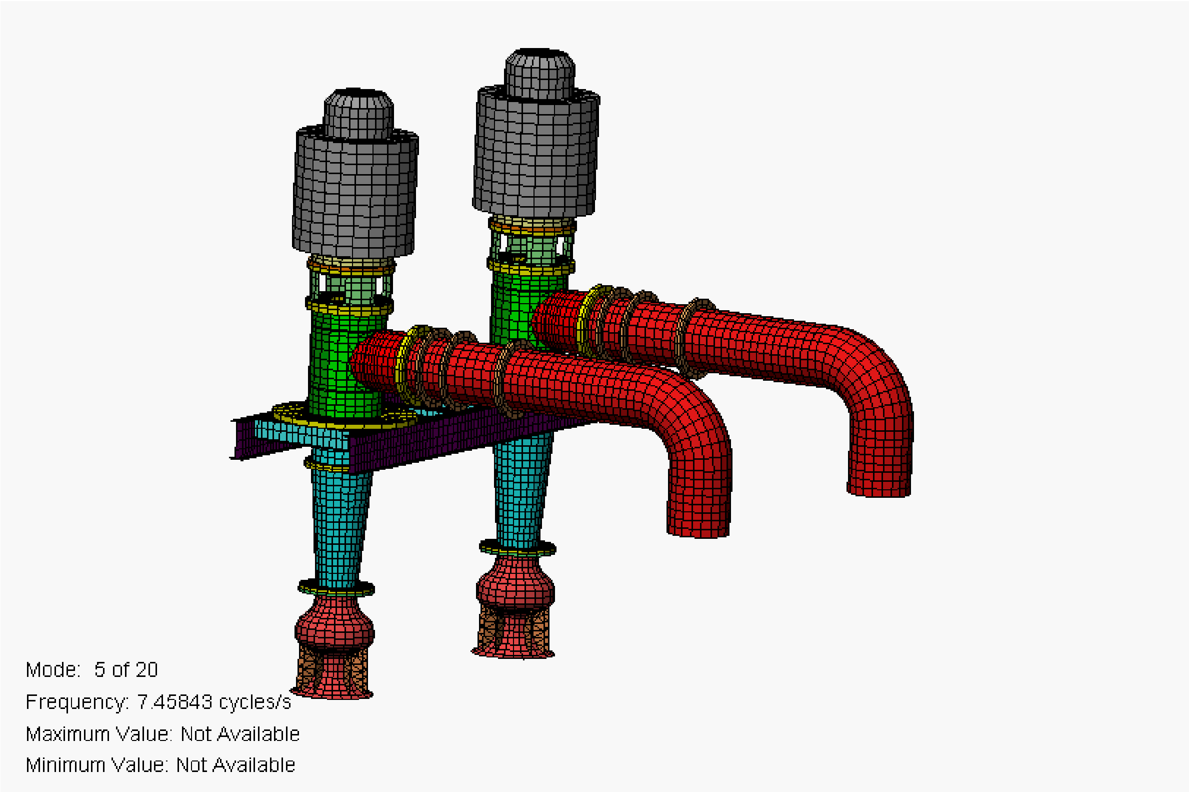
Case Study – Impulse Force Isolation



Mode Shape for Natural Frequency excited by Impulse Force created by Check Valve.

Was Force transmitted to Pump mechanically or hydraulically or

Both?

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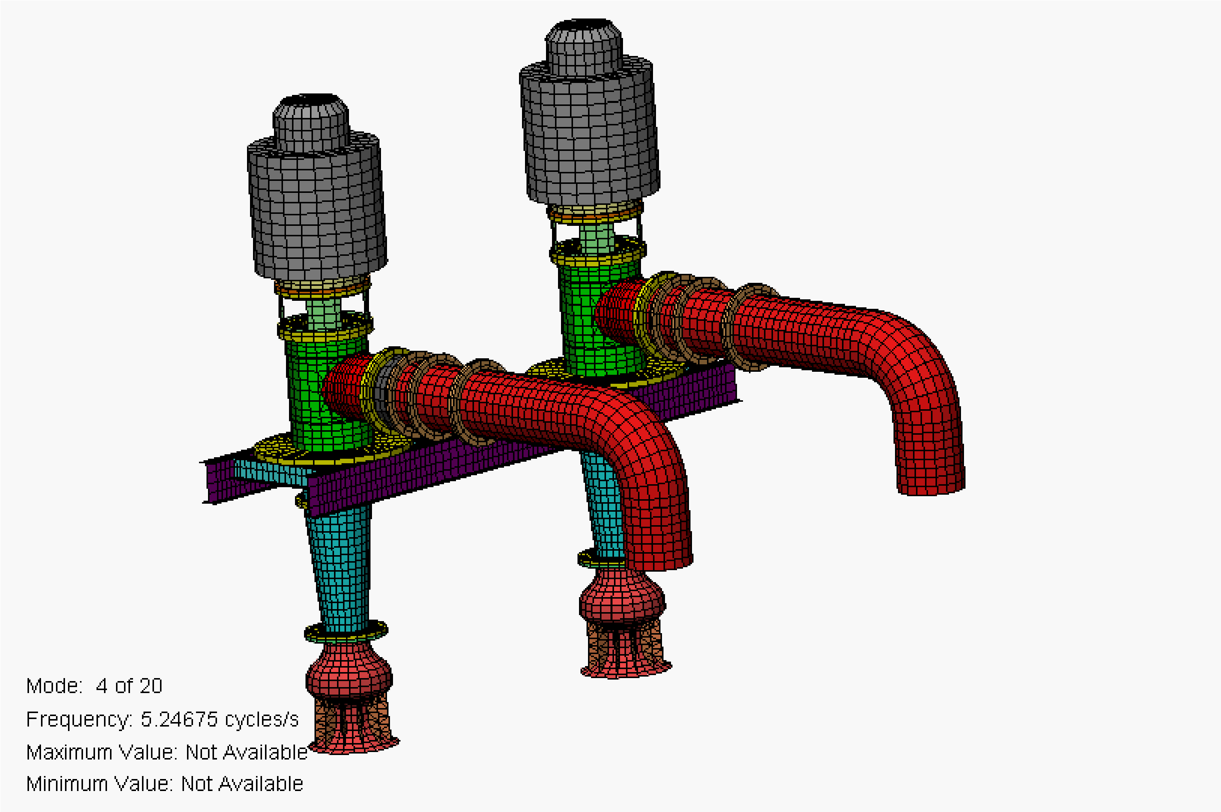
Case Study – Impulse Force Isolation



A flexible rubber spool piece was installed between the pump and check valve to minimize the mechanical transmission of dynamic force. After installation of the flexible spool piece, the maximum vibration level measured during a shutdown event was less than 1.0 ips, an order-of-magnitude reduction from the original configuration.