

### Process 1-2

### Isentropic Compression

#### **Measured Variables:**

- Temperature 1 (T<sub>1</sub>)
- Pressure 1 ( P<sub>1</sub> )
- Temperature 2 (T<sub>2</sub>)
- Cross-sectional Area (A<sub>2</sub>)

The ambient air temperature ( $T_1$ ) and pressure ( $P_1$ ) will be used as the values for state 1. A temperature sensor will be placed in the air stream directly after the compressor to measure the second state temperature ( $T_2$ ). With our measured temperatures we can look at the thermodynamic property table <u>A-17</u>, there we will find our enthalpy values ( $h_1$ ) and ( $h_2$ ) as well as the relative pressures ( $Pr_1$ ) and ( $Pr_2$ ).

First, we will calculate our pressure ( $P_2$ ), absolute volume ( $v_2$ ) and our velocity ( $v_2$ ) using the mass flow rate found in process 6-7.

$Pr_2 = \frac{P_2}{P_1} Pr_1$	$P_2 = [ ]$
$P_2 \mathbf{v}_2 = RT_2$	v <sub>2</sub> = [ ]
$\dot{m} = \frac{A_2 v_2}{v_2}$	<i>v</i> <sub>2</sub> = [ ]

Using the mass flow rate ( $\dot{m}$ ) calculated in process 6-7, as well as ( $\nu_2$ ) we can use the energy balance equation to determine what the work input to the compressor is.

$$\dot{Q}_{in} + \dot{W}_{in} + \dot{m} \left[ h + \frac{v^2}{2} + gz \right] = \dot{Q}_{out} + \dot{W}_{out} + \dot{m} \left[ h + \frac{v^2}{2} + gz \right]$$

We are able to neglect the heat input heat output as well as the work output, the change in potential energy, and the ambient velocity ( $\nu_1$ ) to give us the following equation.

$$\dot{W}_{in} + \dot{m}[h_1] = \dot{m} \left[ h_2 + \frac{v_2^2}{2} \right] \qquad \dot{W}_{in} = []$$

# Process 2-3

# **Steady Flow Diffuser**

#### **Measured Variables:**

- Temperature 2 (T<sub>2</sub>)
- Cross-sectional Area (A<sub>2</sub>)
- Temperature 3 (T<sub>3</sub>)
- Cross-sectional Area (A<sub>3</sub>)

A temperature sensor will be placed just downstream of the diffuser to record our temperature ( $T_3$ ), the cross-section areas of the diffuser inlet and outlet are known ( $A_2$ ) and ( $A_3$ ). With our measured variable ( $T_3$ ) we can look into table <u>A-17</u> to determine our ( $h_3$ ) value.

Since we are looking at a steady flow diffuser, we can neglect heat, work, and potential energy change in our general energy equation and solve for the velocity ( $\nu_3$ ).

$$\dot{m}\left[h_2 + \frac{{v_2}^2}{2}\right] = \dot{m}\left[h_3 + \frac{{v_3}^2}{2}\right] \qquad v_3 = []$$

Next, we want to solve for the pressure ( $P_3$ ) at the downstream end of the diffuser. We will do this by first using ( $\dot{m}$ ) to solve for our ( $v_3$ ). Then we will use ( $v_3$ ) to solve for ( $P_3$ ) in the ideal gas equation.

$$\dot{m} = \frac{A_3 v_3}{v_3} \qquad \qquad v_3 = [ ]$$

$$P_3 \mathbf{v}_3 = RT_3 \qquad \qquad P_3 = [ ]$$

### Process 3-4

# **Combustion Chamber**

#### **Measured Variables:**

- Temperature 3 (T<sub>3</sub>)
- Temperature 4 (T<sub>4</sub>)

Since the mass of the air in the system is so much greater than that of the fuel being added to the system we are allowed to assume that the combustion chamber simply adds heat ( $q_{in}$ ) to the working fluid (air). By measuring the temperature ( $T_4$ ) after the heat addition process with a sensor, we can go to the table <u>A-17</u> and find the enthalpy ( $h_4$ ).

With both enthalpies (  $h_3$  ) and (  $h_4$  ) we can determine the (  $q_{in}$  ) of the system.

 $\dot{q}_{in} = h_4 - h_3$   $\dot{q}_{in} = [$  ]

# Process 4-5

# Steady flow nozzle

#### **Measured Variables:**

- Temperature 4 (T<sub>4</sub>)
- Cross-sectional Area (A<sub>4</sub>)
- Temperature 5 (T<sub>5</sub>)
- Cross-sectional Area (A<sub>5</sub>)

Since the previous step simply added heat to the system, we can assume that ( $v_3$ ) = ( $v_4$ ). A temperature sensor will be placed downstream of the nozzle before the turbine to give us ( $T_5$ ) which in turn will allow us to find ( $h_5$ ).

To solve for the velocity of the working fluid exiting the nozzle we set up the general energy equation as follows.

$$\dot{m}\left[h_4 + \frac{v_4^2}{2}\right] = \dot{m}\left[h_5 + \frac{v_5^2}{2}\right] \qquad v_5 = []$$

Next, we want to find the pressure of the fluid in the system as it exits the nozzle. To do this we must first find the specific volume ( $v_5$ ), then we can use the ideal gas relation to solve for ( $P_5$ ).

$$P_5 \mathbf{v}_5 = RT_5 \qquad \qquad P_5 = [ ]$$

### Process 5-6

### **Isentropic Expansion**

#### **Measured Variables:**

- Temperature 1 (T<sub>5</sub>)
- Temperature 2 (T<sub>6</sub>)

We will measure the temperature of the gas leaving the turbine ( $T_6$ ) with a temperature sensor placed just downstream from the turbine exit. This will allow us to acquire ( $h_6$ ) from table <u>A-17</u> we will also need to obtain our relative pressures ( $Pr_5$ ) and ( $Pr_6$ ) from the table as well.

With the relative pressures and ( $P_5$ ) we can find the pressure of the air after the turbine ( $P_6$ ) using the isentropic compression/expansion relation. We will also calculate the specific volume ( $v_6$ ) and the velocity ( $v_6$ ) to set up our energy balance equation.

$Pr_6 = \frac{P_6}{P_5} Pr_5$	$P_6 = [ ]$
$P_6 \mathbf{v}_6 = RT_6$	v <sub>6</sub> = [ ]
$\dot{m} = \frac{A_6 v_6}{v_6}$	<i>v</i> <sub>6</sub> = [ ]

Next, we want to determine the  $\dot{W}$  of the turbine, to do this we have to set up the general energy equation.

$$\dot{m}\left[h_{5} + \frac{v_{5}^{2}}{2}\right] = \dot{W}_{out} + \dot{m}\left[h_{6} + \frac{v_{6}^{2}}{2}\right] \qquad \qquad \dot{W}_{out} = []$$

### Process 6-7

# Steady flow nozzle

#### **Measured Variables:**

- Temperature 6 (T<sub>6</sub>)
- Cross-sectional Area (A<sub>6</sub>)
- Temperature 7 (T<sub>7</sub>)
- Cross-sectional Area (A<sub>7</sub>)
- Air Velocity (*V*<sub>7</sub>)

The temperature  $(T_7)$  will be taken as the temperature of the air as it leaves the engine. The velocity of the air as it exits the final accelerating nozzle will also be measured. We can use  $(T_6)$  and  $(T_7)$  to find our relative pressures and use isentropic relations.

To determine the mass flow of the system we must first find the pressure at the nozzle exit and the specific volume of the air as it exits the engine ( $v_7$ ). Once that is determined we can solve for the mass flow rate ( $\dot{m}$ ).

$Pr_7 = \frac{P_7}{P_6} Pr_6$	<i>P</i> <sub>7</sub> = [ ]
$P_7 \mathbf{v}_7 = RT_7$	v <sub>7</sub> = [ ]
$\dot{m} = \frac{A_7 v_7}{v_7}$	<i>ṁ</i> = [ ]

# **Calculations**

#### **General Thrust Equation:**

 $F = \dot{m}_1 v_1 - \dot{m}_7 v_7 + (P_1 - P_7) A_7$ 

Thrust (F) is a force that propels the aircraft through the air, the general equation for thrust is shown above. For our calculations we are going to be simplifying the equation, we will treat the mass flow rate is being consistent through the whole engine. While we know this to be untrue because we added fuel to the engine in the combustion chamber. But that fuel addition was so minute compared to the amount of air moving through the engine already that it is allowed to be neglected and replaced by a heat addition process. This simplification keeps the mass flow rate the same throughout the whole engine.

 $F = \dot{m} v_1 - \dot{m} v_7 + (P_1 - P_7) A_7$ 

#### **Engine Temp and Pressure Ratios:**

$$T_7 = T_1 ETR$$

$$P_7 = P_1 EPR$$

The engine temperature ratio (ETR) and the engine pressure ratio (EPR) are useful parameters and will be used in later equations.

#### **Burner Temperature Ratio:**

$$\frac{T_3}{T_4} = \frac{1 + \frac{f \eta_b Q}{c_p T_3}}{1 + f}$$

- Fuel to air ratio (f)
- Burner efficiency ( $\eta_b$ )
- Fuel heating value (Q)

What we are interested in finding is the efficiency factor that accounts for losses in the burner  $(\eta_b)$ . To do this we must figure out the fuel to air ratio (f) because all other variables in the above equation are readily available through our analysis or through research. (f) is easily defined as the mass flow of the fuel divided by the mass flow of the air. First, we need to find the mass flow of the fuel. This can be found by looking at the volumetric flow rate of the fuel and the density of the fuel. Once (f) is found we can use it in the equation and find our efficiency, which will be used to help us determine how to improve the overall engine performance.

#### **Nozzle Performance:**

$$v_7 = \sqrt{2 c_p T_7 \eta_n \left[1 - (1/NPR^{\frac{\Upsilon-1}{\Upsilon}})\right]}$$

- Adiabatic nozzle efficiency (η<sub>n</sub>)
- Nozzle pressure ratio (NPR)
- Specific heat ratio (Υ)

When solving this equation, we first must look at he nozzle pressure ratio. This is the ratio of the nozzle pressure to the static pressure,  $NPR = \frac{P_7}{P_1}$ . With this we can solve for the adiabatic nozzle efficiency ( $\eta_n$ ) using this parameter we will be able to look at how to improve the overall efficiency and operation of the engine.

#### **Turbine Work:**

$$TW = \eta_t c_p T_5 \left[ 1 - TPR^{\frac{Y-1}{Y}} \right]$$

- Turbine work (TW)
- Adiabatic efficiency ( $\eta_t$ )
- Turbine pressure ratio (NPR)
- Specific heat ratio (Υ)

Looking at this equation what we first have to determine is the turbine pressure ratio (NPR), that is simply  $P_5/P_6$ . Then we will solve for the turbine work which is the specific heat multiplied by the change in temperature  $TW = c_p [T_6 - T_5]$ . With this we will solve for the adiabatic efficiency of the turbine which will help us better understand how improve the engines efficiency and operation.

#### **Compressor Work:**

$$CW = \frac{c_p T_1}{\eta_c} \left[ CPR^{\frac{Y-1}{Y}} - 1 \right]$$

- Compressor work (CW)
- Adiabatic efficiency ( $\eta_c$ )
- Compressor pressure ratio (CPR)
- Specific heat ratio (Υ)

We are looking to solve for the adiabatic efficiency of the compressor of the engine ( $\eta_c$ ). To do this we must first figure out the compressor pressure ratio using this equation  $CW = c_p [T_2 - T_1]$ . We can use the adiabatic efficiency to determine how to improve the efficiency and overall operation of the engine.