## Exponential Growth and Decay Functions

### ⟳ Core Concept

**Parent Function for Exponential Growth Functions**

The function $f(x) = b^x$, where $b > 1$, is the parent function for the family of exponential growth functions with base $b$. The graph shows the general shape of an exponential growth function.
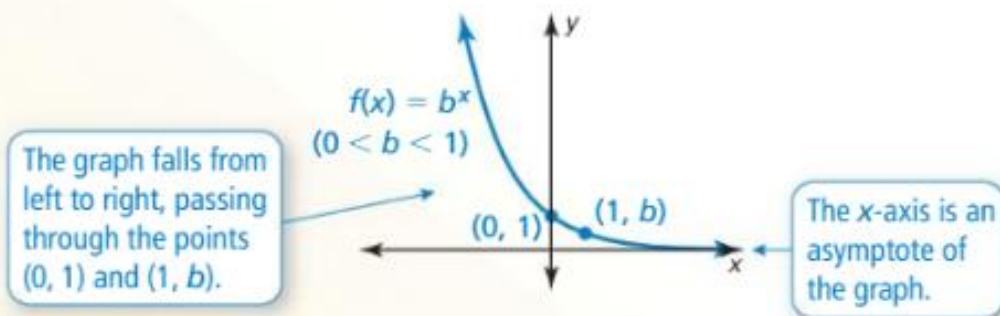
The x-axis is an *asymptote* of the graph. An **asymptote** is a line that a graph approaches more and more closely.

$f(x) = b^x$
$(b > 1)$
$(0, 1)$ $(1, b)$

The graph rises from left to right, passing through the points $(0, 1)$ and $(1, b)$.
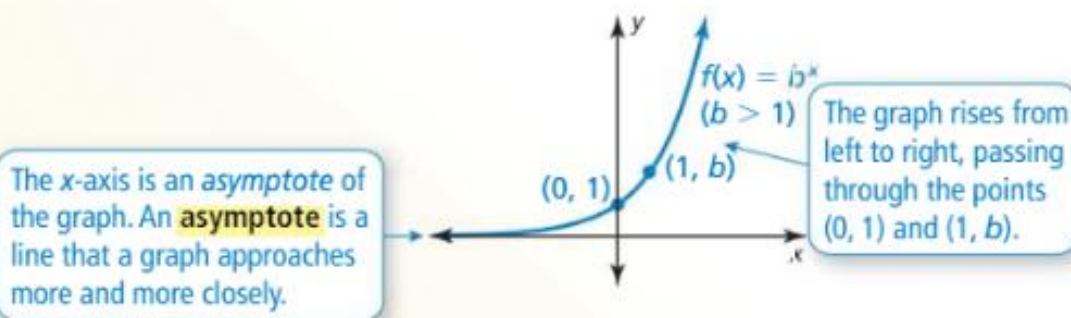
The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

### ⟳ Core Concept

**Parent Function for Exponential Decay Functions**

The function $f(x) = b^x$, where $0 < b < 1$, is the parent function for the family of exponential decay functions with base $b$. The graph shows the general shape of an exponential decay function.

The graph falls from left to right, passing through the points $(0, 1)$ and $(1, b)$.

$f(x) = b^x$
$(0 < b < 1)$
$(0, 1)$ $(1, b)$

The x-axis is an asymptote of the graph.

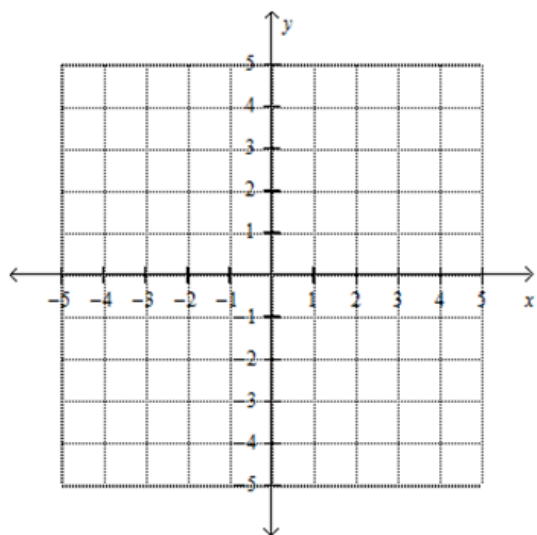The domain of $f(x) = b^x$ is all real numbers. The range is $y > 0$.

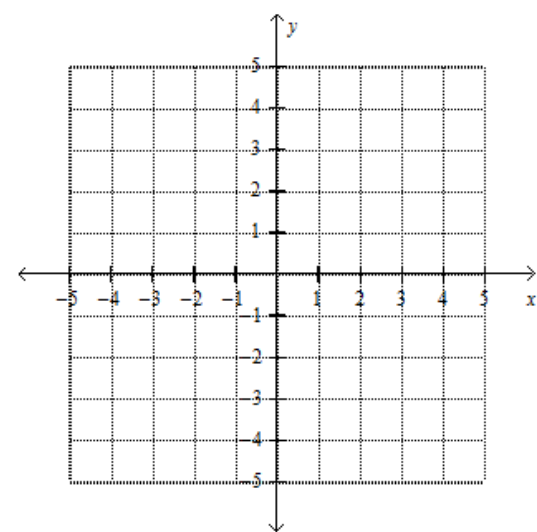EXAMPLE 1 **Graphing Exponential Growth and Decay Functions**

Tell whether each function represents *exponential growth* or *exponential decay*. Then graph the function.

**a.** $y = 2^x$

**b.** $y = \left(\frac{1}{2}\right)^x$

| X | Y |
|---|---|
|   |   |
|   |   |
|   |   |
|   |   |
|   |   |
|   |   |

| X | Y |
|---|---|
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# Exponential Models

Some real-life quantities increase or decrease by a fixed percent each year (or some other time period). The amount $y$ of such a quantity after $t$ years can be modeled by one of these equations.

**Exponential Growth Model**          **Exponential Decay Model**

$$y = a(1 + r)^t \qquad\qquad\qquad y = a(1 - r)^t$$

Note that $a$ is the initial amount and $r$ is the percent increase or decrease written as a decimal. The quantity $1 + r$ is the growth factor, and $1 - r$ is the decay factor.

**EXAMPLE 2**    **Solving a Real-Life Problem**

The value of a car $y$ (in thousands of dollars) can be approximated by the model $y = 25(0.85)^t$, where $t$ is the number of years since the car was new.

**a.** Tell whether the model represents exponential growth or exponential decay.

**b.** Identify the annual percent increase or decrease in the value of the car.

**c.** Estimate when the value of the car will be $8000.

EXAMPLE 3 **Writing an Exponential Model**

In 2000, the world population was about 6.09 billion. During the next 13 years, the world population increased by about 1.18% each year.

a. Write an exponential growth model giving the population $y$ (in billions) $t$ years after 2000. Estimate the world population in 2005.

b. Estimate the year when the world population was 7 billion.

EXAMPLE 4 **Rewriting an Exponential Function**

The amount $y$ (in grams) of the radioactive isotope chromium-51 remaining after $t$ days is $y = a(0.5)^{t/28}$, where $a$ is the initial amount (in grams). What percent of the chromium-51 decays each day?

*Compound interest* is interest paid on an initial investment, called the *principal*, and on previously earned interest. Interest earned is often expressed as an *annual* percent, but the interest is usually compounded more than once per year. So, the exponential growth model $y = a(1 + r)^t$ must be modified for compound interest problems.

## Core Concept

**Compound Interest**

Consider an initial principal $P$ deposited in an account that pays interest at an annual rate $r$ (expressed as a decimal), compounded $n$ times per year. The amount $A$ in the account after $t$ years is given by

$$A = P\left(1 + \frac{r}{n}\right)^{nt}.$$

**EXAMPLE 5**  **Finding the Balance in an Account**

You deposit $9000 in an account that pays 1.46% annual interest. Find the balance after 3 years when the interest is compounded quarterly.