Chapter 6
Exponential and Logarithmic Functions

Section 6-3
Logarithms and Logarithmic Functions

Logarithms

You know that $2^2 = 4$ and $2^3 = 8$. However, for what value of x does $2^x = 6$? Mathematicians define this x-value using a *logarithm* and write $x = \log_2 6$. The definition of a logarithm can be generalized as follows.

Core Concept

Definition of Logarithm with Base *b*

Let b and y be positive real numbers with $b \neq 1$. The **logarithm of *y* with base *b*** is denoted by $\log_b y$ and is defined as

$$\log_b y = x \quad \text{if and only if} \quad b^x = y.$$

The expression $\log_b y$ is read as "log base b of y."

This definition tells you that the equations $\log_b y = x$ and $b^x = y$ are equivalent. The first is in *logarithmic form*, and the second is in *exponential form*.

EXAMPLE 1 Rewriting Logarithmic Equations

Rewrite each equation in exponential form.

a. $\log_2 16 = 4$ **b.** $\log_4 1 = 0$ **c.** $\log_{12} 12 = 1$ **d.** $\log_{1/4} 4 = -1$

EXAMPLE 2 Rewriting Exponential Equations

Rewrite each equation in logarithmic form.

a. $5^2 = 25$ **b.** $10^{-1} = 0.1$ **c.** $8^{2/3} = 4$ **d.** $6^{-3} = \frac{1}{216}$

Parts (b) and (c) of Example 1 illustrate two special logarithm values that you should learn to recognize. Let b be a positive real number such that $b \neq 1$.

Logarithm of 1

$\log_b 1 = 0$ because $b^0 = 1$.

Logarithm of *b* with Base *b*

$\log_b b = 1$ because $b^1 = b$.

EXAMPLE 3 Evaluating Logarithmic Expressions

Evaluate each logarithm.

a. $\log_4 64$ **b.** $\log_5 0.2$ **c.** $\log_{1/5} 125$ **d.** $\log_{36} 6$

A **common logarithm** is a logarithm with base 10. It is denoted by $\log_{10}$ or simply by log. A **natural logarithm** is a logarithm with base e. It can be denoted by $\log_e$ but is usually denoted by ln.

Common Logarithm	**Natural Logarithm**
$\log_{10} x = \log x$	$\log_e x = \ln x$

EXAMPLE 4 Evaluating Common and Natural Logarithms

Evaluate (a) log 8 and (b) ln 0.3 using a calculator. Round your answer to three decimal places.

SOLUTION

Most calculators have keys for evaluating common and natural logarithms.

a. $\log 8 \approx 0.903$

b. $\ln 0.3 \approx -1.204$

Check your answers by rewriting each logarithm in exponential form and evaluating.

```
log(8)
       .903089987
ln(0.3)
     -1.203972804
```

Using Inverse Properties

By the definition of a logarithm, it follows that the logarithmic function $g(x) = \log_b x$ is the inverse of the exponential function $f(x) = b^x$. This means that

$$g(f(x)) = \log_b b^x = x \qquad \text{and} \qquad f(g(x)) = b^{\log_b x} = x.$$

In other words, exponential functions and logarithmic functions "undo" each other.

EXAMPLE 5 Using Inverse Properties

Simplify (a) $10^{\log 4}$ and (b) $\log_5 25^x$.

EXAMPLE 6 Finding Inverse Functions

Find the inverse of each function.

a. $f(x) = 6^x$ **b.** $y = \ln(x + 3)$

Graphing Logarithmic Functions

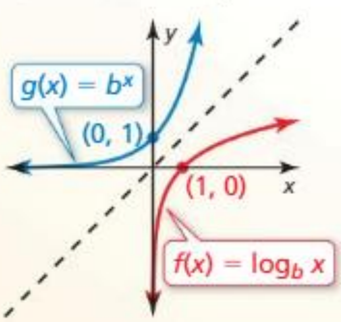
You can use the inverse relationship between exponential and logarithmic functions to graph logarithmic functions.

Core Concept

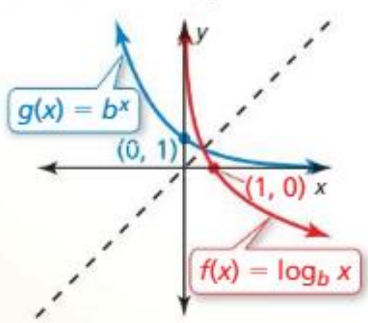
Parent Graphs for Logarithmic Functions

The graph of $f(x) = \log_b x$ is shown below for $b > 1$ and for $0 < b < 1$. Because $f(x) = \log_b x$ and $g(x) = b^x$ are inverse functions, the graph of $f(x) = \log_b x$ is the reflection of the graph of $g(x) = b^x$ in the line $y = x$.

Graph of $f(x) = \log_b x$ for $b > 1$



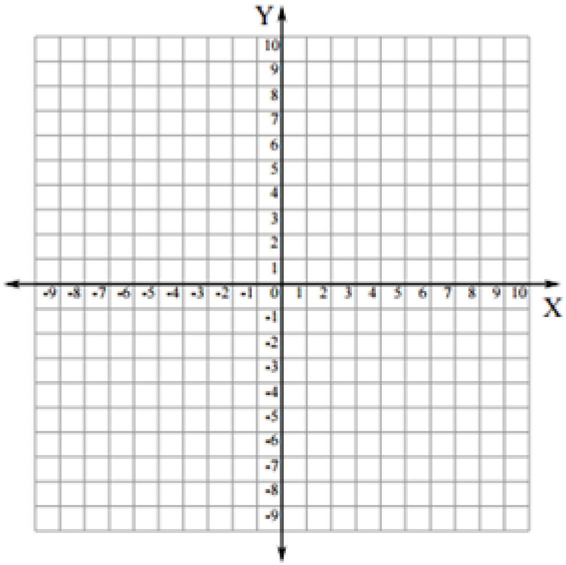
Graph of $f(x) = \log_b x$ for $0 < b < 1$



Note that the y-axis is a vertical asymptote of the graph of $f(x) = \log_b x$. The domain of $f(x) = \log_b x$ is $x > 0$, and the range is all real numbers.

EXAMPLE 7 Graphing a Logarithmic Function

Graph $f(x) = \log_3 x$.



X	Y