Chapter 6
Exponential and Logarithmic Functions

Section 6-5
Properties of Logarithms

Properties of Logarithms

You know that the logarithmic function with base b is the inverse function of the exponential function with base b. Because of this relationship, it makes sense that logarithms have properties similar to properties of exponents.

Core Concept

Properties of Logarithms

Let b, m, and n be positive real numbers with $b \neq 1$.

Product Property $\log_b mn = \log_b m + \log_b n$

Quotient Property $\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property $\log_b m^n = n \log_b m$

EXAMPLE 1 Using Properties of Logarithms

Use $\log_2 3 \approx 1.585$ and $\log_2 7 \approx 2.807$ to evaluate each logarithm.

a. $\log_2 \frac{3}{7}$ **b.** $\log_2 21$ **c.** $\log_2 49$

COMMON ERROR

Note that in general $\log_b \frac{m}{n} \neq \frac{\log_b m}{\log_b n}$ and $\log_b mn \neq (\log_b m)(\log_b n)$.

Rewriting Logarithmic Expressions

You can use the properties of logarithms to expand and condense logarithmic expressions.

EXAMPLE 2 Expanding a Logarithmic Expression

Expand $\ln \frac{5x^7}{y}$.

EXAMPLE 3 Condensing a Logarithmic Expression

Condense $\log 9 + 3 \log 2 - \log 3$.

Change-of-Base Formula

Logarithms with any base other than 10 or e can be written in terms of common or natural logarithms using the *change-of-base formula*. This allows you to evaluate any logarithm using a calculator.

Core Concept

Change-of-Base Formula

If a, b, and c are positive real numbers with $b \neq 1$ and $c \neq 1$, then

$$\log_c a = \frac{\log_b a}{\log_b c}.$$

In particular, $\log_c a = \frac{\log a}{\log c}$ and $\log_c a = \frac{\ln a}{\ln c}$.

Tutorial

EXAMPLE 4 **Changing a Base Using Common Logarithms**

Evaluate $\log_3 8$ using common logarithms.

ANOTHER WAY

In Example 4, $\log_3 8$ can be evaluated using natural logarithms.

$\log_3 8 = \frac{\ln 8}{\ln 3} \approx 1.893$

Notice that you get the same answer whether you use natural logarithms or common logarithms in the change-of-base formula.

Tutorial

EXAMPLE 5 **Changing a Base Using Natural Logarithms**

Evaluate $\log_6 24$ using natural logarithms.

EXAMPLE 6 Solving a Real-Life Problem

For a sound with intensity I (in watts per square meter), the loudness $L(I)$ of the sound (in decibels) is given by the function

$$L(I) = 10 \log \frac{I}{I_0}$$

where I_0 is the intensity of a barely audible sound (about 10^{-12} watts per square meter). An artist in a recording studio turns up the volume of a track so that the intensity of the sound doubles. By how many decibels does the loudness increase?