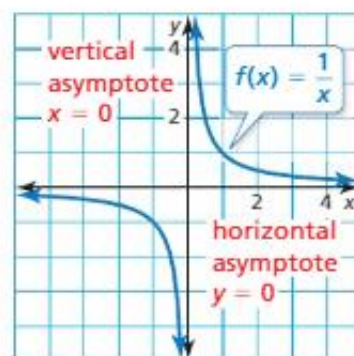## Graphing Simple Rational Functions

A **rational function** has the form $f(x) = \dfrac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. The inverse variation function $f(x) = \dfrac{a}{x}$ is a rational function. The graph of this function when $a = 1$ is shown below.

⑤ Core Concept

Parent Function for Simple Rational Functions

The graph of the parent function $f(x) = \dfrac{1}{x}$ is a hyperbola, which consists of two symmetrical parts called branches. The domain and range are all nonzero real numbers.

Any function of the form $g(x) = \dfrac{a}{x}$ $(a \neq 0)$ has the same asymptotes, domain, and range as the function $f(x) = \dfrac{1}{x}$.
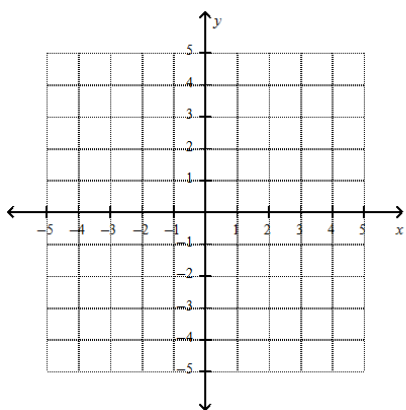

STUDY TIP

Notice that $\dfrac{1}{x} \to 0$ as $x \to \infty$ and as $x \to -\infty$. This explains why $y = 0$ is a horizontal asymptote of the graph of $f(x) = \dfrac{1}{x}$. You can also analyze y-values as x approaches 0 to see why $x = 0$ is a vertical asymptote.

EXAMPLE 1 Graphing a Rational Function of the Form $y = \dfrac{a}{x}$

Graph $g(x) = \dfrac{4}{x}$. Compare the graph with the graph of $f(x) = \dfrac{1}{x}$.

| X	Y

1

Translating Simple Rational Functions

🌀 Core Concept

Graphing Translations of Simple Rational Functions

To graph a rational function of the form $y = \dfrac{a}{x - h} + k$, follow these steps:

Step 1 Draw the asymptotes $x = h$ and $y = k$.

Step 2 Plot points to the left and to the right of the vertical asymptote.

Step 3 Draw the two branches of the hyperbola so that they pass through the plotted points and approach the asymptotes.
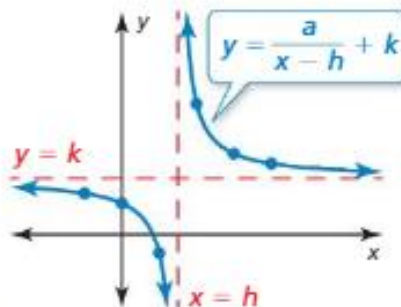
EXAMPLE 2 **Graphing a Translation of a Rational Function**

Graph $g(x) = \dfrac{-4}{x + 2} - 1$. State the domain and range.

X	Y

2
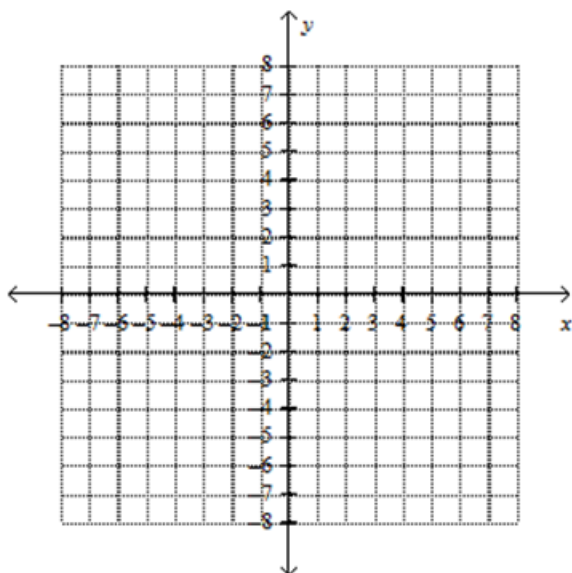
Graphing Other Rational Functions

All rational functions of the form $y = \dfrac{ax + b}{cx + d}$ also have graphs that are hyperbolas.
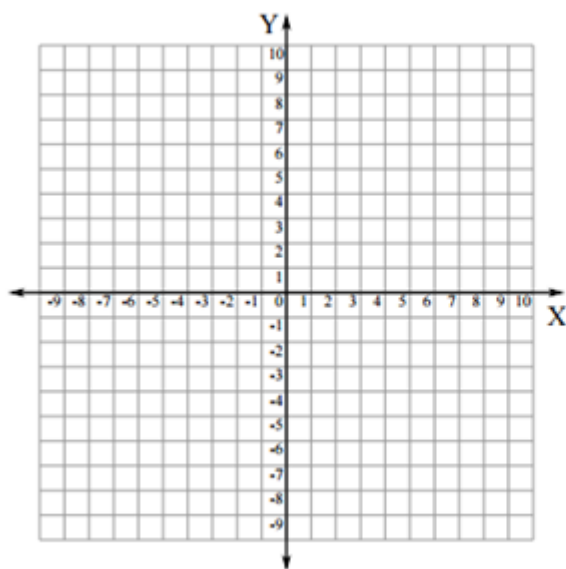
- The vertical asymptote of the graph is the line $x = -\dfrac{d}{c}$ because the function is undefined when the denominator $cx + d$ is zero.

- The horizontal asymptote is the line $y = \dfrac{a}{c}$.

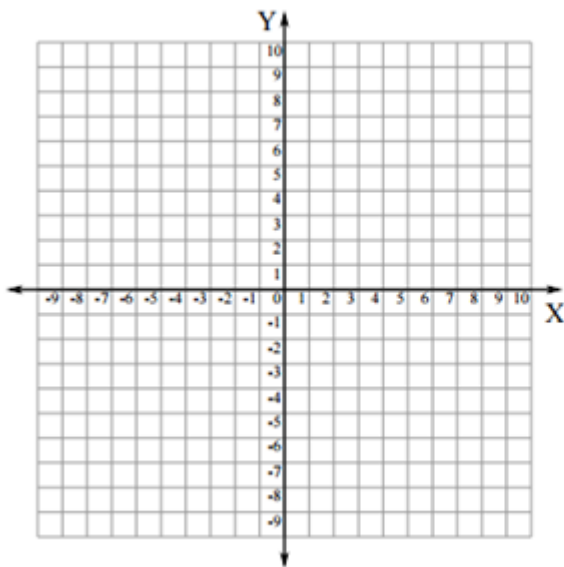**EXAMPLE 3** **Graphing a Rational Function of the Form $y = \dfrac{ax + b}{cx + d}$**

Graph $f(x) = \dfrac{2x + 1}{x - 3}$. State the domain and range.

X	Y

EXAMPLE 4 **Rewriting and Graphing a Rational Function**

Rewrite $g(x) = \dfrac{3x+5}{x+1}$ in the form $g(x) = \dfrac{a}{x-h} + k$. Graph the function. Describe

the graph of g as a transformation of the graph of $f(x) = \dfrac{a}{x}$.

EXAMPLE 5 **Modeling with Mathematics**

A 3-D printer builds up layers of materials to make three-dimensional models. Each deposited layer bonds to the layer below it. A company decides to make small display models of engine components using a 3-D printer. The printer costs $1000. The material for each model costs $50.

- Estimate how many models must be printed for the average cost per model to fall to $90.

- What happens to the average cost as more models are printed?

USING A GRAPHING CALCULATOR

Because the number of models and average cost cannot be negative, choose a viewing window in the first quadrant.