Chapter 8
Sequences and Series

Section 8-3
Analyzing Geometric Sequences and Series

Identifying Geometric Sequences

In a **geometric sequence**, the ratio of any term to the previous term is constant. This constant ratio is called the **common ratio** and is denoted by r.

EXAMPLE 1 Identifying Geometric Sequences

Tell whether each sequence is geometric.

a. 6, 12, 20, 30, 42, . . .

b. 256, 64, 16, 4, 1, . . .

Writing Rules for Geometric Sequences

Core Concept

Rule for a Geometric Sequence

Algebra The nth term of a geometric sequence with first term a_1 and common ratio r is given by:

$$a_n = a_1 r^{n-1}$$

Example The nth term of a geometric sequence with a first term of 2 and a common ratio of 3 is given by:

$$a_n = 2(3)^{n-1}$$

EXAMPLE 2 Writing a Rule for the *n*th Term

Write a rule for the *n*th term of each sequence. Then find a_8.

a. 5, 15, 45, 135, . . .

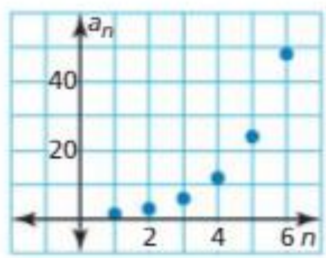
b. 88, −44, 22, −11, . . .

EXAMPLE 3 Writing a Rule Given a Term and Common Ratio

One term of a geometric sequence is $a_4 = 12$. The common ratio is $r = 2$. Write a rule for the nth term. Then graph the first six terms of the sequence.

Use the rule to create a table of values for the sequence. Then plot the points.

n	1	2	3	4	5	6
a_n	1.5	3	6	12	24	48



EXAMPLE 4 Writing a Rule Given Two Terms

Two terms of a geometric sequence are $a_2 = 12$ and $a_5 = -768$. Write a rule for the nth term.

Check

Use the rule to verify that the 2nd term is 12 and the 5th term is -768.

$a_2 = -3(-4)^{2-1}$

$= -3(-4) = 12$ ✓

$a_5 = -3(-4)^{5-1}$

$= -3(256) = -768$ ✓

Finding Sums of Finite Geometric Series

The expression formed by adding the terms of a geometric sequence is called a **geometric series**. The sum of the first n terms of a geometric series is denoted by S_n.

Core Concept

The Sum of a Finite Geometric Series

The sum of the first n terms of a geometric series with common ratio $r \neq 1$ is

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right).$$

EXAMPLE 5 Finding the Sum of a Geometric Series

Find the sum $\sum_{k=1}^{10} 4(3)^{k-1}$.

Check

Use a graphing calculator to check the sum.

```
sum(seq(4*3^(X-1
),X,1,10))
              118096
```

Tutorial

EXAMPLE 6 Solving a Real-Life Problem

You can calculate the monthly payment M (in dollars) for a loan using the formula

$$M = \frac{L}{\sum_{k=1}^{t}\left(\frac{1}{1+i}\right)^k}$$

where L is the loan amount (in dollars), i is the monthly interest rate (in decimal form), and t is the term (in months). Calculate the monthly payment on a 5-year loan for \$20,000 with an annual interest rate of 6%.

USING TECHNOLOGY

Storing the value of $\frac{1}{1.005}$ helps minimize mistakes and also assures an accurate answer. Rounding this value to 0.995 results in a monthly payment of \$386.94.

SOLUTION

Step 1 Substitute for L, i, and t. The loan amount is $L = 20{,}000$, the monthly interest rate is $i = \frac{0.06}{12} = 0.005$, and the term is $t = 5(12) = 60$.

$$M = \frac{20{,}000}{\sum_{k=1}^{60}\left(\frac{1}{1+0.005}\right)^k}$$

Step 2 Notice that the denominator is a geometric series with first term $\frac{1}{1.005}$ and common ratio $\frac{1}{1.005}$. Use a calculator to find the monthly payment.

```
1/1.005→R
        .9950248756
R((1-R^60)/(1-R)
)
        51.72556075
20000/Ans
        386.6560306
```

▶ So, the monthly payment is \$386.66.