Chapter 8
Sequences and Series

Section 8-4
Finding Sums of Infinite Geometric Series

Partial Sums of Infinite Geometric Series

The sum S_n of the first n terms of an infinite series is called a **partial sum**. The partial sums of an infinite geometric series may approach a limiting value.

EXAMPLE 1 Finding Partial Sums

Consider the infinite geometric series

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots$$

Find and graph the partial sums S_n for $n = 1, 2, 3, 4$, and 5. Then describe what happens to S_n as n increases.

SOLUTION

Step 1 Find the partial sums.

$$S_1 = \frac{1}{2} = 0.5$$

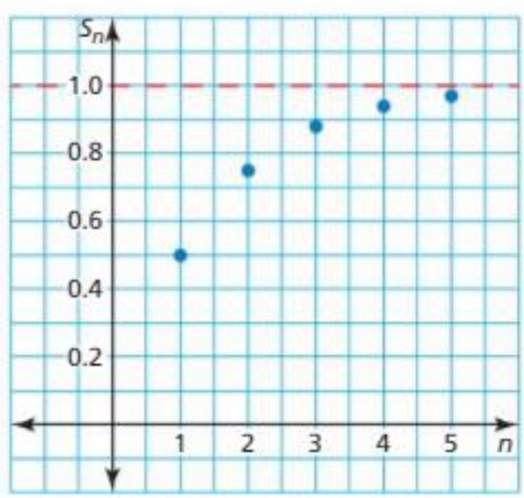
$$S_2 = \frac{1}{2} + \frac{1}{4} = 0.75$$

$$S_3 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} \approx 0.88$$

$$S_4 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \approx 0.94$$

$$S_5 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} \approx 0.97$$

Step 2 Plot the points (1, 0.5), (2, 0.75), (3, 0.88), (4, 0.94), and (5, 0.97). The graph is shown at the right.



▶ From the graph, S_n appears to approach 1 as n increases.

Core Concept

The Sum of an Infinite Geometric Series

The sum of an infinite geometric series with first term a_1 and common ratio r is given by

$$S = \frac{a_1}{1 - r}$$

provided $|r| < 1$. If $|r| \geq 1$, then the series has no sum.

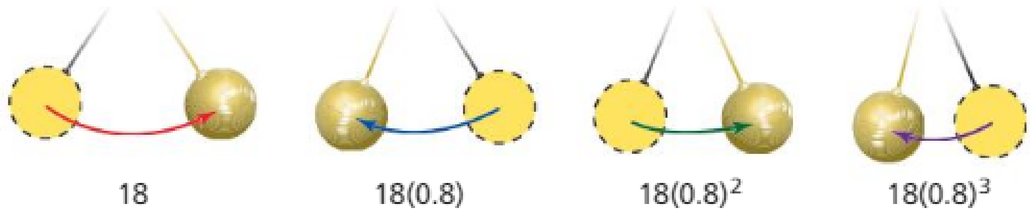
EXAMPLE 2 Finding Sums of Infinite Geometric Series

Find the sum of each infinite geometric series.

a. $\sum_{i=1}^{\infty} 3(0.7)^{i-1}$ **b.** $1 + 3 + 9 + 27 + \cdots$ **c.** $1 - \frac{3}{4} + \frac{9}{16} - \frac{27}{64} + \cdots$

EXAMPLE 3 Solving a Real-Life Problem

A pendulum that is released to swing freely travels 18 inches on the first swing. On each successive swing, the pendulum travels 80% of the distance of the previous swing. What is the total distance the pendulum swings?



EXAMPLE 4 Writing a Repeating Decimal as a Fraction

Write 0.242424 . . . as a fraction in simplest form.