Chapter 8
Sequences and Series

Section 8-5
Using Recursive Rules with Sequences

Evaluating Recursive Rules

So far in this chapter, you have worked with explicit rules for the nth term of a sequence, such as $a_n = 3n - 2$ and $a_n = 7(0.5)^n$. An **explicit rule** gives a_n as a function of the term's position number n in the sequence.

In this section, you will learn another way to define a sequence—by a *recursive rule*. A **recursive rule** gives the beginning term(s) of a sequence and a *recursive equation* that tells how a_n is related to one or more preceding terms.

EXAMPLE 1 Evaluating Recursive Rules

Write the first six terms of each sequence.

a. $a_0 = 1, a_n = a_{n-1} + 4$

b. $f(1) = 1, f(n) = 3 \cdot f(n - 1)$

Writing Recursive Rules

In part (a) of Example 1, the *differences* of consecutive terms of the sequence are constant, so the sequence is arithmetic. In part (b), the *ratios* of consecutive terms are constant, so the sequence is geometric. In general, rules for arithmetic and geometric sequences can be written recursively as follows.

Core Concept

Recursive Equations for Arithmetic and Geometric Sequences

Arithmetic Sequence

$a_n = a_{n-1} + d$, where d is the common difference

Geometric Sequence

$a_n = r \cdot a_{n-1}$, where r is the common ratio

EXAMPLE 2 Writing Recursive Rules

Write a recursive rule for (a) 3, 13, 23, 33, 43, . . . and (b) 16, 40, 100, 250, 625, ...

EXAMPLE 3 Writing Recursive Rules

Write a recursive rule for each sequence.

a. 1, 1, 2, 3, 5, . . .

b. 1, 1, 2, 6, 24, . . .

Translating Between Recursive and Explicit Rules

Tutorial

EXAMPLE 4 Translating from Explicit Rules to Recursive Rules

Write a recursive rule for (a) $a_n = -6 + 8n$ and (b) $a_n = -3\left(\frac{1}{2}\right)^{n-1}$.

EXAMPLE 5 Translating from Recursive Rules to Explicit Rules

Write an explicit rule for each sequence.

a. $a_1 = -5, a_n = a_{n-1} - 2$

b. $a_1 = 10, a_n = 2a_{n-1}$