Chapter 1
Linear Functions

Section 1-4
Solving Linear Systems

1. Solve

$$-2x + 2y = -4$$
$$4x - 9y = 28$$

2. Solve

$$56x + 8y = -32$$
$$y = -7x - 4$$

3. Solve

$$3x + y = 1$$
$$2y + 6x = -18$$

A **linear equation in three variables** x, y, and z is an equation of the form $ax + by + cz = d$, where a, b, and c are not all zero.

The following is an example of a **system of three linear equations** in three variables.

$$3x + 4y - 8z = -3 \qquad \text{Equation 1}$$

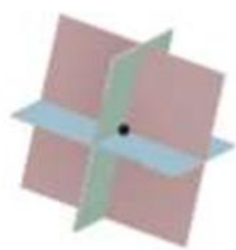
$$x + y + 5z = -12 \qquad \text{Equation 2}$$

$$4x - 2y + z = 10 \qquad \text{Equation 3}$$

A **solution** of such a system is an **ordered triple** (x, y, z) whose coordinates make each equation true.

The graph of a linear equation in three variables is a plane in three-dimensional space. The graphs of three such equations that form a system are three planes whose intersection determines the number of solutions of the system, as shown in the diagrams below.

Exactly One Solution
The planes intersect in a single point, which is the solution of the system.



Infinitely Many Solutions
The planes intersect in a line. Every point on the line is a solution of the system.

The planes could also be the same plane. Every point in the plane is a solution of the system.

No Solution
There are no points in common with all three planes.

Solving Systems of Equations Algebraically

The algebraic methods you used to solve systems of linear equations in two variables can be extended to solve a system of linear equations in three variables.

Core Concept

Solving a Three-Variable System

Step 1 Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

Step 2 Solve the new linear system for both of its variables.

Step 3 Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as $0 = 1$, in any of the steps, the system has no solution.

When you do not obtain a false equation, but obtain an identity such as $0 = 0$, the system has infinitely many solutions.

EXAMPLE 1 Solving a Three-Variable System (One Solution)

Solve the system.

$4x + 2y + 3z = 12$ Equation 1

$2x - 3y + 5z = -7$ Equation 2

$6x - y + 4z = -3$ Equation 3

EXAMPLE 2 Solving a Three-Variable System (No Solution)

Solve the system.

$x + y + z = 2$ Equation 1

$5x + 5y + 5z = 3$ Equation 2

$4x + y - 3z = -6$ Equation 3

EXAMPLE 3 Solving a Three-Variable System (Many Solutions)

Solve the system.

$x - y + z = -3$ Equation 1

$x - y - z = -3$ Equation 2

$5x - 5y + z = -15$ Equation 3

Solve the system. Check your solution, if possible.

1. $x - 2y + z = -11$
$3x + 2y - z = 7$
$-x + 2y + 4z = -9$

2. $x + y - z = -1$
$4x + 4y - 4z = -2$
$3x + 2y + z = 0$

3. $x + y + z = 8$
$x - y + z = 8$
$2x + y + 2z = 16$

4. In Example 3, describe the solutions of the system using an ordered triple in terms of y.

Solving Real-Life Problems

EXAMPLE 4 **Solving a Multi-Step Problem**

An amphitheater charges \$75 for each seat in Section A, \$55 for each seat in Section B, and \$30 for each lawn seat. There are three times as many seats in Section B as in Section A. The revenue from selling all 23,000 seats is \$870,000. How many seats are in each section of the amphitheater?