

Chapter 3
Quadratic Equations and Complex Numbers

Section 3-3
Completing the Square

Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation, you can take the square root of both sides. Be sure to consider the positive and negative square roots.	$x^2 = 15$ $\sqrt{x^2} = \pm\sqrt{15}$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a nonnegative real number, then $x = \pm\sqrt{a}$.

Reading Math

Read $\pm\sqrt{a}$ as “plus or minus square root of a .”

EXAMPLE 1 Solving a Quadratic Equation Using Square Roots

Solve $x^2 - 16x + 64 = 100$ using square roots.

If a quadratic expression of the form $x^2 + bx$ cannot model a square, you can add a term to form a perfect square trinomial. This is called **completing the square**.

IMPORTANT!

To Complete the Square $a=1$ (a must be equal to one).

You can complete the square to solve quadratic equations.

Solving Quadratic Equations $ax^2 + bx + c = 0$ by Completing the Square

1. Collect variable terms on one side of the equation and constants on the other.
2. As needed, divide both sides by a to make the coefficient of the x^2 -term 1.
3. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
4. Factor the variable expression as a perfect square.
5. Take the square root of both sides of the equation.
6. Solve for the values of the variable.

EXAMPLE 2

Making a Perfect Square Trinomial

Find the value of c that makes $x^2 + 14x + c$ a perfect square trinomial. Then write the expression as the square of a binomial.

EXAMPLE 3**Solving $ax^2 + bx + c = 0$ when $a = 1$**

Solve $x^2 - 10x + 7 = 0$ by completing the square.

EXAMPLE 4**Solving $ax^2 + bx + c = 0$ when $a \neq 1$**

Solve $3x^2 + 12x + 15 = 0$ by completing the square.

Writing Quadratic Functions in Vertex Form

Recall that the vertex form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the vertex of the graph of the function. You can write a quadratic function in vertex form by completing the square.

EXAMPLE 5

Writing a Quadratic Function in Vertex Form

Write $y = x^2 - 12x + 18$ in vertex form. Then identify the vertex.



EXAMPLE 6

Modeling with Mathematics

The height y (in feet) of a baseball t seconds after it is hit can be modeled by the function

$$y = -16t^2 + 96t + 3.$$

Find the maximum height of the baseball.

How long does the ball take to hit the ground?