Chapter 3
Quadratic Equations and Complex Numbers

Section 3-4
Using the Quadratic Formula

IMPORTANAT!

To Complete the Square a=1 (the variable “a” must be equal to one). With the Quadratic formula, the variable “a” may be ANY number.

The Quadratic Formula

If $ax^2 + bx + c = 0$ $(a \neq 0)$, then the solutions, or roots, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

You can use the Quadratic Formula to solve any quadratic equation that is written in standard form, including equations with real solutions or complex solutions.

EXAMPLE 1 **Solving an Equation with Two Real Solutions**

Solve $x^2 + 3x = 5$ using the Quadratic Formula.

EXAMPLE 2 Solving an Equation with One Real Solution

Solve $25x^2 - 8x = 12x - 4$ using the Quadratic Formula.

EXAMPLE 3 Solving an Equation with Imaginary Solutions

Solve $-x^2 + 4x = 13$ using the Quadratic Formula.

Analyzing the Discriminant

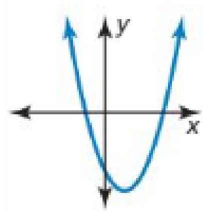
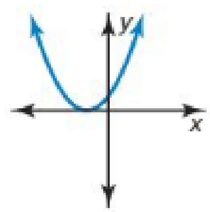
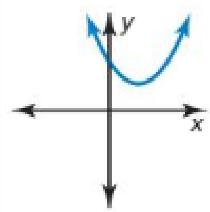
In the Quadratic Formula, the expression $b^2 - 4ac$ is called the **discriminant** of the associated equation $ax^2 + bx + c = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ ⬅ discriminant

You can analyze the discriminant of a quadratic equation to determine the number and type of solutions of the equation.

Core Concept

Analyzing the Discriminant of $ax^2 + bx + c = 0$

Value of discriminant	$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
Number and type of solutions	Two real solutions	One real solution	Two imaginary solutions
Graph of $y = ax^2 + bx + c$	 Two x-intercepts	 One x-intercept	 No x-intercept

EXAMPLE 4 **Analyzing the Discriminant**

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

a. $x^2 - 6x + 10 = 0$ **b.** $x^2 - 6x + 9 = 0$ **c.** $x^2 - 6x + 8 = 0$

Solving Real-Life Problems

The function $h = -16t^2 + h_0$ is used to model the height of a *dropped* object. For an object that is *launched* or *thrown*, an extra term v_0t must be added to the model to account for the object's initial vertical velocity v_0 (in feet per second). Recall that h is the height (in feet), t is the time in motion (in seconds), and h_0 is the initial height (in feet).

$h = -16t^2 + h_0$ Object is dropped.

$h = -16t^2 + v_0t + h_0$ Object is launched or thrown.

As shown below, the value of v_0 can be positive, negative, or zero depending on whether the object is launched upward, downward, or parallel to the ground.

$V_0 > 0$

$V_0 < 0$

$V_0 = 0$

Tutorial

EXAMPLE 6 Modeling a Launched Object

A juggler tosses a ball into the air. The ball leaves the juggler's hand 4 feet above the ground and has an initial vertical velocity of 30 feet per second. The juggler catches the ball when it falls back to a height of 3 feet. How long is the ball in the air?

The table shows five methods for solving quadratic equations. For a given equation, it may be more efficient to use one method instead of another. Suggestions about when to use each method are shown below.

Concept Summary

Methods for Solving Quadratic Equations

Method	When to Use
Graphing	Use when approximate solutions are adequate.
Using square roots	Use when solving an equation that can be written in the form $u^2 = d$, where u is an algebraic expression.
Factoring	Use when a quadratic equation can be factored easily.
Completing the square	Can be used for *any* quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and b is an even number.
Quadratic Formula	Can be used for *any* quadratic equation.