Chapter 3
Quadratic Equations and Complex Numbers

Section 3-5
Solving Nonlinear Systems

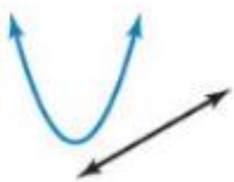
Systems of Nonlinear Equations

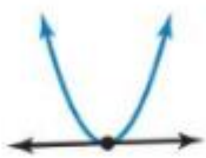
Previously, you solved systems of *linear* equations by graphing, substitution, and elimination. You can also use these methods to solve a system of *nonlinear* equations. In a **system of nonlinear equations**, at least one of the equations is nonlinear. For instance, the nonlinear system shown has a quadratic equation and a linear equation.

$y = x^2 + 2x - 4$ Equation 1 is nonlinear.

$y = 2x + 5$ Equation 2 is linear.

When the graphs of the equations in a system are a line and a parabola, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.


No solution

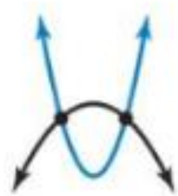

One solution


Two solutions

When the graphs of the equations in a system are a parabola that opens up and a parabola that opens down, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.


No solution


One solution


Two solutions

EXAMPLE 1 Solving a Nonlinear System by Graphing

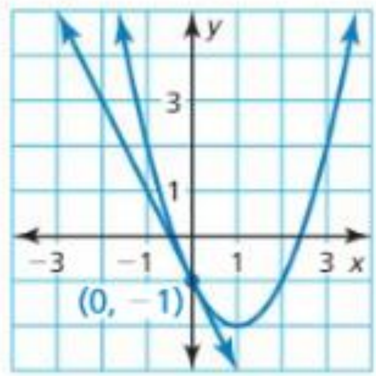
Solve the system by graphing.

$y = x^2 - 2x - 1$ Equation 1

$y = -2x - 1$ Equation 2

SOLUTION

Graph each equation. Then estimate the point of intersection. The parabola and the line appear to intersect at the point $(0, -1)$. Check the point by substituting the coordinates into each of the original equations.



Equation 1

$y = x^2 - 2x - 1$

$-1 \stackrel{?}{=} (0)^2 - 2(0) - 1$

$-1 = -1$ ✓

Equation 2

$y = -2x - 1$

$-1 \stackrel{?}{=} -2(0) - 1$

$-1 = -1$ ✓

▶ The solution is $(0, -1)$.

Remember that graphing is not the best way to solve equations. Sometimes it can be hard to look to see where two graphs intersect.

EXAMPLE 2 Solving a Nonlinear System by Substitution

Tutorial

Solve the system by substitution. $x^2 + x - y = -1$ Equation 1

$x + y = 4$ Equation 2

SOLUTION

Begin by solving for y in Equation 2.

$y = -x + 4$ Solve for y in Equation 2.

Next, substitute $-x + 4$ for y in Equation 1 and solve for x.

$x^2 + x - y = -1$ Write Equation 1.

$x^2 + x - (-x + 4) = -1$ Substitute $-x + 4$ for y.

$x^2 + 2x - 4 = -1$ Simplify.

$x^2 + 2x - 3 = 0$ Write in standard form.

$(x + 3)(x - 1) = 0$ Factor.

$x + 3 = 0$ or $x - 1 = 0$ Zero-Product Property

$x = -3$ or $x = 1$ Solve for x.

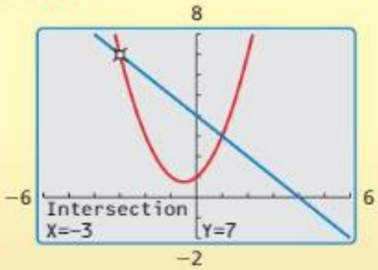
To solve for y, substitute $x = -3$ and $x = 1$ into the equation $y = -x + 4$.

$y = -x + 4 = -(-3) + 4 = 7$ Substitute -3 for x.

$y = -x + 4 = -1 + 4 = 3$ Substitute 1 for x.

▶ The solutions are $(-3, 7)$ and $(1, 3)$. Check the solutions by graphing the system.

Check



EXAMPLE 3 Solving a Nonlinear System by Elimination

Tutorial

Solve the system by elimination. $x^2 - 5x - y = -2$ Equation 1

$x^2 + 2x + y = 0$ Equation 2

SOLUTION

Add the equations to eliminate the y-term and obtain a quadratic equation in x.

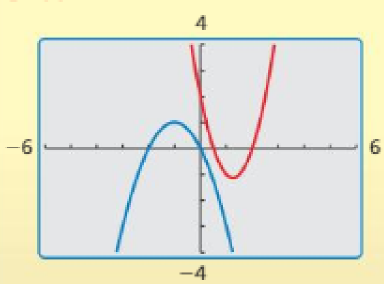
$$\begin{array}{rl} 2x^2 - 5x - y &= -2 \\ x^2 + 2x + y &= 0 \\ \hline 3x^2 - 3x \quad &= -2 \end{array}$$ Add the equations.

$3x^2 - 3x + 2 = 0$ Write in standard form.

$x = \dfrac{3 \pm \sqrt{-15}}{6}$ Use the Quadratic Formula.

▶ Because the discriminant is negative, the equation $3x^2 - 3x + 2 = 0$ has no real solution. So, the original system has no real solution. You can check this by graphing the system and seeing that the graphs do not appear to intersect.

Check

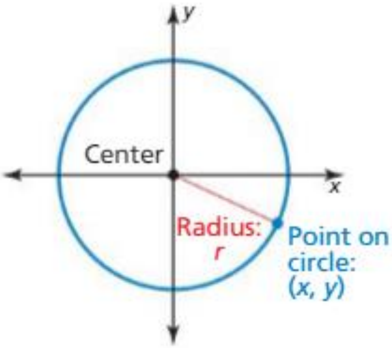


Solve the system using any method. Explain your choice of method.

1. $y = -x^2 + 4$
$y = -4x + 8$

2. $x^2 + 3x + y = 0$
$2x + y = 5$

3. $2x^2 + 4x - y = -2$
$x^2 + y = 2$

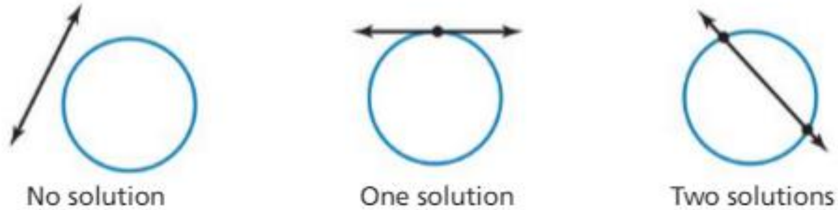


Some nonlinear systems have equations of the form

$$x^2 + y^2 = r^2.$$

This equation is the standard form of a circle with center (0, 0) and radius r.

When the graphs of the equations in a system are a line and a circle, the graphs can intersect in zero, one, or two points. So, the system can have zero, one, or two solutions, as shown.



EXAMPLE 4 Solving a Nonlinear System by Substitution

Solve the system by substitution.

$x^2 + y^2 = 10$ Equation 1

$y = -3x + 10$ Equation 2

SOLUTION

Substitute $-3x + 10$ for y in Equation 1 and solve for x.

$x^2 + y^2 = 10$	Write Equation 1.
$x^2 + (-3x + 10)^2 = 10$	Substitute $-3x + 10$ for y.
$x^2 + 9x^2 - 60x + 100 = 10$	Expand the power.
$10x^2 - 60x + 90 = 0$	Write in standard form.
$x^2 - 6x + 9 = 0$	Divide each side by 10.
$(x - 3)^2 = 0$	Perfect Square Trinomial Pattern
$x = 3$	Zero-Product Property

COMMON ERROR

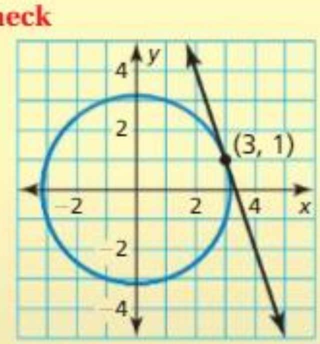
You can also substitute $x = 3$ in Equation 1 to find y. This yields two *apparent* solutions, (3, 1) and (3, −1). However, (3, −1) is *not* a solution because it does not satisfy Equation 2. You can also see (3, −1) is not a solution from the graph.

To find the y-coordinate of the solution, substitute $x = 3$ in Equation 2.

$$y = -3(3) + 10 = 1$$

▶ The solution is (3, 1). Check the solution by graphing the system. You can see that the line and the circle intersect only at the point (3, 1).

Check



Solve the system.

4. $x^2 + y^2 = 16$

$y = -x + 4$

5. $x^2 + y^2 = 4$

$y = x + 4$

6. $x^2 + y^2 = 1$

$y = \frac{1}{2}x + \frac{1}{2}$