Chapter 3
Quadratic Equations and Complex Numbers

Section 3-6
Quadratic Inequalities

Graphing Quadratic Inequalities in Two Variables

A **quadratic inequality in two variables** can be written in one of the following forms, where a, b, and c are real numbers and $a \neq 0$.

$$y < ax^2 + bx + c \qquad y > ax^2 + bx + c$$

$$y \leq ax^2 + bx + c \qquad y \geq ax^2 + bx + c$$

The graph of any such inequality consists of all solutions (x, y) of the inequality.

Previously, you graphed linear inequalities in two variables. You can use a similar procedure to graph quadratic inequalities in two variables.

Core Concept

Graphing a Quadratic Inequality in Two Variables

To graph a quadratic inequality in one of the forms above, follow these steps.

Step 1 Graph the parabola with the equation $y = ax^2 + bx + c$. Make the parabola *dashed* for inequalities with $<$ or $>$ and *solid* for inequalities with $\leq$ or $\geq$.

Step 2 Test a point (x, y) inside the parabola to determine whether the point is a solution of the inequality.

Step 3 Shade the region inside the parabola if the point from Step 2 is a solution. Shade the region outside the parabola if it is not a solution.

Remember to use the vertex formula ($x = -\frac{b}{2a}$) to graph a quadratic in Standard Form.

EXAMPLE 1 Graphing a Quadratic Inequality in Two Variables

Graph $y < -x^2 - 2x - 1$.

SOLUTION

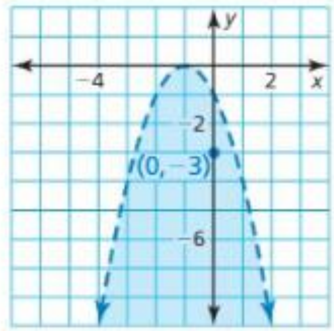
Step 1 Graph $y = -x^2 - 2x - 1$. Because the inequality symbol is $<$, make the parabola dashed.

Step 2 Test a point inside the parabola, such as $(0, -3)$.

$$y < -x^2 - 2x - 1$$
$$-3 \overset{?}{<} -0^2 - 2(0) - 1$$
$$-3 < -1 \checkmark$$

So, $(0, -3)$ is a solution of the inequality.

Step 3 Shade the region inside the parabola.



LOOKING FOR STRUCTURE

Notice that testing a point is less complicated when the x-value is 0 (the point is on the y-axis).

EXAMPLE 2 Graphing a System of Quadratic Inequalities

Graph the system of quadratic inequalities.

$y < -x^2 + 3$ Inequality 1

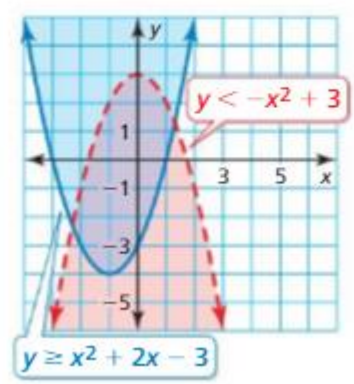
$y \geq x^2 + 2x - 3$ Inequality 2

SOLUTION

Step 1 Graph $y < -x^2 + 3$. The graph is the red region inside (but not including) the parabola $y = -x^2 + 3$.

Step 2 Graph $y \geq x^2 + 2x - 3$. The graph is the blue region inside and including the parabola $y = x^2 + 2x - 3$.

Step 3 Identify the purple region where the two graphs overlap. This region is the graph of the system.



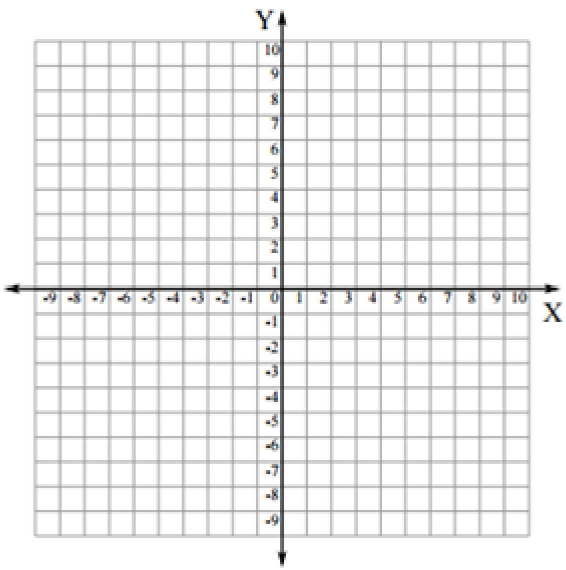
Check

Check that a point in the solution region, such as $(0, 0)$, is a solution of the system.

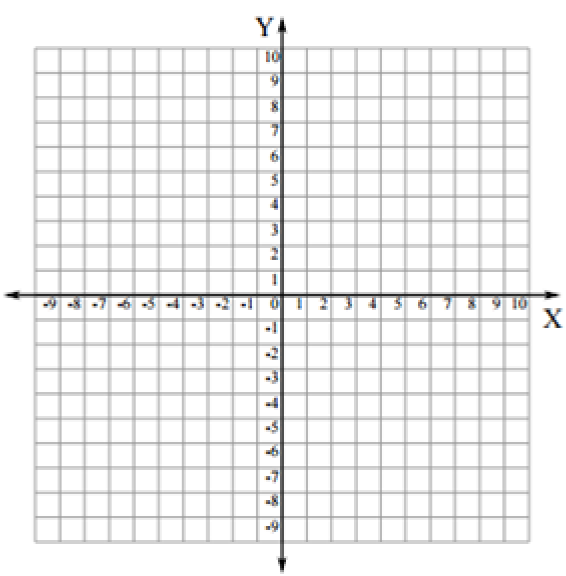
$$y < -x^2 + 3$$
$$0 \overset{?}{<} -0^2 + 3$$
$$0 < 3 \checkmark$$
$$y \geq x^2 + 2x - 3$$
$$0 \overset{?}{\geq} 0^2 + 2(0) - 3$$
$$0 \geq -3 \checkmark$$

Graph the inequality.

1. $y \geq x^2 + 2x - 8$
2. $y \leq 2x^2 - x - 1$
3. $y > -x^2 + 2x + 4$
4. Graph the system of inequalities consisting of $y \leq -x^2$ and $y > x^2 - 3$.



X	Y



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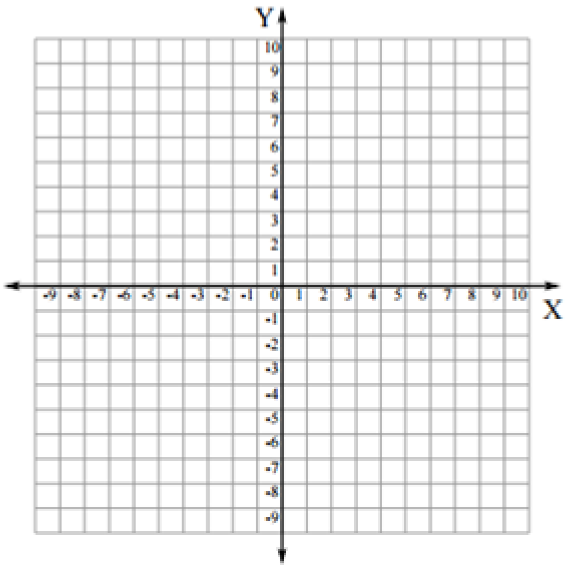
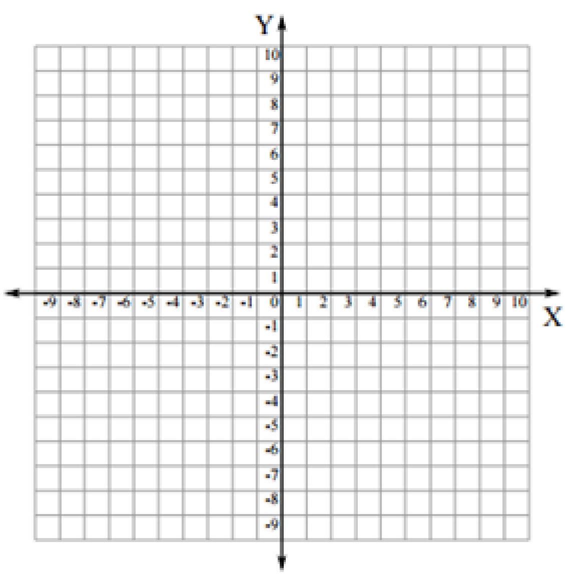


X	Y



X	Y

Solving Quadratic Inequalities in One Variable

A **quadratic inequality in one variable** can be written in one of the following forms, where a, b, and c are real numbers and $a \neq 0$.

$ax^2 + bx + c < 0$ $\quad$ $ax^2 + bx + c > 0$ $\quad$ $ax^2 + bx + c \leq 0$ $\quad$ $ax^2 + bx + c \geq 0$

You can solve quadratic inequalities using algebraic methods or graphs.

EXAMPLE 4 Solving a Quadratic Inequality Algebraically

Solve $x^2 - 3x - 4 < 0$ algebraically.

SOLUTION

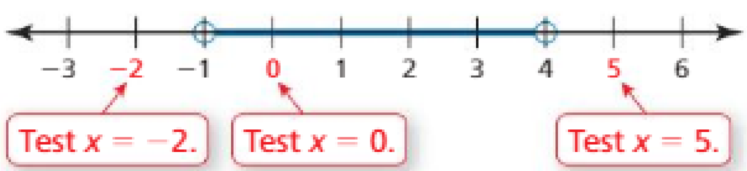
First, write and solve the equation obtained by replacing $<$ with $=$.

$x^2 - 3x - 4 = 0$ — Write the related equation.

$(x - 4)(x + 1) = 0$ — Factor.

$x = 4$ or $x = -1$ — Zero-Product Property

The numbers -1 and 4 are the *critical values* of the original inequality. Plot -1 and 4 on a number line, using open dots because the values do not satisfy the inequality. The critical x-values partition the number line into three intervals. Test an x-value in each interval to determine whether it satisfies the inequality.



$(-2)^2 - 3(-2) - 4 = 6 \not< 0$ $\quad$ $0^2 - 3(0) - 4 = -4 < 0$ ✓ $\quad$ $5^2 - 3(5) - 4 = 6 \not< 0$

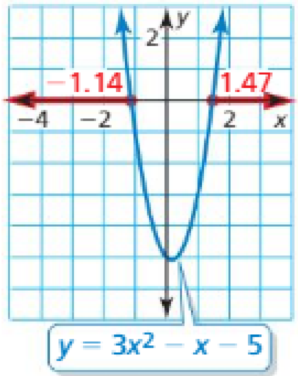
So, the solution is $-1 < x < 4$.

Another way to solve $ax^2 + bx + c < 0$ is to first graph the related function $y = ax^2 + bx + c$. Then, because the inequality symbol is $<$, identify the x-values for which the graph lies *below* the x-axis. You can use a similar procedure to solve quadratic inequalities that involve $\leq$, $>$, or $\geq$.

EXAMPLE 5 Solving a Quadratic Inequality by Graphing

Solve $3x^2 - x - 5 \geq 0$ by graphing.

SOLUTION



The solution consists of the x-values for which the graph of $y = 3x^2 - x - 5$ lies on or above the x-axis. Find the x-intercepts of the graph by letting $y = 0$ and using the Quadratic Formula to solve $0 = 3x^2 - x - 5$ for x.

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(-5)}}{2(3)} \qquad a = 3, b = -1, c = -5$$

$$x = \frac{1 \pm \sqrt{61}}{6} \qquad \text{Simplify.}$$

The solutions are $x \approx -1.14$ and $x \approx 1.47$. Sketch a parabola that opens up and has -1.14 and 1.47 as x-intercepts. The graph lies on or above the x-axis to the left of (and including) $x = -1.14$ and to the right of (and including) $x = 1.47$.

The solution of the inequality is approximately $x \leq -1.14$ or $x \geq 1.47$.

Solve the inequality.

5. $2x^2 + 3x \leq 2$

6. $-3x^2 - 4x + 1 < 0$

7. $2x^2 + 2 > -5x$