Chapter 4
Polynomial Functions

Section 4-1
Graphing Polynomial Functions

Polynomial Functions

Recall that a monomial is a number, a variable, or the product of a number and one or more variables with whole number exponents. A **polynomial** is a monomial or a sum of monomials. A **polynomial function** is a function of the form

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n \neq 0$, the exponents are all whole numbers, and the coefficients are all real numbers. For this function, a_n is the leading coefficient, n is the degree, and a_0 is the constant term. A polynomial function is in *standard form* when its terms are written in descending order of exponents from left to right.

You are already familiar with some types of polynomial functions, such as linear and quadratic. Here is a summary of common types of polynomial functions.

Common Polynomial Functions			
Degree	**Type**	**Standard Form**	**Example**
0	Constant	$f(x) = a_0$	$f(x) = -14$
1	Linear	$f(x) = a_1x + a_0$	$f(x) = 5x - 7$
2	Quadratic	$f(x) = a_2x^2 + a_1x + a_0$	$f(x) = 2x^2 + x - 9$
3	Cubic	$f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^3 - x^2 + 3x$
4	Quartic	$f(x) = a_4x^4 + a_3x^3 + a_2x^2 + a_1x + a_0$	$f(x) = x^4 + 2x - 1$

EXAMPLE 1 Identifying Polynomial Functions

Decide whether each function is a polynomial function. If so, write it in standard form and state its degree, type, and leading coefficient.

a. $f(x) = -2x^3 + 5x + 8$

b. $g(x) = -0.8x^3 + \sqrt{2}x^4 - 12$

c. $h(x) = -x^2 + 7x^{-1} + 4x$

d. $k(x) = x^2 + 3^x$

EXAMPLE 2 Evaluating a Polynomial Function

Evaluate $f(x) = 2x^4 - 8x^2 + 5x - 7$ when $x = 3$.

The **end behavior** of a function's graph is the behavior of the graph as x approaches positive infinity ($+\infty$) or negative infinity ($-\infty$). For the graph of a polynomial function, the end behavior is determined by the function's degree and the sign of its leading coefficient.

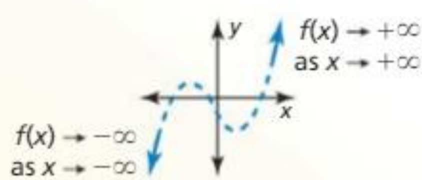
READING

The expression "$x \to +\infty$" is read as "x approaches positive infinity."

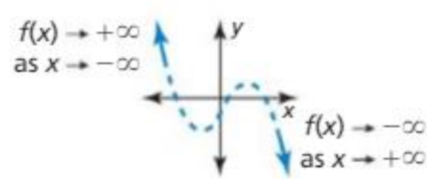
Core Concept

End Behavior of Polynomial Functions

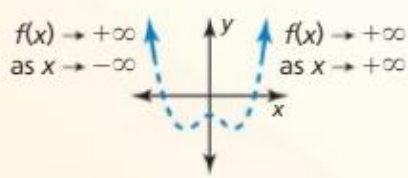
Degree: odd
Leading coefficient: positive



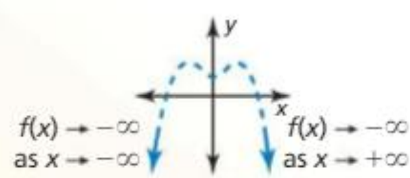
Degree: odd
Leading coefficient: negative



Degree: even
Leading coefficient: positive



Degree: even
Leading coefficient: negative

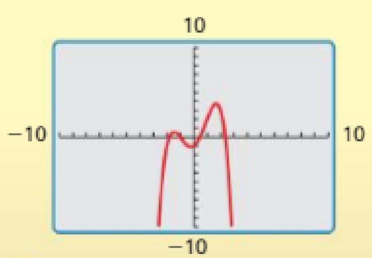


EXAMPLE 3 Describing End Behavior

Tutorial

Describe the end behavior of the graph of $f(x) = -0.5x^4 + 2.5x^2 + x - 1$.

Check



Evaluate the function for the given value of x.

5. $f(x) = 3x^5 - x^4 - 6x + 10;\ x = -2$

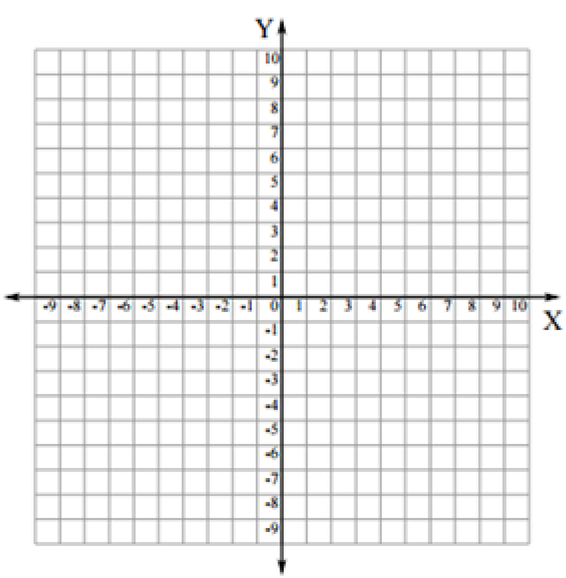
6. Describe the end behavior of the graph of $f(x) = 0.25x^3 - x^2 - 1$.

Graphing Polynomial Functions

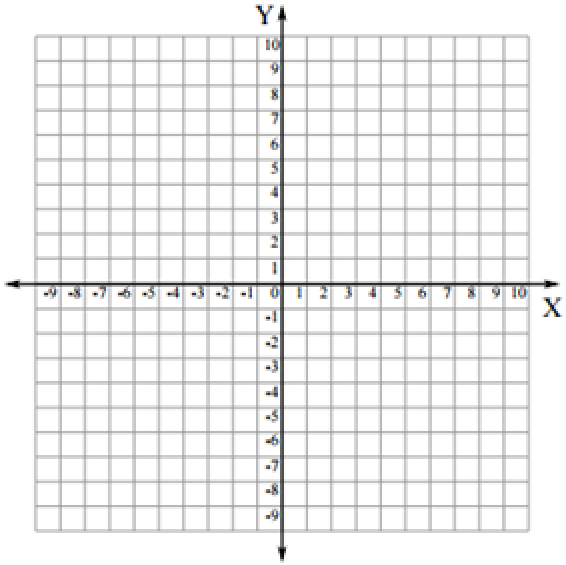
To graph a polynomial function, first plot points to determine the shape of the graph's middle portion. Then connect the points with a smooth continuous curve and use what you know about end behavior to sketch the graph.

EXAMPLE 4 Graphing Polynomial Functions

Graph (a) $f(x) = -x^3 + x^2 + 3x - 3$ and (b) $f(x) = x^4 - x^3 - 4x^2 + 4$.



X	Y



X	Y

EXAMPLE 6 Solving a Real-Life Problem

The estimated number V (in thousands) of electric vehicles in use in the United States can be modeled by the polynomial function

$$V(t) = 0.151280t^3 - 3.28234t^2 + 23.7565t - 2.041$$

where t represents the year, with $t = 1$ corresponding to 2001.

a. Use a graphing calculator to graph the function for the interval $1 \le t \le 10$. Describe the behavior of the graph on this interval.

b. What was the average rate of change in the number of electric vehicles in use from 2001 to 2010?

c. Do you think this model can be used for years before 2001 or after 2010? Explain your reasoning.