Chapter 4
Polynomial Functions

Section 4-2
Adding, Subtracting, and Multiplying Polynomials

Adding and Subtracting Polynomials

Recall that the set of integers is *closed* under addition and subtraction because every sum or difference results in an integer. To add or subtract polynomials, you add or subtract the coefficients of like terms. Because adding or subtracting polynomials results in a polynomial, the set of polynomials is closed under addition and subtraction.

Tutorial

EXAMPLE 1 Adding Polynomials

a. Add $3x^3 + 2x^2 - x - 7$ and $x^3 - 10x^2 + 8$

b. Add $9y^3 + 3y^2 - 2y + 1$ and $-5y^2 + y - 4$

EXAMPLE 2 Subtracting Polynomials

a. Subtract $2x^3 + 6x^2 - x + 1$ from $8x^3 - 3x^2 - 2x + 9$

b. Subtract $3z^2 + z - 4$ from $2z^2 + 3z$

EXAMPLE 3 Multiplying Polynomials

a. Multiply $-x^2 + 2x + 4$ and $x - 3$

b. Multiply $y + 5$ and $3y^2 - 2y + 2$

Remember to FOIL when you multiply.

REMEMBER

Product of Powers Property

$a^m \cdot a^n = a^{m+n}$

a is a real number and m and n are integers.

EXAMPLE 4 Multiplying Three Binomials

Multiply $x - 1$, $x + 4$, and $x + 5$ in a horizontal format.

COMMON ERROR

In general,

$(a \pm b)^2 \neq a^2 \pm b^2$

and

$(a \pm b)^3 \neq a^3 \pm b^3$.

Core Concept

Special Product Patterns

Sum and Difference

$(a + b)(a - b) = a^2 - b^2$ $\qquad$ $(x + 3)(x - 3) = x^2 - 9$

Square of a Binomial

$(a + b)^2 = a^2 + 2ab + b^2$ $\qquad$ $(y + 4)^2 = y^2 + 8y + 16$

$(a - b)^2 = a^2 - 2ab + b^2$ $\qquad$ $(2t - 5)^2 = 4t^2 - 20t + 25$

Cube of a Binomial $\qquad$ **Example**

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ $\qquad$ $(z + 3)^3 = z^3 + 9z^2 + 27z + 27$

$(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$ $\qquad$ $(m - 2)^3 = m^3 - 6m^2 + 12m - 8$

EXAMPLE 5 Proving a Polynomial Identity

a. Prove the polynomial identity for the cube of a binomial representing a sum:

$$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3.$$

b. Use the cube of a binomial in part (a) to calculate 11^3.

EXAMPLE 6 Using Special Product Patterns

Find each product.

a. $(4n + 5)(4n - 5)$ $\qquad$ **b.** $(9y - 2)^2$ $\qquad$ **c.** $(ab + 4)^3$

REMEMBER

Power of a Product Property

$(ab)^m = a^m b^m$

a and b are real numbers and m is an integer.

Pascal's Triangle

Consider the expansion of the binomial $(a + b)^n$ for whole number values of n. When you arrange the coefficients of the variables in the expansion of $(a + b)^n$, you will see a special pattern called **Pascal's Triangle**. Pascal's Triangle is named after French mathematician Blaise Pascal (1623–1662).

Core Concept

Pascal's Triangle

In Pascal's Triangle, the first and last numbers in each row are 1. Every number other than 1 is the sum of the closest two numbers in the row directly above it. The numbers in Pascal's Triangle are the same numbers that are the coefficients of binomial expansions, as shown in the first six rows.

	n	$(a+b)^n$	Binomial Expansion	Pascal's Triangle
0th row	0	$(a+b)^0 =$	1	1
1st row	1	$(a+b)^1 =$	$1a + 1b$	1 1
2nd row	2	$(a+b)^2 =$	$1a^2 + 2ab + 1b^2$	1 2 1
3rd row	3	$(a+b)^3 =$	$1a^3 + 3a^2b + 3ab^2 + 1b^3$	1 3 3 1
4th row	4	$(a+b)^4 =$	$1a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + 1b^4$	1 4 6 4 1
5th row	5	$(a+b)^5 =$	$1a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + 1b^5$	1 5 10 10 5 1

EXAMPLE 7 Using Pascal's Triangle to Expand Binomials

Use Pascal's Triangle to expand (a) $(x - 2)^5$ and (b) $(3y + 1)^3$.