**Chapter 4**
**Polynomial Functions**

**Section 4-3**
**Dividing Polynomials**

## Long Division of Polynomials

When you divide a polynomial $f(x)$ by a nonzero polynomial divisor $d(x)$, you get a quotient polynomial $q(x)$ and a remainder polynomial $r(x)$.

$$\frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)}$$

The degree of the remainder must be less than the degree of the divisor. When the remainder is 0, the divisor *divides evenly* into the dividend. Also, the degree of the divisor is less than or equal to the degree of the dividend $f(x)$. One way to divide polynomials is called **polynomial long division**.

**EXAMPLE 1** Using Polynomial Long Division

Divide $2x^4 + 3x^3 + 5x - 1$ by $x^2 + 3x + 2$.

**Divide using polynomial long division.**

**1.** $(x^3 - x^2 - 2x + 8) \div (x - 1)$

## Synthetic Division

There is a shortcut for dividing polynomials by binomials of the form $x - k$. This shortcut is called **synthetic division**. This method is shown in the next example.

**For Synthetic Division, the Divisor needs to be Linear**

**EXAMPLE 2** Using Synthetic Division

Divide $-x^3 + 4x^2 + 9$ by $x - 3$.

**EXAMPLE 3** Using Synthetic Division

Divide $3x^3 - 2x^2 + 2x - 5$ by $x + 1$.

**Divide using synthetic division.**

**4.** $(2x^3 - x - 7) \div (x + 3)$

## Core Concept

### The Remainder Theorem

If a polynomial $f(x)$ is divided by $x - k$, then the remainder is $r = f(k)$.

The Remainder Theorem tells you that synthetic division can be used to evaluate a polynomial function. So, to evaluate $f(x)$ when $x = k$, divide $f(x)$ by $x - k$. The remainder will be $f(k)$.

### EXAMPLE 4 Evaluating a Polynomial

Use synthetic division to evaluate $f(x) = 5x^3 - x^2 + 13x + 29$ when $x = -4$.

**Use synthetic division to evaluate the function for the indicated value of *x*.**

**5.** $f(x) = 4x^2 - 10x - 21; x = 5$

**6.** $f(x) = 5x^4 + 2x^3 - 20x - 6; x = 2$