Chapter 4
Polynomial Functions

Section 4-4
Factoring Polynomials

Factoring Polynomials

Previously, you factored quadratic polynomials. You can also factor polynomials with degree greater than 2. Some of these polynomials can be *factored completely* using techniques you have previously learned. A factorable polynomial with integer coefficients is **factored completely** when it is written as a product of unfactorable polynomials with integer coefficients.

EXAMPLE 1 Finding a Common Monomial Factor

Factor each polynomial completely.

a. $x^3 - 4x^2 - 5x$ **b.** $3y^5 - 48y^3$ **c.** $5z^4 + 30z^3 + 45z^2$

Core Concept

Special Factoring Patterns

Sum of Two Cubes

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$64x^3 + 1 = (4x)^3 + 1^3$$
$$= (4x + 1)(16x^2 - 4x + 1)$$

Difference of Two Cubes

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

$$27x^3 - 8 = (3x)^3 - 2^3$$
$$= (3x - 2)(9x^2 + 6x + 4)$$

EXAMPLE 2 Factoring the Sum or Difference of Two Cubes

Factor (a) $x^3 - 125$ and (b) $16s^5 + 54s^2$ completely.

For some polynomials, you can **factor by grouping** pairs of terms that have a common monomial factor. The pattern for factoring by grouping is shown below.

$$ra + rb + sa + sb = r(a + b) + s(a + b)$$
$$= (r + s)(a + b)$$

EXAMPLE 3 Factoring by Grouping

Factor $z^3 + 5z^2 - 4z - 20$ completely.

An expression of the form $au^2 + bu + c$, where u is an algebraic expression, is said to be in **quadratic form**. The factoring techniques you have studied can sometimes be used to factor such expressions.

LOOKING FOR STRUCTURE

The expression $16x^4 - 81$ is in quadratic form because it can be written as $u^2 - 81$ where $u = 4x^2$.

Tutorial

EXAMPLE 4 Factoring Polynomials in Quadratic Form

Factor (a) $16x^4 - 81$ and (b) $3p^8 + 15p^5 + 18p^2$ completely.

READING

In other words, $x - k$ is a factor of $f(x)$ if and only if k is a zero of f.

Core Concept

The Factor Theorem

A polynomial $f(x)$ has a factor $x - k$ if and only if $f(k) = 0$.

EXAMPLE 5 Determining Whether a Linear Binomial Is a Factor

Determine whether (a) $x - 2$ is a factor of $f(x) = x^2 + 2x - 4$ and (b) $x + 5$ is a factor of $f(x) = 3x^4 + 15x^3 - x^2 + 25$.

EXAMPLE 6 Factoring a Polynomial

Show that $x + 3$ is a factor of $f(x) = x^4 + 3x^3 - x - 3$. Then factor $f(x)$ completely.

ANOTHER WAY

Notice that you can factor $f(x)$ by grouping.

$f(x) = x^3(x + 3) - 1(x + 3)$

$= (x^3 - 1)(x + 3)$

$= (x + 3)(x - 1) \cdot (x^2 + x + 1)$

Because the x-intercepts of the graph of a function are the zeros of the function, you can use the graph to approximate the zeros. You can check the approximations using the Factor Theorem.

EXAMPLE 7 Real-Life Application

During the first 5 seconds of a roller coaster ride, the function $h(t) = 4t^3 - 21t^2 + 9t + 34$ represents the height h (in feet) of the roller coaster after t seconds. How long is the roller coaster at or below ground level in the first 5 seconds?