Chapter 5
Rational Exponents and Radical Functions

Section 5-2
Properties of Rational Exponents and Radicals

EXAMPLE 1 Using Properties of Exponents

Use the properties of rational exponents to simplify each expression.

a. $7^{1/4} \cdot 7^{1/2} =$

b. $(6^{1/2} \cdot 4^{1/3})^2 =$

c. $(4^5 \cdot 3^5)^{-1/5} =$

d. $\dfrac{5}{5^{1/3}} = \dfrac{5^1}{5^{1/3}} =$

e. $\left(\dfrac{42^{1/3}}{6^{1/3}}\right)^2 =$

EXAMPLE 2 Using Properties of Radical

Use the properties of radicals to simplify each expressio

a. $\sqrt[3]{12} \cdot \sqrt[3]{18} =$

b. $\dfrac{\sqrt[4]{80}}{\sqrt[4]{5}} =$

EXAMPLE 3 Writing Radicals in Simplest Form

Write each expression in simplest form.

a. $\sqrt[3]{135}$

b. $\dfrac{\sqrt[5]{7}}{\sqrt[5]{8}}$

For a denominator that is a sum or difference involving square roots, multiply both the numerator and denominator by the **conjugate** of the denominator. The expressions

$$a\sqrt{b} + c\sqrt{d} \quad \text{and} \quad a\sqrt{b} - c\sqrt{d}$$

are conjugates of each other, where a, b, c, and d are rational numbers.

EXAMPLE 4 Writing a Radical Expression in Simplest Form

Write $\dfrac{1}{5 + \sqrt{3}}$ in simplest form.

EXAMPLE 5 Adding and Subtracting Like Radicals and Roots

Simplify each expression.

a. $\sqrt[4]{10} + 7\sqrt[4]{10}$ **b.** $2(8^{1/5}) + 10(8^{1/5})$ **c.** $\sqrt[3]{54} - \sqrt[3]{2}$

The properties of rational exponents and radicals can also be applied to expressions involving variables. Because a variable can be positive, negative, or zero, sometimes absolute value is needed when simplifying a variable expression.

	Rule	Example
When *n* is odd	$\sqrt[n]{x^n} = x$	$\sqrt[7]{5^7} = 5$ and $\sqrt[7]{(-5)^7} = -5$
When *n* is even	$\sqrt[n]{x^n} = \lvert x \rvert$	$\sqrt[4]{3^4} = 3$ and $\sqrt[4]{(-3)^4} = 3$

Absolute value is not needed when all variables are assumed to be positive.

EXAMPLE 6 Simplifying Variable Expressions

Simplify each expression.

a. $\sqrt[3]{64y^6}$

b. $\sqrt[4]{\dfrac{x^4}{y^8}}$

EXAMPLE 7 Writing Variable Expressions in Simplest Form

Write each expression in simplest form. Assume all variables are positive.

a. $\sqrt[5]{4a^8b^{14}c^5}$ **b.** $\dfrac{x}{\sqrt[3]{y^8}}$ **c.** $\dfrac{14xy^{1/3}}{2x^{3/4}z^{-6}}$

EXAMPLE 8 Adding and Subtracting Variable Expressions

Perform each indicated operation. Assume all variables are positive.

a. $5\sqrt{y} + 6\sqrt{y}$

b. $12\sqrt[3]{2z^5} - z\sqrt[3]{54z^2}$