**Chapter 5**
**Rational Exponents and Radical Functions**

**Section 5-4**
**Solving Radical Equations and Inequalities**

## Solving Equations

Equations with radicals that have variables in their radicands are called **radical equations**. An example of a radical equation is $2\sqrt{x+1} = 4$.

### Core Concept

#### Solving Radical Equations

To solve a radical equation, follow these steps:

**Step 1** Isolate the radical on one side of the equation, if necessary.

**Step 2** Raise each side of the equation to the same exponent to eliminate the radical and obtain a linear, quadratic, or other polynomial equation.

**Step 3** Solve the resulting equation using techniques you learned in previous chapters. Check your solution.

### EXAMPLE 1 Solving Radical Equations

Solve (a) $2\sqrt{x+1} = 4$ and (b) $\sqrt[3]{2x-9} - 1 = 2$.

Raising each side of an equation to the same exponent may introduce solutions that are *not* solutions of the original equation. These solutions are called **extraneous solutions**. When you use this procedure, you should always check each apparent solution in the *original* equation.

## EXAMPLE 3 Solving an Equation with an Extraneous Solution

Solve $x + 1 = \sqrt{7x + 15}$.

## EXAMPLE 4 Solving an Equation with Two Radicals

Solve $\sqrt{x+2}+1=\sqrt{3-x}$.

## EXAMPLE 5 Solving an Equation with a Rational Exponent

Solve $(2x)^{3/4}+2=10$.

**EXAMPLE 6** **Solving an Equation with a Rational Exponent**

Solve $(x + 30)^{1/2} = x$.

## Solving Radical Inequalities

To solve a simple radical inequality of the form $\sqrt[n]{u} < d$, where $u$ is an algebraic expression and $d$ is a nonnegative number, raise each side to the exponent $n$. This procedure also works for $>$, $\leq$, and $\geq$. Be sure to consider the possible values of the radicand.

**EXAMPLE 7** **Solving a Radical Inequality**

Solve $3\sqrt{x - 1} \leq 12$.

Solve (a) $2\sqrt{x} - 3 \geq 3$ and (b) $4\sqrt[3]{x+1} < 8$.