

**Chapter 5**  
**Rational Exponents and Radical Functions**

**Section 5-5**  
**Performing Function Operations**

## Operations on Functions

You have learned how to add, subtract, multiply, and divide polynomial expressions. These operations can also be defined for functions.

### Core Concept

#### Operations on Functions

Let  $f$  and  $g$  be any two functions. A new function can be defined by performing any of the four basic operations on  $f$  and  $g$ .

Operation	Definition	Example: $f(x) = 5x$ , $g(x) = x + 2$
Addition	$(f + g)(x) = f(x) + g(x)$	$(f + g)(x) = 5x + (x + 2) = 6x + 2$
Subtraction	$(f - g)(x) = f(x) - g(x)$	$(f - g)(x) = 5x - (x + 2) = 4x - 2$
Multiplication	$(fg)(x) = f(x) \cdot g(x)$	$(fg)(x) = 5x(x + 2) = 5x^2 + 10x$
Division	$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$	$\left(\frac{f}{g}\right)(x) = \frac{5x}{x + 2}$

The domains of the sum, difference, product, and quotient functions consist of the  $x$ -values that are in the domains of both  $f$  and  $g$ . Additionally, the domain of the quotient does not include  $x$ -values for which  $g(x) = 0$ .

#### **EXAMPLE 1** Adding Two Functions

Let  $f(x) = 3\sqrt{x}$  and  $g(x) = -10\sqrt{x}$ . Find  $(f + g)(x)$  and state the domain. Then evaluate the sum when  $x = 4$ .

**EXAMPLE 2** Subtracting Two Functions

Let  $f(x) = 3x^3 - 2x^2 + 5$  and  $g(x) = x^3 - 3x^2 + 4x - 2$ . Find  $(f - g)(x)$  and state the domain. Then evaluate the difference when  $x = -2$ .

**EXAMPLE 3** Multiplying Two Functions

Let  $f(x) = x^2$  and  $g(x) = \sqrt{x}$ . Find  $(fg)(x)$  and state the domain. Then evaluate the product when  $x = 9$ .

**EXAMPLE 4** Dividing Two Functions

Let  $f(x) = 6x$  and  $g(x) = x^{3/4}$ . Find  $\left(\frac{f}{g}\right)(x)$  and state the domain. Then evaluate the quotient when  $x = 16$ .

**EXAMPLE 5****Performing Function Operations Using Technology**

Let  $f(x) = \sqrt{x}$  and  $g(x) = \sqrt{9 - x^2}$ . Use a graphing calculator to evaluate  $(f + g)(x)$ ,  $(f - g)(x)$ ,  $(fg)(x)$ , and  $\left(\frac{f}{g}\right)(x)$  when  $x = 2$ . Round your answers to two decimal places.

**SOLUTION**

**EXAMPLE 6****Solving a Real-Life Problem**

For a white rhino, heart rate  $r$  (in beats per minute) and life span  $s$  (in minutes) are related to body mass  $m$  (in kilograms) by the functions

$$r(m) = 241m^{-0.25}$$

and

$$s(m) = (6 \times 10^6)m^{0.2}.$$

- a. Find  $(rs)(m)$ .
- b. Explain what  $(rs)(m)$  represents.

**SOLUTION**