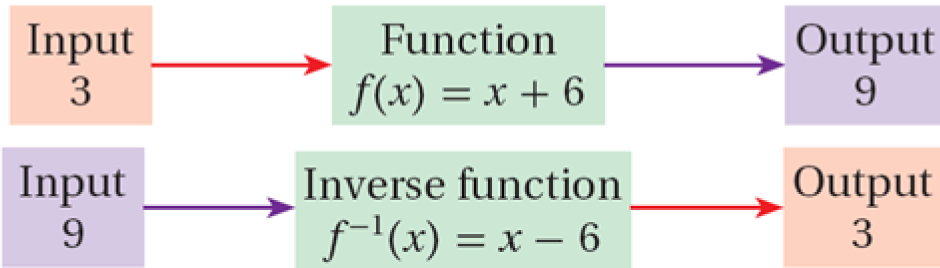
Chapter 5
Rational Exponents and Radical Functions

Section 5-6
Inverse of a Function

Functions that undo each other are **inverse functions**.



To find the inverse function, switch x and y, and then solve for y.

EXAMPLE 1 Writing a Formula for the Input of a Function

Let $f(x) = 2x + 3$.

a. Solve $y = f(x)$ for x.

b. Find the input when the output is -7.

Notice that these steps *undo* each other. Functions that undo each other are called **inverse functions**. In Example 1, you can use the equation solved for x to write the inverse of f by switching the roles of x and y.

$f(x) = 2x + 3$ original function $\qquad g(x) = \dfrac{x-3}{2}$ inverse function

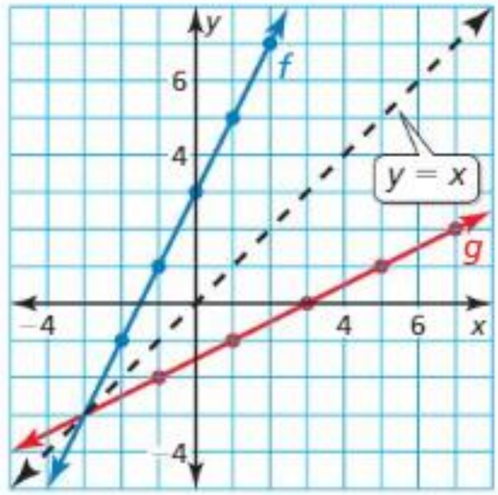
Because inverse functions interchange the input and output values of the original function, the domain and range are also interchanged.

Original function: $f(x) = 2x + 3$

x	-2	-1	0	1	2
y	-1	1	3	5	7

Inverse function: $g(x) = \dfrac{x-3}{2}$

x	-1	1	3	5	7
y	-2	-1	0	1	2



The graph of an inverse function is a *reflection* of the graph of the original function. The *line of reflection* is $y = x$. To find the inverse of a function algebraically, switch the roles of x and y, and then solve for y.

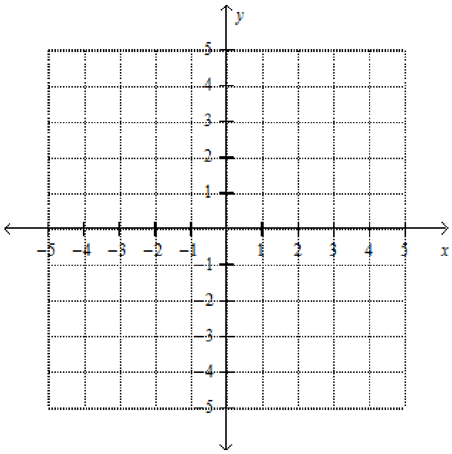
EXAMPLE 2 Finding the Inverse of a Linear Function

Find the inverse of $f(x) = 3x - 1$.

Find the inverse of the function. Then graph the function and its inverse.

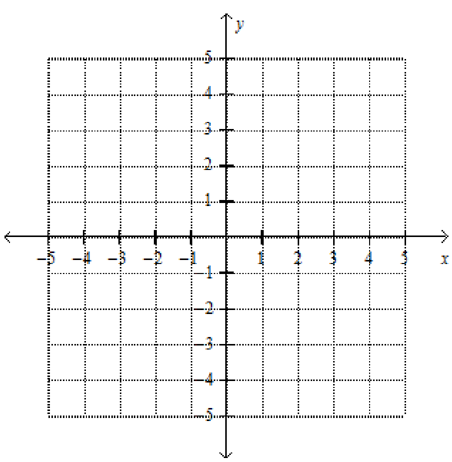
4. $f(x) = 2x$

6. $f(x) = \frac{1}{3}x - 2$



X	Y

X	Y

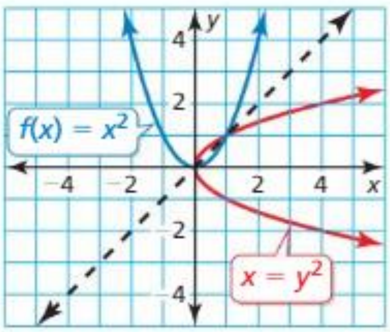


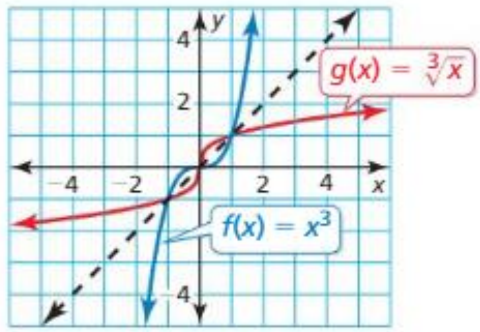
X	Y

X	Y

Inverses of Nonlinear Functions

In the previous examples, the inverses of the linear functions were also functions. However, inverses are not always functions. The graphs of $f(x) = x^2$ and $f(x) = x^3$ are shown along with their reflections in the line $y = x$. Notice that the inverse of $f(x) = x^3$ is a function, but the inverse of $f(x) = x^2$ is *not* a function.





When the domain of $f(x) = x^2$ is *restricted* to only nonnegative real numbers, the inverse of f *is* a function.

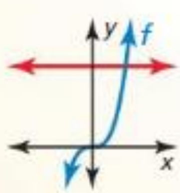
You can use the graph of a function f to determine whether the inverse of f is a function by applying the *horizontal line test*.

Core Concept

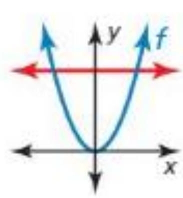
Horizontal Line Test

The inverse of a function f is also a function if and only if no horizontal line intersects the graph of f more than once.

Inverse is a function

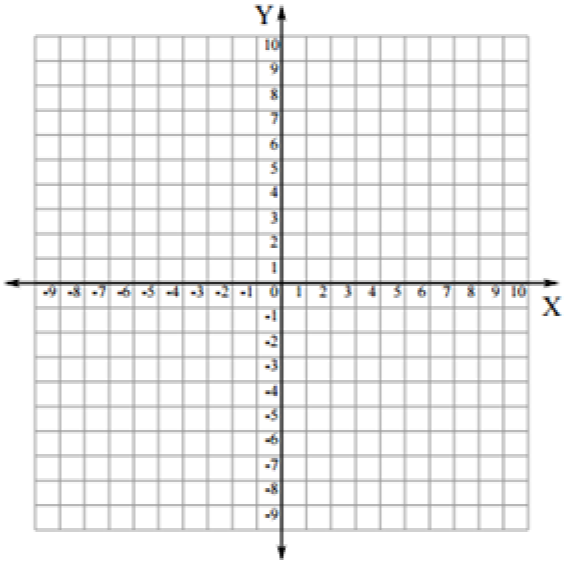


Inverse is not a function



EXAMPLE 3 Finding the Inverse of a Quadratic Function

Find the inverse of $f(x) = x^2, x \geq 0$. Then graph the function and its inverse.

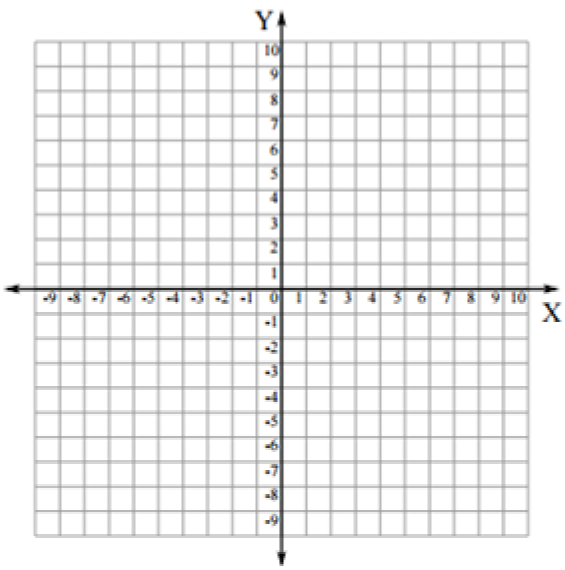


X	Y

X	Y

EXAMPLE 4 Finding the Inverse of a Cubic Function

Consider the function $f(x) = 2x^3 + 1$. Determine whether the inverse of f is a function. Then find the inverse.

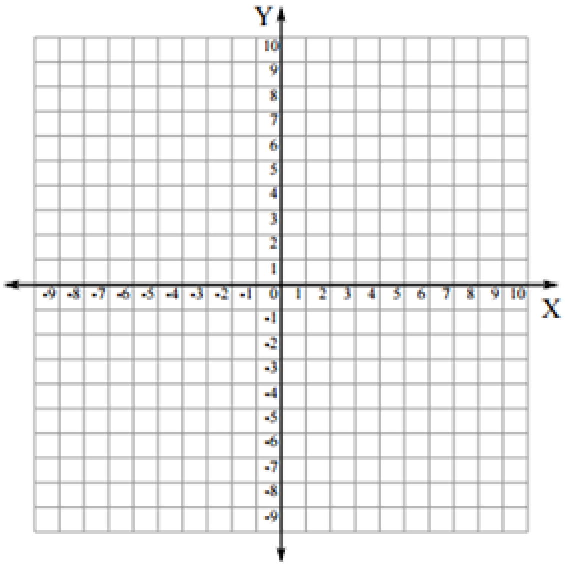


X	Y

X	Y

EXAMPLE 5 Finding the Inverse of a Radical Function

Consider the function $f(x) = 2\sqrt{x - 3}$. Determine whether the inverse of f is a function. Then find the inverse.



X	Y

X	Y

Let f and g be inverse functions. If $f(a) = b$, then $g(b) = a$. So, in general,

$$f(g(x)) = x \qquad \text{and} \qquad g(f(x)) = x.$$

EXAMPLE 6 Verifying Functions Are Inverses

Verify that $f(x) = 3x - 1$ and $g(x) = \dfrac{x + 1}{3}$ are inverse functions.