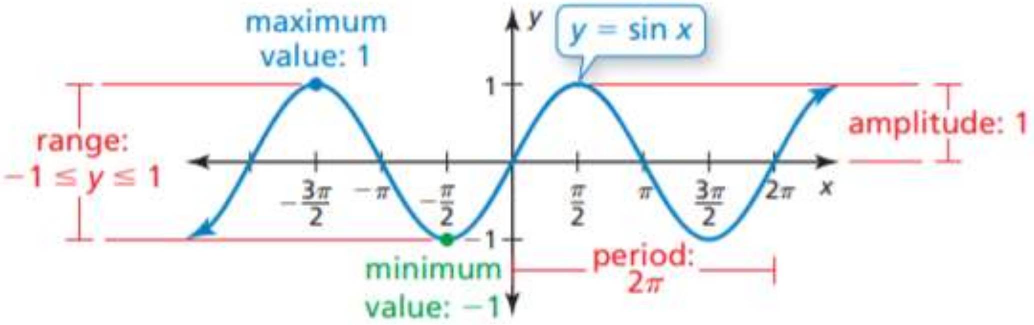
Chapter 9
Trigonometric Ratios and Functions

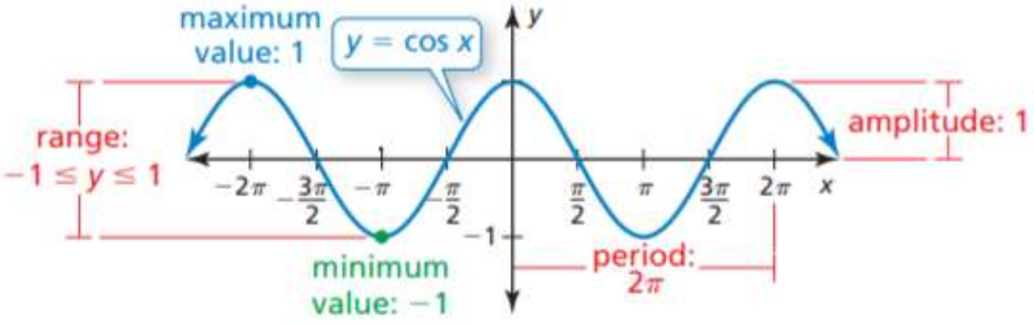
Section 9-4
Graphing Sine and Cosine Functions

Exploring Characteristics of Sine and Cosine Functions

In this lesson, you will learn to graph sine and cosine functions. The graphs of sine and cosine functions are related to the graphs of the parent functions $y = \sin x$ and $y = \cos x$, which are shown below.

x	-2π	$-\frac{3\pi}{2}$	$-\pi$	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$y = \sin x$	0	1	0	-1	0	1	0	-1	0
$y = \cos x$	1	0	-1	0	1	0	-1	0	1





Stretching and Shrinking Sine and Cosine Functions

The graphs of $y = a \sin bx$ and $y = a \cos bx$ represent transformations of their parent functions. The value of a indicates a vertical stretch ($a > 1$) or a vertical shrink ($0 < a < 1$) and changes the amplitude of the graph. The value of b indicates a horizontal stretch ($0 < b < 1$) or a horizontal shrink ($b > 1$) and changes the period of the graph.

$$y = a \sin bx$$

$$y = a \cos bx$$

vertical stretch or shrink by a factor of a ↑ ↑ horizontal stretch or shrink by a factor of $\frac{1}{b}$

Core Concept

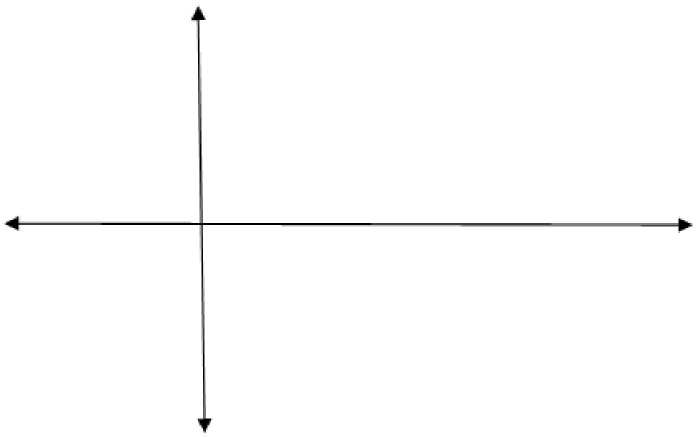
Amplitude and Period

The amplitude and period of the graphs of $y = a \sin bx$ and $y = a \cos bx$, where a and b are nonzero real numbers, are as follows:

$$\text{Amplitude} = |a| \qquad \text{Period} = \frac{2\pi}{|b|}$$

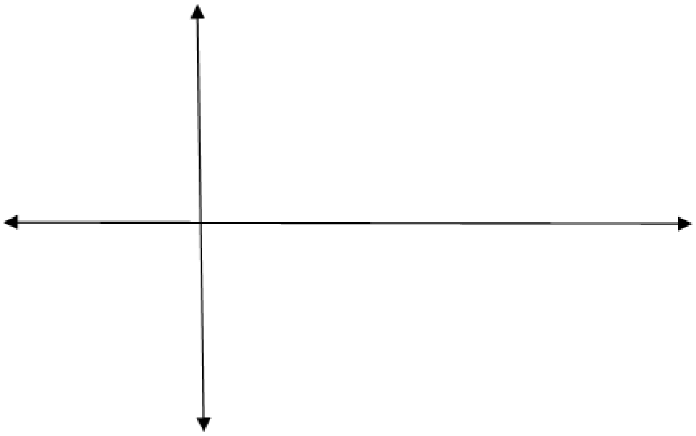
EXAMPLE 1 Graphing a Sine Function

Identify the amplitude and period of $g(x) = 4 \sin x$. Then graph the function and describe the graph of g as a transformation of the graph of $f(x) = \sin x$.



EXAMPLE 2 Graphing a Cosine Function

Identify the amplitude and period of $g(x) = \frac{1}{2} \cos 2\pi x$. Then graph the function and describe the graph of g as a transformation of the graph of $f(x) = \cos x$.



Translating Sine and Cosine Functions

The graphs of $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$ represent translations of $y = a \sin bx$ and $y = a \cos bx$. The value of k indicates a translation up $(k > 0)$ or down $(k < 0)$. The value of h indicates a translation left $(h < 0)$ or right $(h > 0)$. A horizontal translation of a periodic function is called a **phase shift**.

Core Concept

Graphing $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$

To graph $y = a \sin b(x - h) + k$ or $y = a \cos b(x - h) + k$ where $a > 0$ and $b > 0$, follow these steps:

Step 1 Identify the amplitude a, the period $\frac{2\pi}{b}$, the horizontal shift h, and the vertical shift k of the graph.

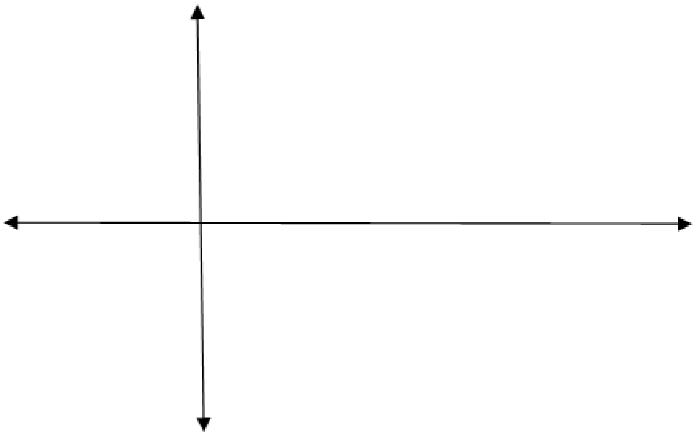
Step 2 Draw the horizontal line $y = k$, called the **midline** of the graph.

Step 3 Find the five key points by translating the key points of $y = a \sin bx$ or $y = a \cos bx$ horizontally h units and vertically k units.

Step 4 Draw the graph through the five translated key points.

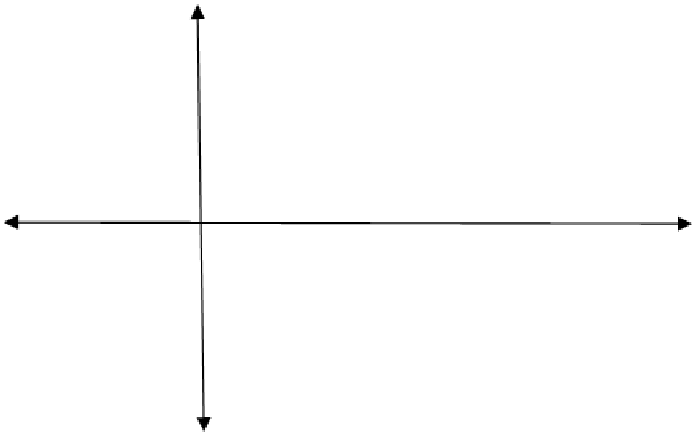
EXAMPLE 3 Graphing a Vertical Translation

Graph $g(x) = 2 \sin 4x + 3$.



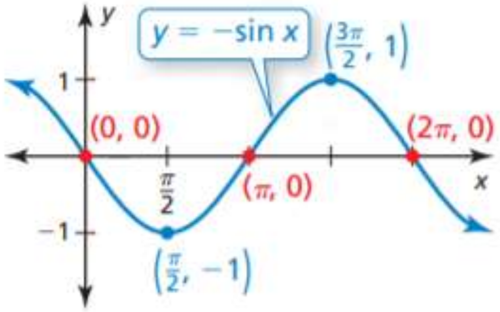
EXAMPLE 4 Graphing a Horizontal Translation

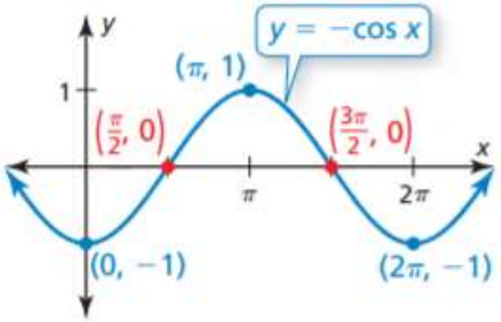
Graph $g(x) = 5\cos\frac{1}{2}(x - 3\pi)$.



Reflecting Sine and Cosine Functions

You have graphed functions of the form $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$, where $a > 0$ and $b > 0$. To see what happens when $a < 0$, consider the graphs of $y = -\sin x$ and $y = -\cos x$.

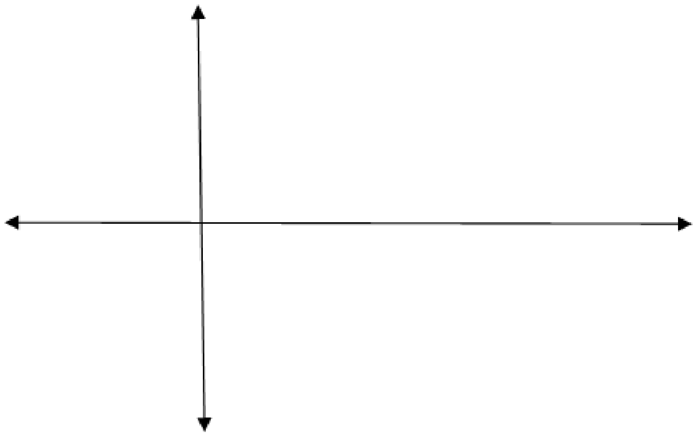




The graphs are reflections of the graphs of $y = \sin x$ and $y = \cos x$ in the x-axis. In general, when $a < 0$, the graphs of $y = a \sin b(x - h) + k$ and $y = a \cos b(x - h) + k$ are reflections of the graphs of $y = |a| \sin b(x - h) + k$ and $y = |a| \cos b(x - h) + k$, respectively, in the midline $y = k$.

EXAMPLE 5 Graphing a Reflection

Graph $g(x) = -2 \sin \frac{2}{3}\left(x - \frac{\pi}{2}\right)$.



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