Chapter 9
Trigonometric Ratios and Functions

Section 9-5
Graphing Other Trigonometric Functions

Exploring Tangent and Cotangent Functions

The graphs of tangent and cotangent functions are related to the graphs of the parent functions $y = \tan x$ and $y = \cot x$, which are graphed below.

← x approaches $-\frac{\pi}{2}$ —— | —— x approaches $\frac{\pi}{2}$ →

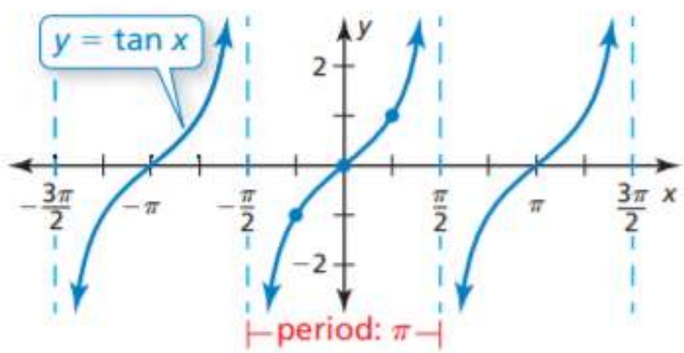
x	$-\frac{\pi}{2}$	-1.57	-1.5	$-\frac{\pi}{4}$	0	$\frac{\pi}{4}$	1.5	1.57	$\frac{\pi}{2}$
$y = \tan x$	Undef.	-1256	-14.10	-1	0	1	14.10	1256	Undef.

← $\tan x$ approaches $-\infty$ —— | —— $\tan x$ approaches ∞ →

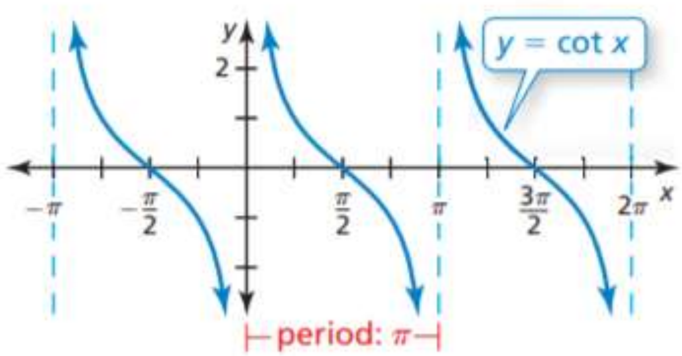
Because $\tan x = \frac{\sin x}{\cos x}$, $\tan x$ is undefined for x-values at which $\cos x = 0$, such as

$x = \pm\frac{\pi}{2} \approx \pm 1.571.$

The table indicates that the graph has asymptotes at these values. The table represents one cycle of the graph, so the period of the graph is π.



You can use a similar approach to graph $y = \cot x$. Because $\cot x = \frac{\cos x}{\sin x}$, $\cot x$ is undefined for x-values at which $\sin x = 0$, which are multiples of π. The graph has asymptotes at these values. The period of the graph is also π.



Core Concept

Characteristics of $y = \tan x$ and $y = \cot x$

The functions $y = \tan x$ and $y = \cot x$ have the following characteristics.

- The domain of $y = \tan x$ is all real numbers except odd multiples of $\frac{\pi}{2}$. At these x-values, the graph has vertical asymptotes.
- The domain of $y = \cot x$ is all real numbers except multiples of π. At these x-values, the graph has vertical asymptotes.
- The range of each function is all real numbers. So, the functions do not have maximum or minimum values, and the graphs do not have an amplitude.
- The period of each graph is π.
- The x-intercepts for $y = \tan x$ occur when $x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$.
- The x-intercepts for $y = \cot x$ occur when $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2}, \ldots$.

Graphing Tangent and Cotangent Functions

The graphs of $y = a \tan bx$ and $y = a \cot bx$ represent transformations of their parent functions. The value of a indicates a vertical stretch ($a > 1$) or a vertical shrink ($0 < a < 1$). The value of b indicates a horizontal stretch ($0 < b < 1$) or a horizontal shrink ($b > 1$) and changes the period of the graph.

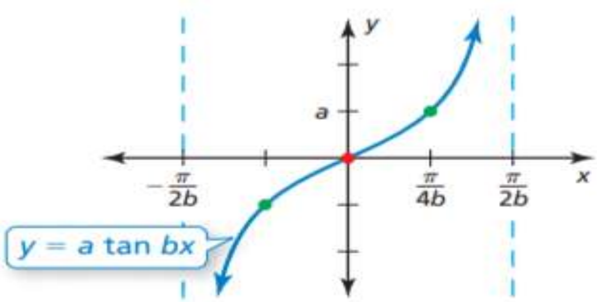
Core Concept

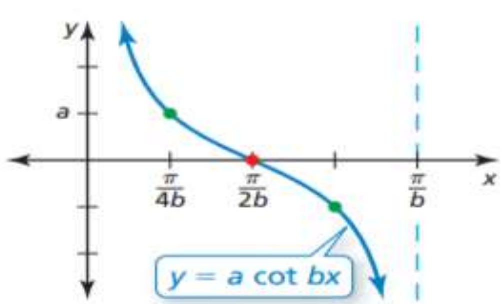
Period and Vertical Asymptotes of $y = a \tan bx$ and $y = a \cot bx$

The period and vertical asymptotes of the graphs of $y = a \tan bx$ and $y = a \cot bx$, where a and b are nonzero real numbers, are as follows.

- The period of the graph of each function is $\frac{\pi}{|b|}$.
- The vertical asymptotes for $y = a \tan bx$ are at odd multiples of $\frac{\pi}{2|b|}$.
- The vertical asymptotes for $y = a \cot bx$ are at multiples of $\frac{\pi}{|b|}$.

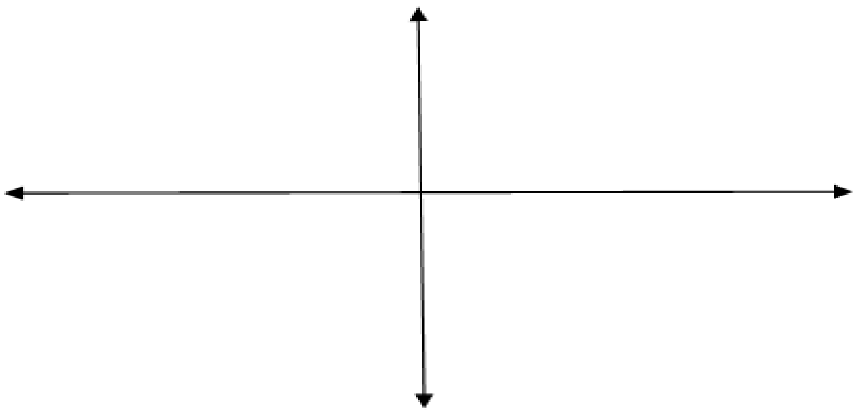
Each graph below shows five key x-values that you can use to sketch the graphs of $y = a \tan bx$ and $y = a \cot bx$ for $a > 0$ and $b > 0$. These are the x-intercept, the x-values where the asymptotes occur, and the x-values halfway between the x-intercept and the asymptotes. At each halfway point, the value of the function is either a or $-a$.





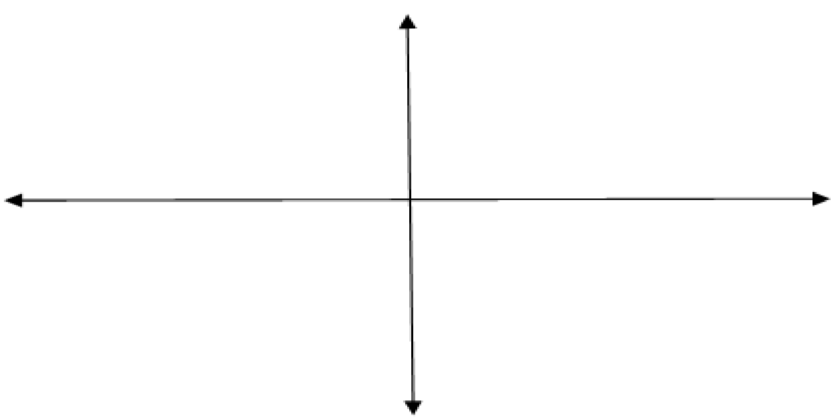
EXAMPLE 1 Graphing a Tangent Function

Graph one period of $g(x) = 2 \tan 3x$. Describe the graph of g as a transformation of the graph of $f(x) = \tan x$.



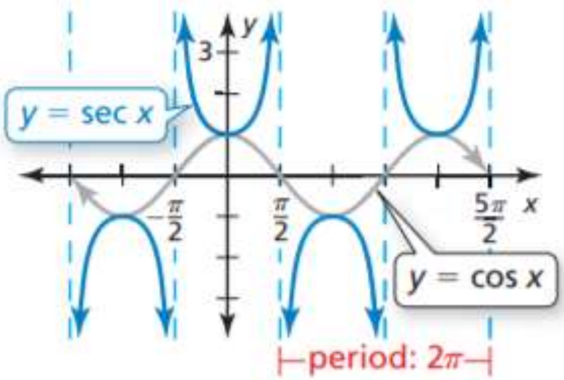
EXAMPLE 2 Graphing a Cotangent Function

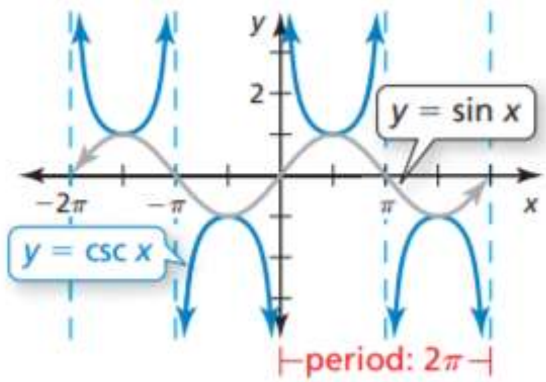
Graph one period of $g(x) = \cot \frac{1}{2}x$. Describe the graph of g as a transformation of the graph of $f(x) = \cot x$.



Graphing Secant and Cosecant Functions

The graphs of secant and cosecant functions are related to the graphs of the parent functions $y = \sec x$ and $y = \csc x$, which are shown below.





Core Concept

Characteristics of y = sec x and y = csc x

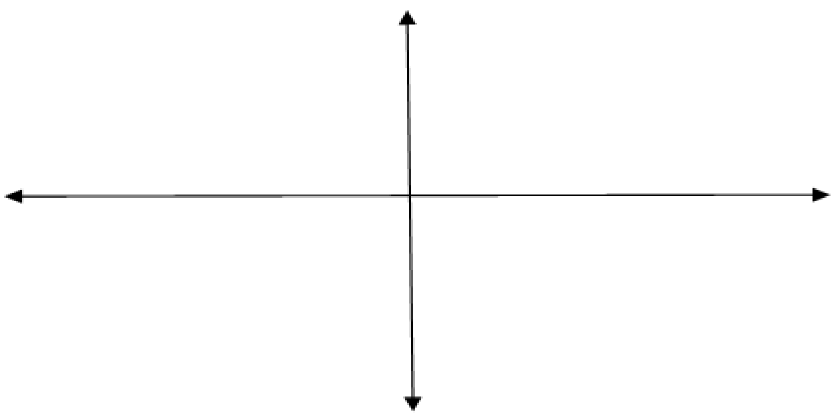
The functions $y = \sec x$ and $y = \csc x$ have the following characteristics.

- The domain of $y = \sec x$ is all real numbers except odd multiples of $\frac{\pi}{2}$. At these x-values, the graph has vertical asymptotes.
- The domain of $y = \csc x$ is all real numbers except multiples of π. At these x-values, the graph has vertical asymptotes.
- The range of each function is $y \leq -1$ and $y \geq 1$. So, the graphs do not have an amplitude.
- The period of each graph is 2π.

To graph $y = a \sec bx$ or $y = a \csc bx$, first graph the function $y = a \cos bx$ or $y = a \sin bx$, respectively. Then use the asymptotes and several points to sketch a graph of the function. Notice that the value of b represents a horizontal stretch or shrink by a factor of $\frac{1}{b}$, so the period of $y = a \sec bx$ and $y = a \csc bx$ is $\frac{2\pi}{|b|}$.

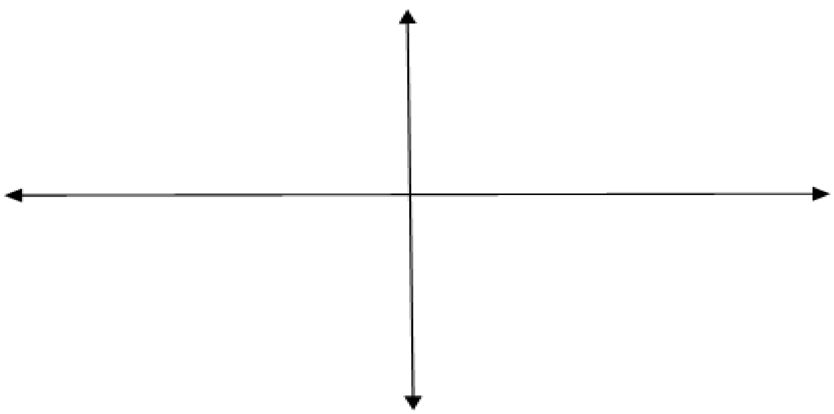
EXAMPLE 3 Graphing a Secant Function

Graph one period of $g(x) = 2 \sec x$. Describe the graph of g as a transformation of the graph of $f(x) = \sec x$.



EXAMPLE 4 Graphing a Cosecant Function

Graph one period of $g(x) = \frac{1}{2} \csc \pi x$. Describe the graph of g as a transformation of the graph of $f(x) = \csc x$.



Section 9-5 Homework #5-23 odd