

Describing Transformations of Polynomial Functions

You can transform graphs of polynomial functions in the same way you transformed graphs of linear functions, absolute value functions, and quadratic functions. Examples of transformations of the graph of $f(x) = x^4$ are shown below.

Core Concept

T-chart

Transformation	$f(x)$ Notation	Examples
Horizontal Translation Graph shifts left or right.	$f(x - h)$	$g(x) = (x - 5)^4$ 5 units right $g(x) = (x + 2)^4$ 2 units left
Vertical Translation Graph shifts up or down.	$f(x) + k$	$g(x) = x^4 + 1$ 1 unit up $g(x) = x^4 - 4$ 4 units down
Reflection Graph flips over x - or y -axis.	$f(-x)$ $-f(x)$	$g(x) = (-x)^4 = x^4$ over y -axis $g(x) = -x^4$ over x -axis
Horizontal Stretch or Shrink $0 < a < 1$ $a > 1$ Graph stretches away from or shrinks toward y -axis.	$f(ax)$	$g(x) = (2x)^4$ shrink by a factor of $\frac{1}{2}$ $g(x) = (\frac{1}{2}x)^4$ stretch by a factor of 2
Vertical Stretch or Shrink $a > 1$ $0 < a < 1$ Graph stretches away from or shrinks toward x -axis.	$a \cdot f(x)$	$g(x) = 8x^4$ stretch by a factor of 8 $g(x) = \frac{1}{4}x^4$ shrink by a factor of $\frac{1}{4}$

Add to each x
 Subtract from each x
 Add to y
 Subtract from y
 Change sign of x
 Change sign of y
 } multiply x by factor
 } multiply y by factor

Transformation	$f(x)$ Notation	Example	Adjustment To T-Chart
Shift Left	$f(x+h)$	If $f(x) = x^2$ and then $f(x) = (x+3)^2$. Shifts left 3-units	Subtract from each x-value
Shift Right	$f(x-h)$	If $g(x) = x^2$ and then $g(x) = (x-3)^2$. Shifts right 3-units	Add to each x-value
Shift Up	$f(x) + k$	If $f(x) = x^2$ then $f(x) = x^2 + 2$. Shifts up 2 units	Add to each y-value
Shift Down	$f(x) - k$	If $f(x) = x^2$ then $f(x) = x^2 - 3$. Shifts down 3-units	Subtract from each y-value
Reflect x-axis	$-f(x)$	If $f(x) = x^2 + 5$ then $f(x) = -x^2 - 5$. Reflect on x-axis	Change sign of each y-value
Reflect y-axis	$f(-x)$	If $f(x) = x+3 $ then $f(x) = -x+3 $. Reflect on y-axis	Change sign of each x-value
Vertical Stretch	$a \cdot f(x)$	When $a > 1$ it's a vertical stretch. If $f(x) = x^2$ then $f(x) = 5x^2$. factor is 5	Multiply each y-value by factor
Vertical Shrink	$a \cdot f(x)$	When $0 < a < 1$ it's vertical shrink. If $f(x) = x^2$ then $f(x) = \frac{1}{3}x^2$ then factor $\frac{1}{3}$	Multiply each y-value by factor
Horizontal Stretch	$f(ax)$	If $g(x) = x^2$ then $g(x) = (\frac{1}{2}x)^2$. $0 < a < 1$ stretch by factor 2	Multiply each x-value by factor
Horizontal Shrink	$f(ax)$	If $g(x) = x^2$ then $g(x) = (4x)^2$. $a > 1$ shrink by factor $\frac{1}{4}$	Multiply each x-value by factor