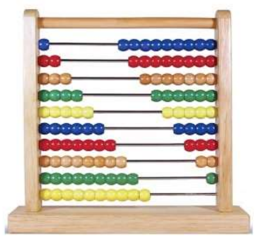
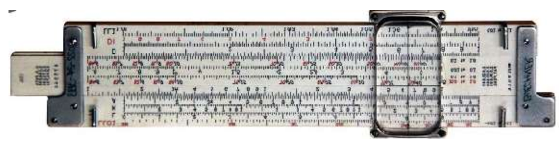
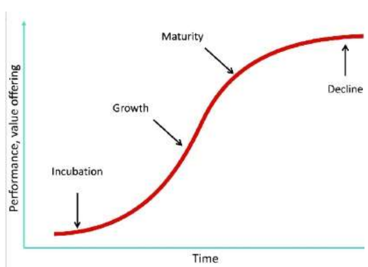
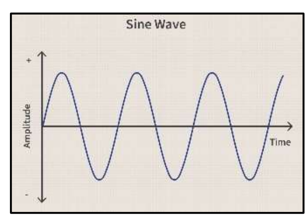
Mathematical Models for Fun and Prophet
or
Does 1+1=two?









By: Larry Zaleski

Presented to the Hagerstown Torch Club

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About the Author

Larry Zaleski was born and raised in New Jersey. He grew up 'a free range kid,' which allowed him to become familiar with the farms, fields, woods, streams and wildlife of his native State. Larry holds BS and MS degrees in Biology from Northern Arizona University. He has worked primarily within the federal government in biologically oriented programs within the Departments of Commerce (as a high-sea's biological technician), Interior (with the U.S. Fish and Wildlife Service Division of Law Enforcement), and Agriculture (as an inspector, officer, and trainer). For 30 years, Larry worked within USDA's Animal and Plant Health Inspection Service holding various training posts at their Professional Development Center in Frederick, MD. These positions included instructional designer, project leader, supervisor, and senior training specialist. Larry has written technical manuals, managed, designed, developed, and delivered both e-learning and classroom courses covering a wide range of scientific, technical, and managerial topics. He retired October, 2015 and now resides in Hagerstown, Maryland.

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Introduction

As a species, we humans have advanced on many fronts: scientifically, socially, and technologically. But why? What sets us apart from other animals, some of whom have larger brains, have their own social organization, exhibit complex behaviors, and in some cases possess significant intelligence?

In fact, this is possible because we have a cascading collection of traits that permit us to excel.

First our brains. Our brains are large and complex. This allows us to learn, store information, perform sophisticated behaviors, and think abstractly. While not unique, our brains are fused with a body structure that extends its reach.

Second, our bodies. Our bodies have an especially useful organization. We walk upright freeing our forelimbs. Our forelimbs have hands with opposable thumbs, allowing us to manipulate objects, and – in partnership with our brains – to make and use tools. Tools, in turn, extend our abilities to manipulate the world beyond the limitations of our bodies.

Third, our ability to speak. Language allows us to communicate, to share information, and to coordinate our actions.

Fourth, writing. Writing is a tool that extends the power of language and overcomes the limitations of memory. Distance, complexity, and time are no longer barriers to communication nor the storage of information. Writing is no doubt humanity's greatest and most powerful invention and a tribute to human intellect.

Finally, we have one more trick, a universal, written-only extension to language called mathematics (Helmenstine). Mathematics enables us to track events and describe nature with unrivaled clarity. Mathematics is a supercharger for the human mind.

Because of this combination, humans are a superspecies. In contrast, other animals with big brains, but no hands – like whales and elephants – cannot build complex tools. Consequently, their capabilities are limited. Similarly, species with complex brains and nimble hands, but no language, such as Chimpanzees and Orangutangs, might create tools, but are limited in how they share and store that knowledge.

Without a complex language, communication is limited. Without writing, information and knowledge remains shallow and temporary. And without mathematics we would be limited to general concepts, unable to achieve the deep understanding of nature necessary for science, medicine, and engineering.

The Power of Mathematics

So why is mathematics so useful? The secret lies in its incorporation in models. But what are models and where do they come from?

Models are a set of assumptions that people use to understand some aspect of our infinitely varied world (Meadows et al.). Every day, people observe the world around them, and in the process notice patterns. For example:

- Daylight is shorter in winter than in summer
- Humvees use more gasoline than Honda Civics
- Rocks are usually heavier than wood of the same volume
- It's colder in winter than in summer
- The Buffalo Bills are a better football team than the New York Jets

We use these patterns to form '*mental models*' to understand the world and direct our lives. Mental models are often unconscious, and part of our personality. They inform our beliefs, guide our decisions, and direct our actions. Mental models are sometimes called "rules of thumb" – broadly accurate guides or principles based on intuition, experience, or practice rather than theory. Because mental models exist in the mind, they are abstract.

But to paraphrase Lord Kelvin, *'Unless you measure things, you don't know what you're talking about'* (AZ Quotes). While a bit harsh, Lord Kelvin made an important observation: Numerical measurements represent patterns in accurate and useful ways.

There are many topics for which the qualitative nature of spoken language is inadequate. Physics, finance, economics, chemistry, biology, and engineering all contain subjects that must be described quantitatively.

Scientist and professionals, like everyone else, observe events and then organize them into patterns. However, they often go a step further by making measurements. This allows them to express patterns in abstract mathematical terms such as points, lines, and geometric shapes (Figure 1). These abstractions now represent physical objects and processes such as velocity, acceleration, and

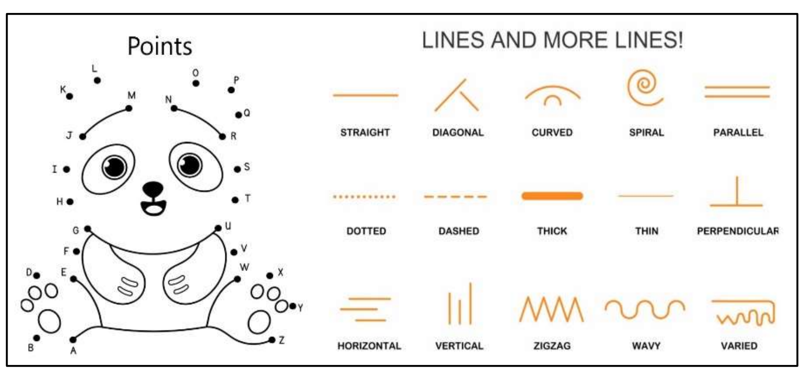

Figure 1. Points, lines, and more.

force, and they describe relative relationships (Allendoerfer and Oakley). In this way, scientists and other professionals convert mental models into mathematical models.

But numbers, points, and lines have no meaning, so how do they represent the real world? To understand, we must first comprehend what we mean by abstract. The standard definition of the word 'abstract' is: ***"Existing in thought or as an idea but not having a physical or concrete existence."***

Applying this idea to mathematical models requires a refinement: ***"Abstraction is the process of <u>reducing</u> an issue to its fundamental elements."***

The purpose of reduction is to present a problem free of irrelevant distractions thus simplifying the solution and widening the span of application (Saari). But reduction departs from reality. Figure 2 demonstrates an abstract mathematical model used to determine the height of a wall using only two measurements, the angle of the ladder and its distance to the wall.

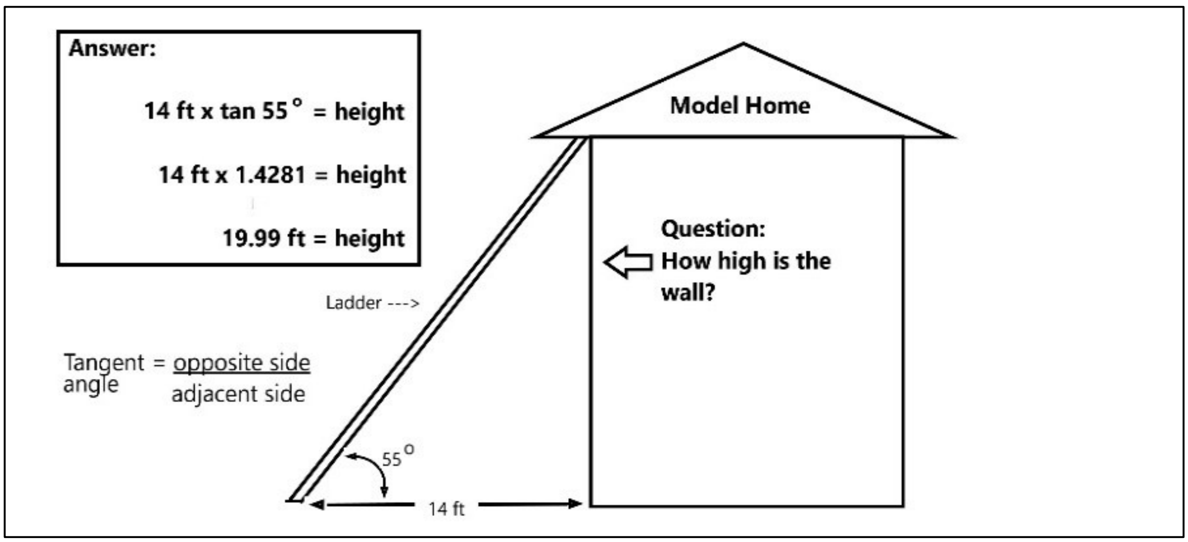

Figure 2. A mathematical model using a trigonometric function to determine the height of a wall. The ladder leaning against the wall forms a right triangle. The model is a simplification providing only two relevant factors—distance to the base of the ladder (14 ft) and the angle of the ladder (55°). The image does not include color, doors, windows, building materials, or other features. It is an abstraction, not reality, yet it allows us to answer the question – the wall is 19.99 ft high or about 20 ft. It applies to mansions and shanties alike.

The numbers 14 and 55 are assigned "*dimensions*." The number 14 becomes 14 feet, and 55 becomes 55 degrees. Consequently, they now represent real world features that can be manipulated using a known trigonometric relationship. Mathematics can <u>describe</u> reality because reality often aligns with mathematical functions, but mathematics is <u>not</u> reality. Alone, mathematics describes nothing. Mathematical relationships must always be matched to a system and confirmed by observation. Once verified, mathematical descriptions may be applied broadly and confidently.

Examples of Patterns

Patterns can take many forms. Here, I will limit these to linear, exponential, sine, and sigmoid functions. These patterns are illustrated in Figures 3, 4, 5, and 6 respectively.

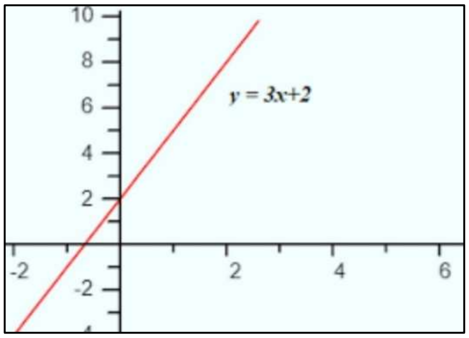

Figure 3. A linear function. Linear functions show constant change or velocity. Linear patterns have the formula y = (slope)x + y intercept.

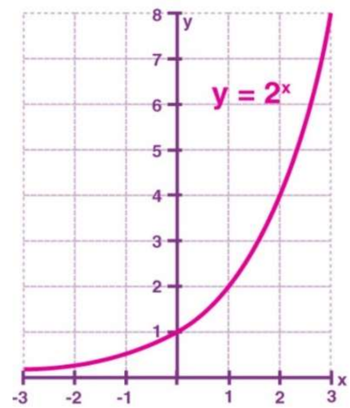

Figure 4. An exponential function. Exponential functions show acceleration or deceleration (speeding up or slowing down) resulting in curved lines (changing slope). Exponential functions are formed by raising some number to the power of x, for example $y = 2^x$. Often, the base "e" is used, which is the base of the natural logarithm and is approximately equal to 2.71828.

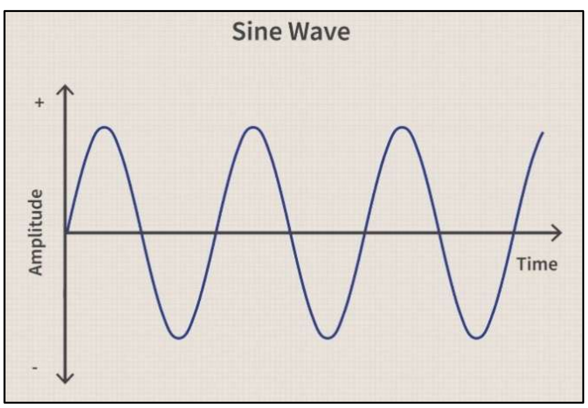

Figure 5. A sine function represents oscillations of constant amplitude given by a sine function. The formula is y=a[sin(bx)], where: a is known as the amplitude (height) of the sine wave, b is known as the periodicity (wavelength), and x is time. Sine waves are used to represent sound, radio frequency, day/night cycle, biological populations, and the stock market.

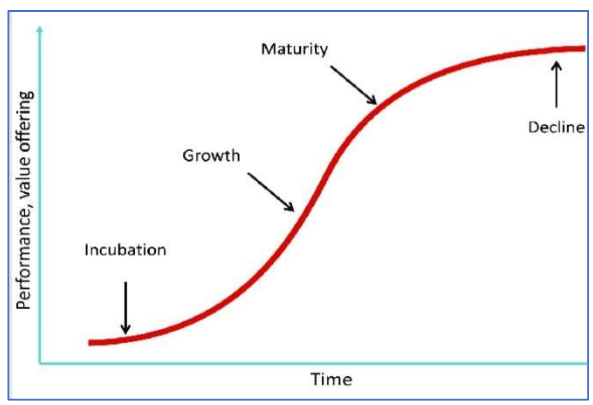

Figure 6. A sigmoid function resembles an "S." This pattern is common in engineering, business, and biology. The pattern starts with an incubation phase, moves to an exponential growth phase, and tops out with a steady state or declining phase. The general formula is $f(x) = x/(1 + |x|^k)^{1/k}$. where k refers to the carrying capacity of the system.

The stock market's business cycle represents a real-world example of the sine function (Fidelity, Hurst). Like many real-world patterns, the model for the stock market is accurate, but imprecise (Figure 7).

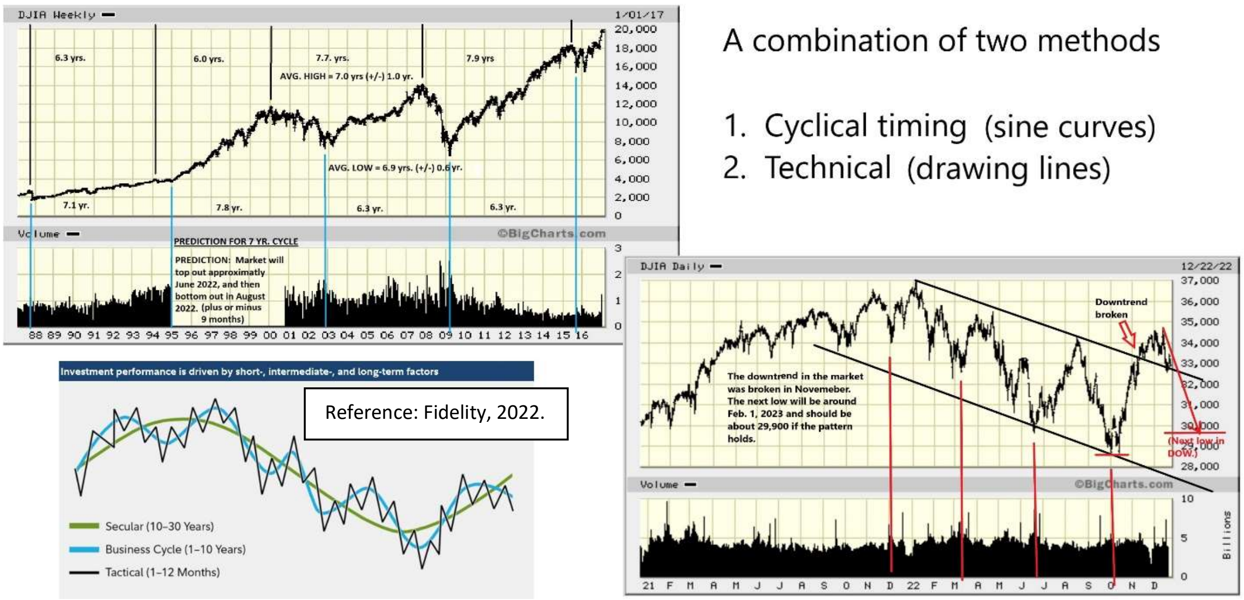

Figure 7. The stock market moves in a series of sine curves of different durations superimposed atop one another. The resulting model is accurate but imprecise (± the measured variation) and the market highs are out of phase with the market lows (like a wave breaking). In the longest cycle shown, the pattern for the lows is 6.9 years ± 7 months, and the pattern for the highs is 7 years ± 12 months. According to the model, the 6.9 year cycle downtrend (analogous to radio wavelength) in the Dow Jones Industrials was broken in November 2022 (meaning the market will be in uptrend until the next 7 year cycle high predicted to occur in 2029±1 year (Disclaimer: Not intended as market advice)).

Computational Aids

Most people learn mathematics using paper and pencil. And while this is still necessary, it is brutally tedious and prone to error. Consequently, computational aids were soon invented. The abacus, for example, was invented around 2400 B.C. by the Babylonians to count large numbers (Abacus). And soon after logarithms were discovered, William Oughtred invented the slide rule, around the year 1630 A.D. (Slide Rule). Slide rules were in common use until the late 1970's and took us to the moon.

Today, pocket calculators, and computers running electronic spreadsheets and dedicated computer programs remove much of the drudgery (Figure 7). These computational aids are crucial when solving problems with many steps and repetitions, which were previously impractical or impossible to attempt. Once vetted, computer programs save time, money, and reduce error. Like all tools, they extend human ability.

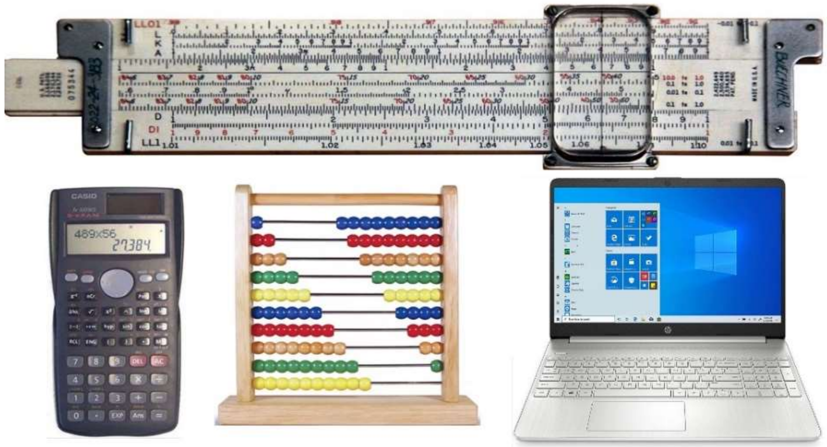

Figure 8. Computational aids. (Top) Slide Rule. (Bottom, from left to right) Calculator, Abacus, and Laptop computer.

<u>Computer Models</u>

Computer models are mathematical models consisting of equations and algorithms used to mimic the behavior of the system being modeled. An algorithm is a process or set of rules followed when making calculations or other problem-solving operations (Figure 8). The computer performs the calculations according to an algorithm.

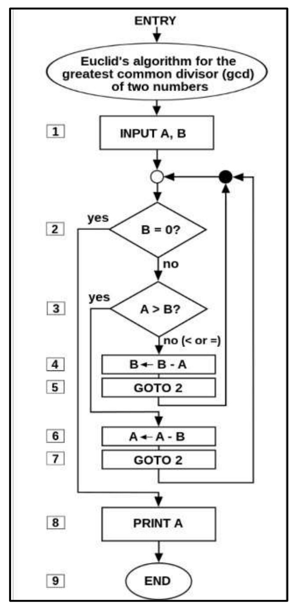

Figure 9. Example of an algorithm (Algorithm).

Algorithms are communicated to the computer using a written programming language or by organizing an electronic spreadsheet. A computer simulation, is simply the act of running the model on a computer to simulate the behavior, or outcome of real-world systems ((Computer Simulation). Well-designed computer models can have startling predictive ability (Figure 9). Examples include weather prediction, financial operations, ballistics, aerodynamics, biological relationships, and recreational games.

Documentation is a strength of mathematical models. Their algorithms and assumptions are written down allowing detailed inspection and editing resulting in continual improvement, something people often fail to do with mental models. The ability to evaluate the model is crucial because all models are only as good as their data, organization, and assumptions – Garbage in, Garbage out.

Kinds of Simulations

Figure 10. Well-designed computer models can have startling predictive ability.

Simulations can be large or small and range from weather prediction, predicting the effect of interest rates on loan payments, flight simulation, and children's games. Simulations typically allow users to change the value of variables on different computer runs enabling "what if" games.

"What if" scenarios provide insights into how the system being manipulated works, and permits projecting outcomes through time.

Recreational games like "Mario" can gross $millions for their creators.

Summary

In review, this presentation covered the following:

1. Models are the way we perceive the universe. Everything we think and believe, and almost every rational and many irrational actions that we take are based on models. Models come in two broad types – mental and mathematical, both are abstract.

2. Mental models are qualitative. They consist of general observations, feelings, and beliefs. They tend to be unconscious and deeply embedded in our personalities. Consequently, they are habitual. And while they can be improved, they often become dogma accurate or not.

3. Mathematical models are quantitative. They are based on observation and measurement, and are characterized by planning and documentation. Like mental models, mathematical models can be wrong, but because they are documented, if they fail, they can be re-evaluated by the author or others, then rejected or improved.

4. Mathematical models extend our abilities, as do other tools that humans make.

5. Finally, since numbers are abstract and can be written in many ways, any symbols that represent the concepts are equally valid. So, yes, 1+1=t-w-o.

For the remainder of this presentation, I will demonstrate several Excel-based models that I have made for fun and profit:

- Kinetic energy of a meteor strike – The end is near
- Future college cost calculator – Help!
- Amortization – The effect of changing interest rates and loan duration
- Stock market prediction – Advice 5¢
- Population dynamics – Procreation by the numbers
- The magic 8 ball – Predictions from the macronancer (Figure 10).

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