Activity 1

Objective

To find the number of subsets of a given set and verify that if a set has n number of elements, then the total number of subsets is 2^n.

Material Required

Paper, different coloured pencils.

Method of Construction

1. Take the empty set (say) A_0 which has no element.
2. Take a set (say) A_1 which has one element (say) a_1.
3. Take a set (say) A_2 which has two elements (say) a_1 and a_2.
4. Take a set (say) A_3 which has three elements (say) a_1, a_2 and a_3.

Demonstration

1. Represent A_0 as in Fig. 1.1

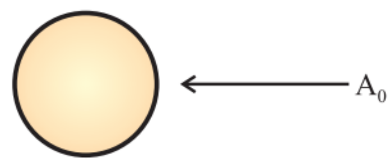

Fig. 1.1

   Here the possible subsets of A_0 is A_0 itself only, represented symbolically by ϕ. The number of subsets of A_0 is $1 = 2^0$.

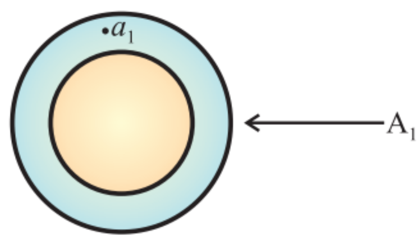

Fig. 1.2

2. Represent A_1 as in Fig. 1.2. Here the subsets of A_1 are ϕ, $\{a_1\}$. The number of subsets of A_1 is $2 = 2^1$

3. Represent A_2 as in Fig. 1.3

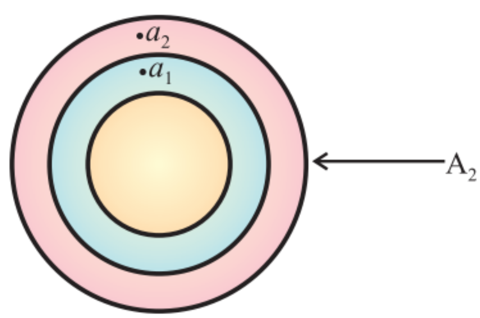

Fig. 1.3

   Here the subsets of A_2 are ϕ, $\{a_1\}$, $\{a_2\}$, $\{a_1, a_2\}$. The number of subsets of A_2 is $4 = 2^2$.

4. Represent A_3 as in Fig. 1.4

   Here the subsets of A_3 are ϕ, $\{a_1\}$, $\{a_2\}$, $\{a_3)$,$\{a_1, a_2\}$, $\{a_2, a_3)$,$\{a_3, a_1)$ and $\{a_1, a_2, a_3\}$. The number of subsets of A_3 is $8 = 2^3$.

5. Continuing this way, the number of subsets of set A containing n elements $a_1, a_2, ..., a_n$ is 2^n.

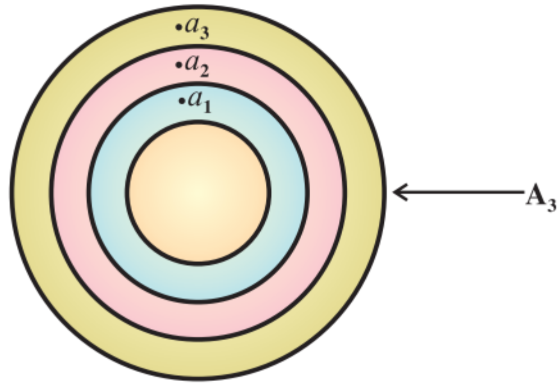

Fig. 1.4

Observation

1. The number of subsets of A_0 is ____________ $= 2^{\square}$
2. The number of subsets of A_1 is ____________ $= 2^{\square}$
3. The number of subsets of A_2 is ____________ $= 2^{\square}$
4. The number of subsets of A_3 is ____________ $= 2^{\square}$
5. The number of subsets of A_{10} is $= 2^{\square}$
6. The number of subsets of A_n is $= 2^{\square}$

Application

The activity can be used for calculating the number of subsets of a given set.

Activity 3

OBJECTIVE

To represent set theoretic operations using Venn diagrams.

MATERIAL REQUIRED

Hardboard, white thick sheets of paper, pencils, colours, scissors, adhesive.

METHOD OF CONSTRUCTION

1. Cut rectangular strips from a sheet of paper and paste them on a hardboard. Write the symbol U in the left/right top corner of each rectangle.
2. Draw circles A and B inside each of the rectangular strips and shade/colour different portions as shown in Fig. 3.1 to Fig. 3.10.

DEMONSTRATION

1. U denotes the universal set represented by the rectangle.
2. Circles A and B represent the subsets of the universal set U as shown in the figures 3.1 to 3.10.
3. A' denote the complement of the set A, and B' denote the complement of the set B as shown in the Fig. 3.3 and Fig. 3.4.
4. Coloured portion in Fig. 3.1. represents $A \cup B$.

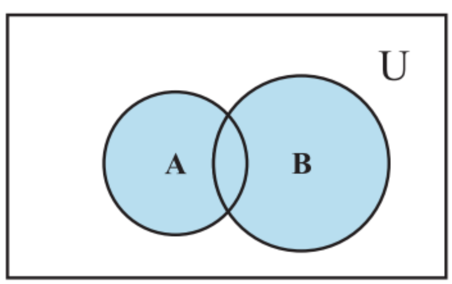

Fig. 3.1

5. Coloured portion in Fig. 3.2. represents A ∩ B.

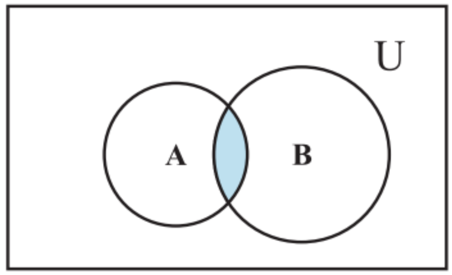

Fig. 3.2

6. Coloured portion in Fig. 3.3 represents A′

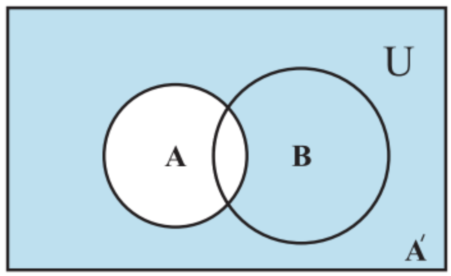

Fig. 3.3

7. Coloured portion in Fig. 3.4 represents B′

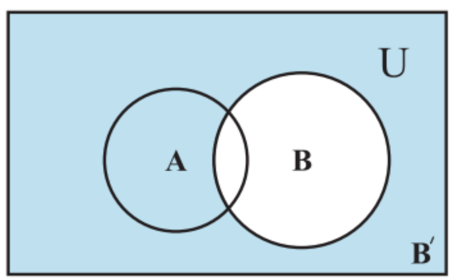

Fig. 3.4

8. Coloured portion in Fig. 3.5 represents (A ∩ B)′

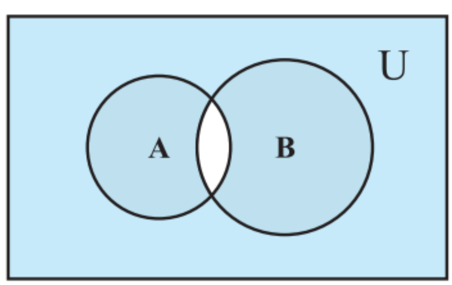

Fig. 3.5

9. Coloured portion in Fig. 3.6 represents $(A \cup B)'$

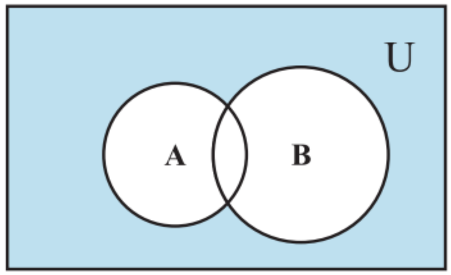

Fig. 3.6

10. Coloured portion in Fig. 3.7 represents $A' \cap B$ which is same as B – A.

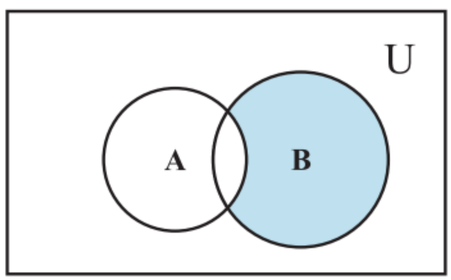

Fig. 3.7

11. Coloured portion in Fig. 3.8 represents $A' \cup B$.

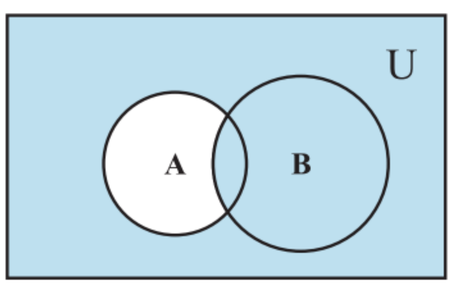

Fig. 3.8

12. Fig. 3.9 shows $A \cap B = \phi$

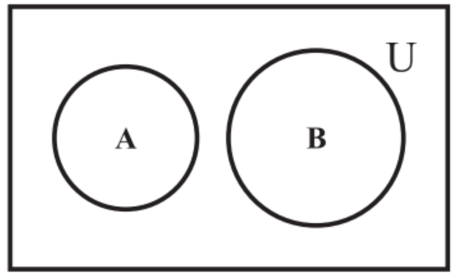

Fig. 3.9

13. Fig. 3.10 shows $A \subset B$

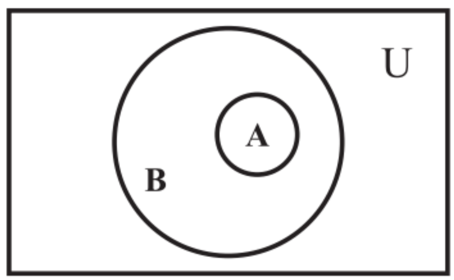

Fig. 3.10

OBSERVATION

1. Coloured portion in Fig. 3.1, represents _______________
2. Coloured portion in Fig. 3.2, represents _______________
3. Coloured portion in Fig. 3.3, represents _______________
4. Coloured portion in Fig. 3.4, represents _______________
5. Coloured portion in Fig. 3.5, represents _______________
6. Coloured portion in Fig. 3.6, represents _______________
7. Coloured portion in Fig. 3.7, represents _______________
8. Coloured portion in Fig. 3.8, represents _______________
9. Fig. 3.9, shows that $(A \cap B)$ = _______________
10. Fig. 3.10, represents A _______________ B.

APPLICATION

Set theoretic representation of Venn diagrams are used in Logic and Mathematics.

Activity 7

OBJECTIVE

To verify the relation between the degree measure and the radian measure of an angle.

MATERIAL REQUIRED

Bangle, geometry box, protractor, thread, marker, cardboard, white paper.

METHOD OF CONSTRUCTION

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Draw a circle using a bangle on the white paper.
3. Take a set square and place it in two different positions to find diameters PQ and RS of the circle as shown in the Fig.7.1 and 7.2

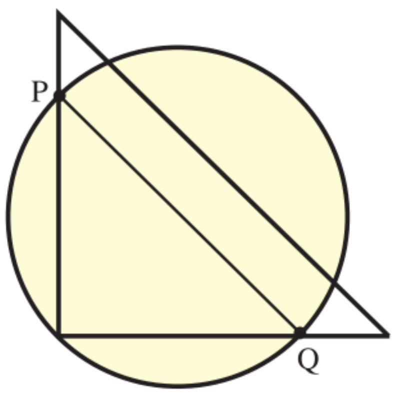

Fig. 7.1

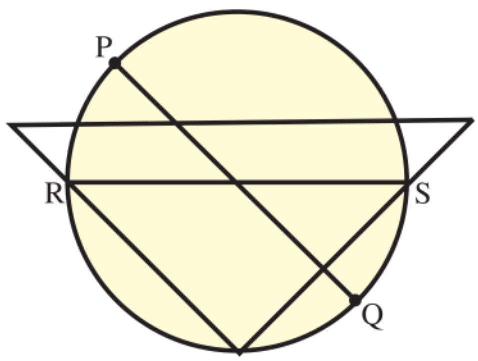

Fig. 7.2

4. Let PQ and RS intersect at C. The point C will be the centre of the circle (Fig. 7.3).
5. Clearly CP = CR = CS = CQ = radius.

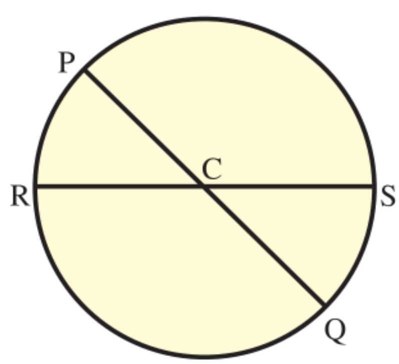

Fig. 7.3

Demonstration

1. Let the radius of the circle be r and l be an arc subtending an angle θ at the centre C, as shown in Fig. 7.4. $\theta = \frac{l}{r}$ radians.

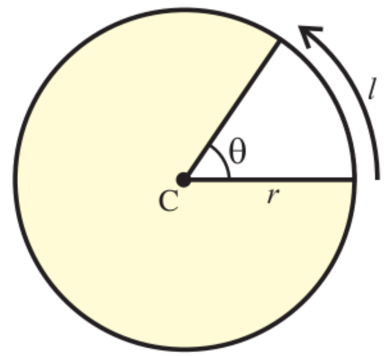

Fig. 7.4

2. If Degree measure of $\theta = \frac{l}{2\pi r} \times 360$ degrees

Then $\frac{l}{r}$ radians $= \frac{l}{2\pi r} \times 360$ degrees

or 1 radian $= \frac{180}{\pi}$ degrees = 57.27 degrees.

Observation

Using thread, measure arc lengths RP, PS, RQ, QS and record them in the table given below :

S.No	Arc	length of arc (l)	radius of circle (r)	Radian measure
1.	$\overset{\frown}{RP}$	--------	--------	$\angle RCP = \frac{\overset{\frown}{RP}}{r} =$ __
2.	$\overset{\frown}{PS}$	--------	--------	$\angle PCS = \frac{\overset{\frown}{PS}}{r} =$ __
3.	$\overset{\frown}{SQ}$	--------	--------	$\angle SCQ = \frac{\overset{\frown}{SQ}}{r} =$ __
4.	$\overset{\frown}{QR}$	--------	--------	$\angle QCR = \frac{\overset{\frown}{QR}}{r} =$ __

2. Using protractor, measure the angle in degrees and complete the table.

Angle	Degree measure	Radian Measure	Ratio = $\frac{\textit{Degree measure}}{\textit{Radian measure}}$
$\angle$ RCP	--------	--------	--------
$\angle$ PCS	--------	--------	--------
$\angle$ QCS	--------	--------	--------
$\angle$ QCR	--------	--------	--------

3. The value of one radian is equal to ________ degrees.

Application

This result is useful in the study of trigonometric functions.

Activity 8

OBJECTIVE

To find the values of sine and cosine functions in second, third and fourth quadrants using their given values in first quadrant.

MATERIAL REQUIRED

Cardboard, white chart paper, ruler, coloured pens, adhesive, steel wires and needle.

METHOD OF CONSTRUCTION

1. Take a cardboard of convenient size and paste a white chart paper on it.
2. Draw a unit circle with centre O on chart paper.
3. Through the centre of the circle, draw two perpendicular lines $X'OX$ and YOY' representing *x*-axis and *y*-axis, respectively, as shown in Fig.8.1.

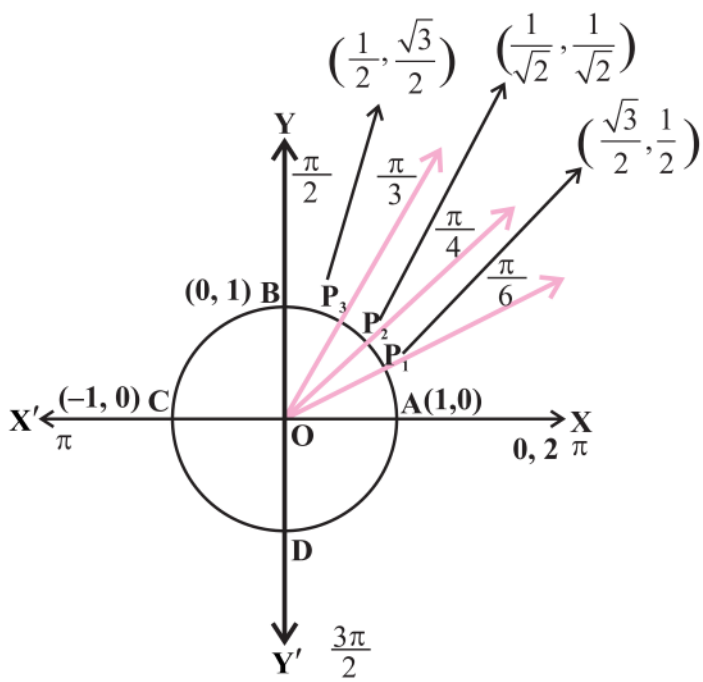

Fig. 8.1

4. Mark the points as A, B, C and D, where the circle cuts the x-axis and y-axis, respectively, as shown in Fig. 8.1.

5. Through O, draw angles P_1OX, P_2OX, and P_3OX of measures $\frac{\pi}{6}, \frac{\pi}{4}$ and $\frac{\pi}{3}$, respectively.

6. Take a needle of unit length. Fix one end of it at the centre of the circle and the other end to move freely along the circle.

DEMONSTRATION

1. The coordinates of the point P_1 are $\left(\sqrt{\frac{3}{2}}, \frac{1}{2}\right)$ because its x-coordinate is $\cos\frac{\pi}{6}$ and y-coordinate is $\sin\frac{\pi}{6}$. The coordinates of the points P_2 and P_3 are $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, respectively.

2. To find the value of sine or cosine of some angle in the second quadrant (say) $\frac{2\pi}{3}$, rotate the needle in anti clockwise direction making an angle P_4OX of measure $\frac{2\pi}{3} = 120^\circ$ with the positive direction of x-axis.

3. Look at the position OP_4 of the needle in

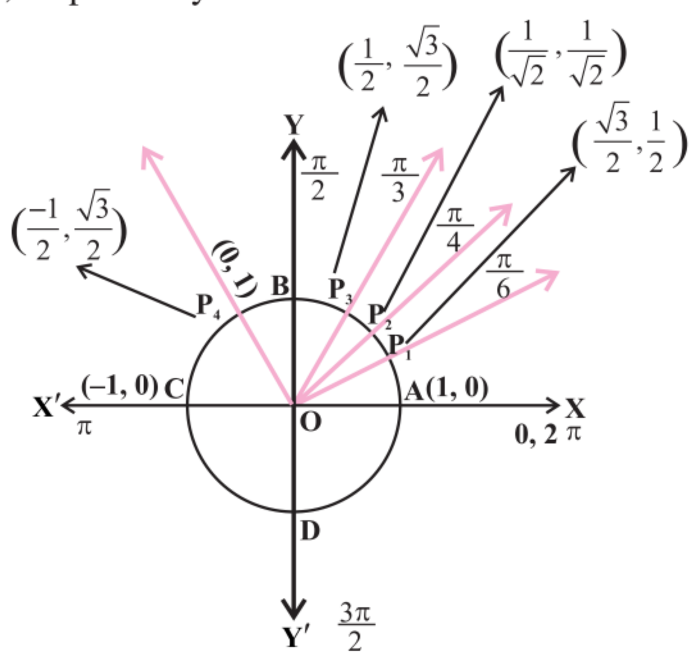

Fig. 8.2

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Fig.8.2. Since $\frac{2\pi}{3} = \pi - \frac{\pi}{3}$, OP_4 is the mirror image of OP_3 with respect to y-axis. Therefore, the coordinate of P_4 are $\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$. Thus $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$ and $\cos\frac{2\pi}{3} = -\frac{1}{2}$.

4. To find the value of sine or cosine of some angle say, $\pi + \frac{\pi}{3} = \frac{4\pi}{3}$, i.e., $\frac{-2\pi}{3}$ (say) in the third quadrant, rotate the needle in anti clockwise direction making as an angle of $\frac{4\pi}{3}$ with the positive direction of x-axis.

5. Look at the new position OP_5 of the needle, which is shown in Fig. 8.3. Point P_5 is the mirror image of the point P_4 (since $\angle P_4OX' = P_5OX'$) with respect to x-axis. Therefore, co-ordinates of P_5 are $\left(-\frac{1}{2}, \frac{-\sqrt{3}}{2}\right)$ and hence

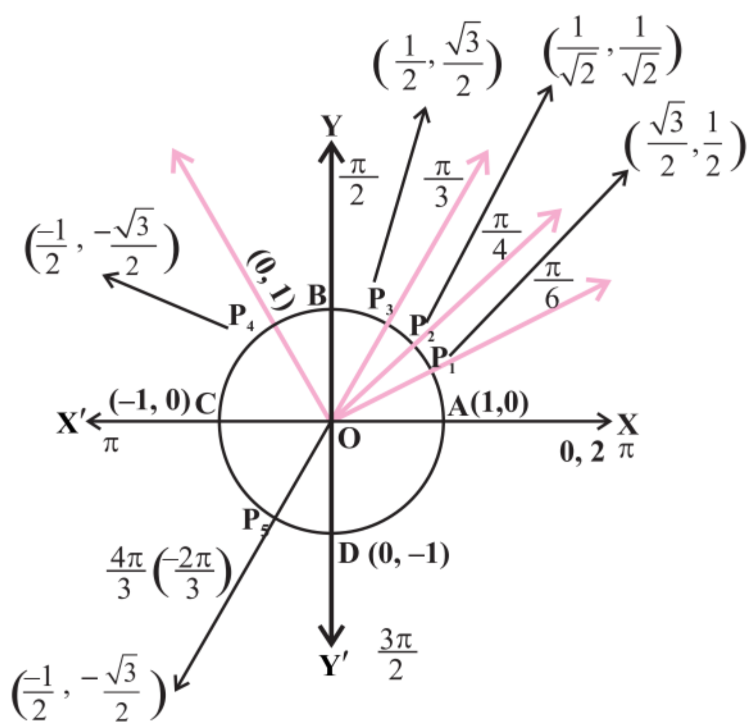

Fig. 8.3

$$\sin\left(-\frac{2\pi}{3}\right) = \sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2} \text{ and } \cos\left(-\frac{2\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = -\frac{1}{2}.$$

6. To find the value of sine or cosine of some angle in the fourth quadrant, say $\frac{7\pi}{4}$, rotate the needle in anti clockwise direction making an angle of $\frac{7\pi}{4}$ with the positive direction of x-axis represented by OP_6, as shown in Fig. 8.4. Angle $\frac{7\pi}{4}$ in anti clockwise direction = Angle $-\frac{\pi}{4}$ in the clockwise direction.

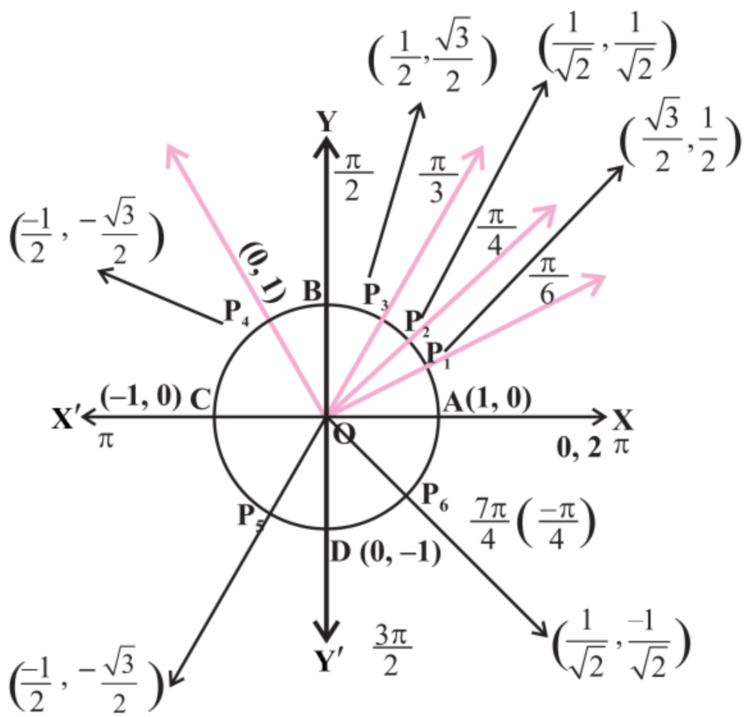

Fig. 8.4

From Fig. 8.4, P_6 is the mirror image of P_2 with respect to x-axis. Therefore, coordinates of P_6 are $\left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$.

Thus $\sin\left(\frac{7\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$

and $\cos\left(\frac{7\pi}{4}\right) = \cos\left(-\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

8. To find the value of sine or cosine of some angle, which is greater than one revolution, say $\frac{13\pi}{6}$, rotate the needle in anti clockwise direction since $\frac{13\pi}{6} = 2\pi + \frac{\pi}{6}$, the needle will reach at the position OP_1. Therefore, $\sin\left(\frac{13\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ and $\cos\left(\frac{13\pi}{6}\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$.

OBSERVATION

1. Angle made by the needle in one complete revolution is _________.
2. $\cos\frac{\pi}{6} =$ _________ $= \cos\left(-\frac{\pi}{6}\right)$

   $\sin\frac{\pi}{6} =$ _________ $= \sin(2\pi +$ ______$)$.
3. sine function is non-negative in _______ and _______ quadrants.
4. cosine function is non-negative in ______ and _____ quadrants.

APPLICATION

1. The activity can be used to get the values for tan, cot, sec, and cosec functions also.
2. From this activity students may learn that
   sin (– θ) = – sin θ and cos (– θ) = – cos θ

   This activity can be applied to other trigonometric functions also.

Activity 9

Objective

To prepare a model to illustrate the values of sine function and cosine function for different angles which are multiples of $\frac{\pi}{2}$ and π.

Material Required

A stand fitted with 0º-360º protractor and a circular plastic sheet fixed with handle which can be rotated at the centre of the protractor.

Method of Construction

1. Take a stand fitted with 0º-360º protractor.
2. Consider the radius of protractor as 1 unit.

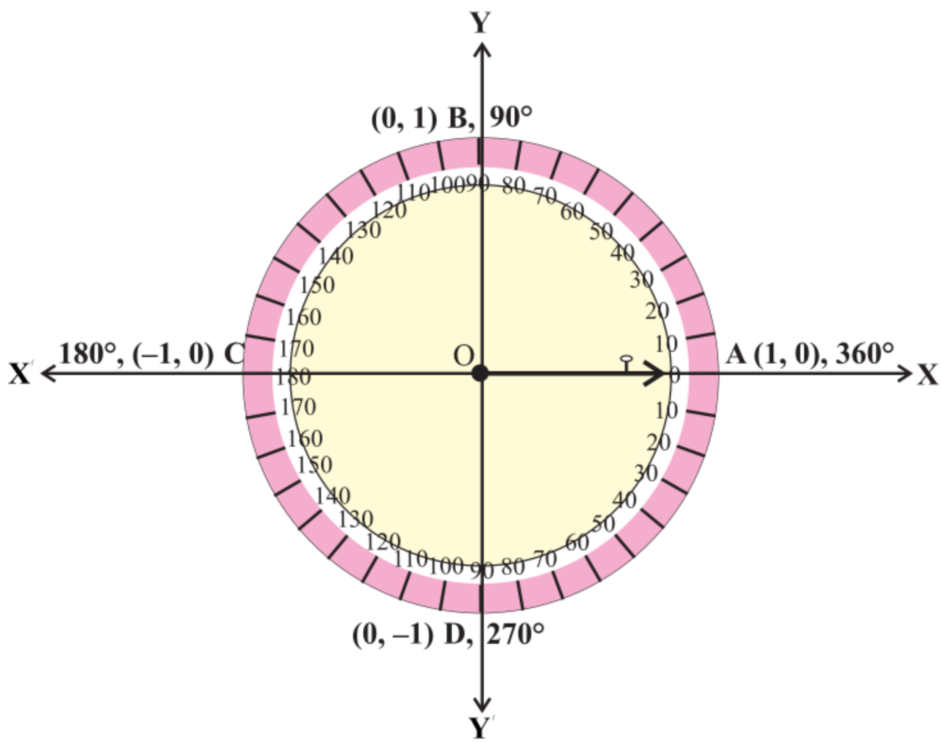

Fig. 9

3. Draw two lines, one joining 0º-180º line and another 90º-270º line, obviously perpendicular to each other.
4. Mark the ends of 0°-180° line as (1,0) at 0°, (–1, 0) at 180° and that of 90° - 270° line as (0,1) at 90° and (0, –1) at 270°
5. Take a plastic circular plate and mark a line to indicate its radius and fix a handle at the outer end of the radius.
6. Fix the plastic circular plate at the centre of the protractor.

Demonstration

1. Move the circular plate in anticlock wise direction to make different angles like 0, $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$, 2π etc.
2. Read the values of sine and cosine function for these angles and their multiples from the perpendicular lines.

Observation

1. When radius line of circular plate is at 0º indicating the point A (1,0), cos 0 = ______ and sin 0 = _______.
2. When radius line of circular plate is at 90º indicating the point B (0, 1), $\cos\frac{\pi}{2}=$ _______ and $\sin\frac{\pi}{2}=$ _______ .
3. When radius line of circular plate is at 180º indicating the point C (–1,0), cos π = ______ and sin π = _________.
4. When radius line of circular plate is at 270º indicating the point D (0, – 1) which means $\cos\frac{3\pi}{2}=$ _______ and $\sin\frac{3\pi}{2}=$ _______
5. When radius line of circular plate is at 360º indicating the point again at A (1,0), cos 2 π = _______ and sin 2 π = ________.

Now fill in the table :

Trigonometric function	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π	$\frac{7\pi}{2}$	4π
$\sin\theta$	–	–	–	–	–	–	–	–	–
$\cos\theta$	–	–	–	–	–	–	–	–	–

APPLICATION

This activity can be used to determine the values of other trigonometric functions for angles being multiple of $\frac{\pi}{2}$ and π.

Activity 10

OBJECTIVE

To plot the graphs of sin x, sin $2x$, $2\sin x$ and $\sin\frac{x}{2}$, using same coordinate axes.

MATERIAL REQUIRED

Plyboard, squared paper, adhesive, ruler, coloured pens, eraser.

METHOD OF CONSTRUCTION

1. Take a plywood of size 30 cm × 30 cm.
2. On the plywood, paste a thick graph paper of size 25 cm × 25 cm.
3. Draw two mutually perpendicular lines on the squared paper, and take them as coordinate axes.
4. Graduate the two axes as shown in the Fig. 10.
5. Prepare the table of ordered pairs for sin x, sin $2x$, $2\sin x$ and $\sin\frac{x}{2}$ for different values of x shown in the table below:

T. ratios	**0°**	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{5\pi}{12}$	$\frac{\pi}{2}$	$\frac{7\pi}{12}$	$\frac{2\pi}{3}$	$\frac{9\pi}{12}$	$\frac{5\pi}{6}$	$\frac{11\pi}{12}$	π
sin x	0	0.26	0.50	0.71	0.86	0.97	1.00	0.97	0.86	0.71	0.50	0.26	0
sin $2x$	0	0.50	0.86	1.00	0.86	0.50	0	–0.5	–0.86	–1.0	–0.86	–0.50	0
2 sin x	0	0.52	1.00	1.42	1.72	1.94	2.00	1.94	1.72	1.42	1.00	0.52	0
$\sin\frac{x}{2}$	0	0.13	0.26	0.38	0.50	0.61	0.71	0.79	0.86	0.92	0.97	0.99	1.00

DEMONSTRATION

1. Plot the ordered pair $(x, \sin x)$, $(x, \sin 2x)$, $(x, \sin \frac{x}{2})$ and $(x, 2\sin x)$ on the same axes of coordinates, and join the plotted ordered pairs by free hand curves in different colours as shown in the Fig.10.

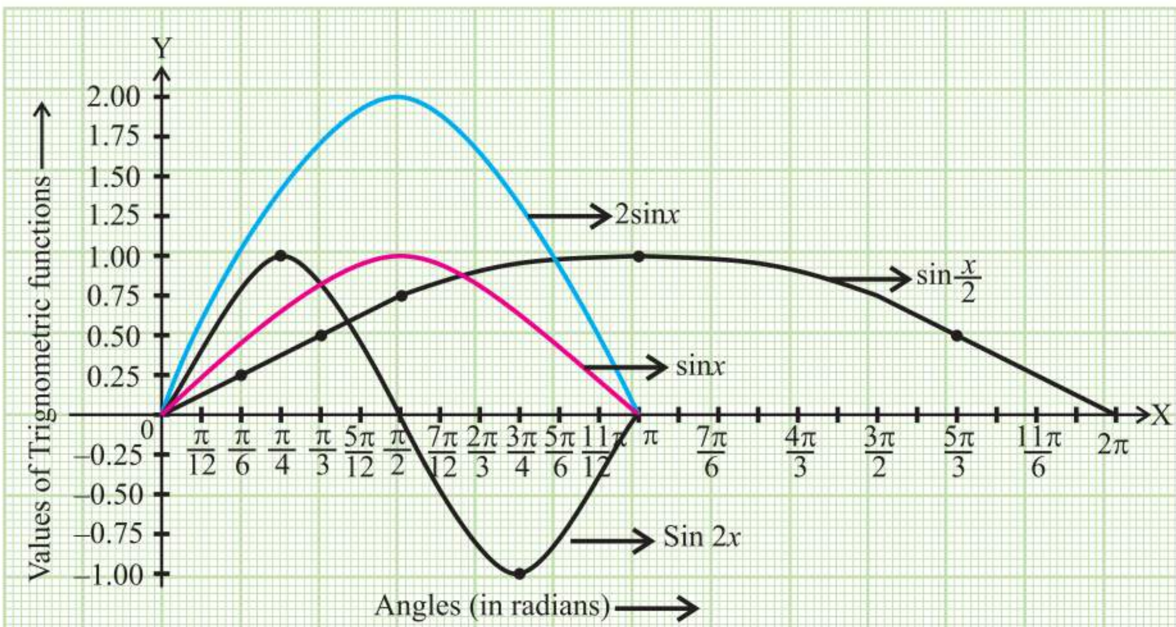

Fig. 10

OBSERVATION

1. Graphs of sin x and 2 sin x are of same shape but the maximum height of the graph of sin x is ________ the maximum height of the graph of __________.
2. The maximum height of the graph of sin $2x$ is __________. It is at x = __________.
3. The maximum height of the graph of 2 sin x is __________ . It is at x = __________ .

4. The maximum height of the graph of $\sin \frac{x}{2}$ is ___________ . It is at $\frac{x}{2}$ = _________ .

5. At x = ________, $\sin x = 0$, at x = _______, $\sin 2x = 0$ and at x = ________, $\sin\frac{x}{2} = 0$.

6. In the interval $[0, \pi]$, graphs of $\sin x$, $2 \sin x$ and $\sin \frac{x}{2}$ are _______ x - axes and some portion of the graph of $\sin 2x$ lies _______ x-axes.

7. Graphs of $\sin x$ and $\sin 2x$ intersect at x = _______ in the interval $(0, \pi)$

8. Graphs of $\sin x$ and $\sin \frac{x}{2}$ intersect at x = ________ in the interval $(0, \pi)$.

Application

This activity may be used in comparing graphs of a trigonometric function of multiples and submultiples of angles.

Activity 14

OBJECTIVE

To find the number of ways in which three cards can be selected from given five cards.

MATERIAL REQUIRED

Cardboard sheet, white paper sheets, sketch pen, cutter.

METHOD OF CONSTRUCTION

1. Take a cardboard sheet and paste white paper on it.
2. Cut out 5 identical cards of convenient size from the cardboard.
3. Mark these cards as C_1, C_2, C_3, C_4 and C_5.

DEMONSTRATION

1. Select one card from the given five cards.
2. Let the first selected card be C_1. Then other two cards from the remaining four cards can be : C_2C_3, C_2C_4, C_2C_5, C_3C_4, C_3C_5 and C_4C_5.Thus, the possible selections are : $C_1C_2C_3$, $C_1C_2C_4$, $C_1C_2C_5$,$C_1C_3C_4$, $C_1C_3C_5$, $C_1C_4C_5$. Record these on a paper sheet.
3. Let the first selected card be C_2. Then the other two cards from the remaining 4 cards can be : C_1C_3, C_1C_4, C_1C_5, C_3C_4, C_3C_5, C_4C_5.Thus, the possible selections are: $C_2C_1C_3$, $C_2C_1C_4$, $C_2C_1C_5$,$C_2C_3C_4$, $C_2C_3C_5$, $C_2C_4C_5$. Record these on the same paper sheet.
4. Let the first selected card be C_3. Then the other two cards can be : C_1C_2, C_1C_4, C_1C_5, C_2C_4, C_2C_5, C_4C_5.Thus, the possible selections are : $C_3C_1C_2$, $C_3C_1C_4$, $C_3C_1C_5$,$C_3C_2C_4$, $C_3C_2C_5$, $C_3C_4C_5$. Record them on the same paper sheet.
5. Let the first selected card be C_4. Then the other two cards can be : C_1C_2, C_1C_3, C_2C_3, C_1C_5, C_2C_5, C_3C_5 Thus, the possible selections are: $C_4C_1C_2$, $C_4C_1C_3$,$C_4C_2C_3$, $C_4C_1C_5$, $C_4C_2C_5$, $C_4C_3C_5$. Record these on the same paper sheet.

6. Let the first selected card be C_5. Then the other two cards can be: C_1C_2, C_1C_3, C_1C_4, C_2C_3, C_2C_4, C_3C_4 Thus, the possible selections are: $C_5C_1C_2$,$C_5C_1C_3$, $C_5C_1C_4$, $C_5C_2C_3$, $C_5C_2C_4$, $C_5C_3C_4$. Record these on the same paper sheet.
7. Now look at the paper sheet on which the possible selectios are listed. Here, there are in all 30 possible selections and each of the selection is repeated thrice. Therefore, the number of distinct selection $= 30 \div 3 = 10$ which is same as $5C_3$.

OBSERVATION

1. $C_1C_2C_3$, $C_2C_1C_3$ and $C_3C_1C_2$ represent the _______ selection.
2. $C_1C_2C_4$, _____________, ____________ represent the same selection.
3. Among $C_2C_1C_5$, $C_1C_2C_5$, $C_1C_2C_3$, ________ and ________ represent the same selection.
4. $C_2C_1C_5$, $C_1C_2C_3$, represent _______ selections.
5. Among $C_3C_1C_5$, $C_1C_4C_3$, $C_5C_3C_4$, $C_4C_2C_5$, $C_2C_4C_3$, $C_1C_3C_5$

   $C_3C_1C_5$, ________ represent the same selections.

   $C_3C_1C_5$, $C_1C_4C_5$, ______, _____, represent different selections.

APPLICATION

Activities of this type can be used in understanding the general formula for finding the number of possible selections when *r* objects are selected from given *n* distinct objects, i.e., ${}^nC_r = \frac{n!}{r!(n-r)!}$.

Activity 15

Objective

To construct a Pascal's Triangle and to write binomial expansion for a given positive integral exponent.

Material Required

Drawing board, white paper, matchsticks, adhesive.

Method of Construction

1. Take a drawing board and paste a white paper on it.
2. Take some matchsticks and arrange them as shown in Fig.15.

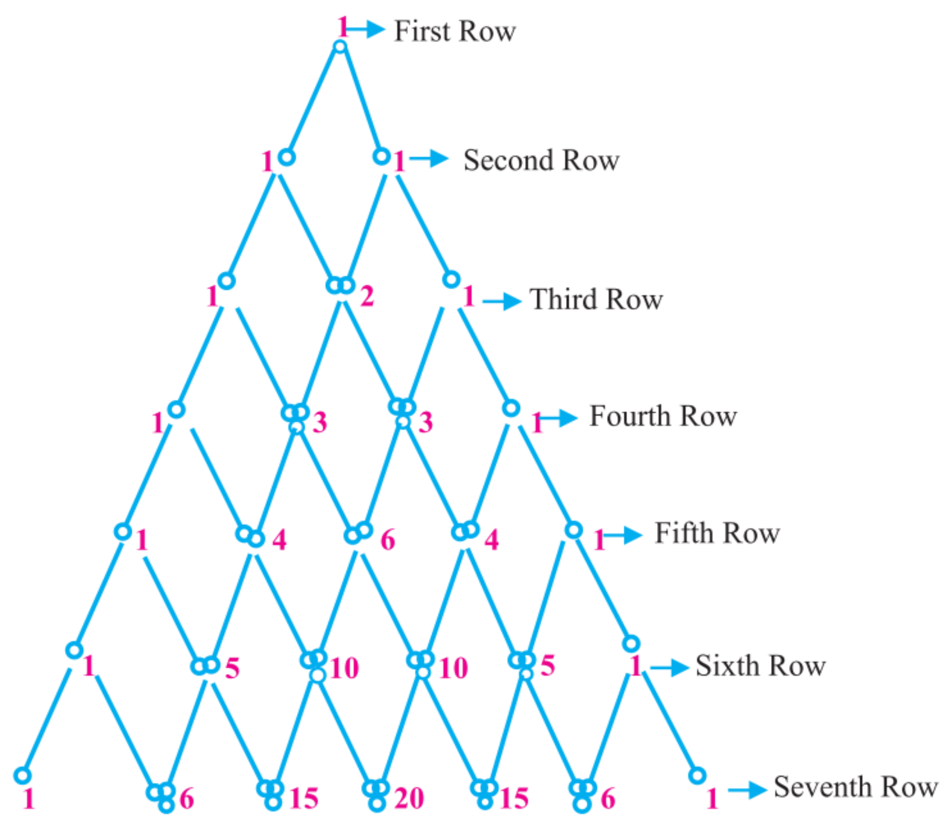

Fig. 15

3. Write the numbers as follows:

   1 (first row)
   1 1 (second row)
   1 2 1 (third row)
   1 3 3 1 (fourth row), 1 4 6 4 1 (fifth row) and so on (see Fig. 15).

4. To write binomial expansion of $(a + b)^n$, use the numbers given in the $(n + 1)^{th}$ row.

DEMONSTRATION

1. The above figure looks like a triangle and is referred to as Pascal's Triangle.
2. Numbers in the second row give the coefficients of the terms of the binomial expansion of $(a + b)^1$. Numbers in the third row give the coefficients of the terms of the binomial expansion of $(a + b)^2$, numbers in the fourth row give coefficients of the terms of binomial expansion of $(a + b)^3$. Numbers in the fifth row give coefficients of the terms of binomial expansion of $(a + b)^4$ and so on.

OBSERVATION

1. Numbers in the fifth row are ___________, which are coefficients of the binomial expansion of ___________.
2. Numbers in the seventh row are _____________, which are coefficients of the binomial expansion of _______.
3. $(a + b)^3 =$ ___ $a^3 +$ ___$a^2b +$ ___$ab^2 +$ ___b^3
4. $(a + b)^5 =$ ___ + ___ + ___ + ___ + ___ + ___.
5. $(a + b)^6 =$ ___$a^6 +$ ___$a^5b +$ ___$a^4b^2 +$ ___$a^3b^3 +$ ___$a^2b^4 +$ ___$ab^5 +$ ___b^6.
6. $(a + b)^8 =$ ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___.
7. $(a + b)^{10} =$ ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + ___ + __.

APPLICATION

The activity can be used to write binomial expansion for $(a + b)^n$, where n is a positive integer.

Activity 21

Objective

To construct different types of conic sections.

Material Required

Transparent sheet, scissors, hard-board, adhesive, white paper.

Method of Construction

1. Take a hardboard of convenient size and paste a white paper on it.
2. Cut a transparent sheet in the shape of sector of a circle and fold it to obtain a right circular cone as shown in Fig.21.1.
3. Form 4 more such cones of the same size using transparent sheet. Put these cones on a hardboard.
4. Cut these cones with a transparent plane sheet in different positions as shown in Fig. 21.2 to Fig. 21.5.

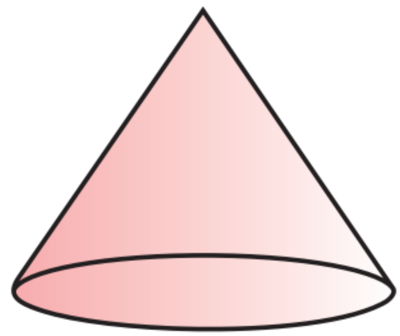

Fig 21.1

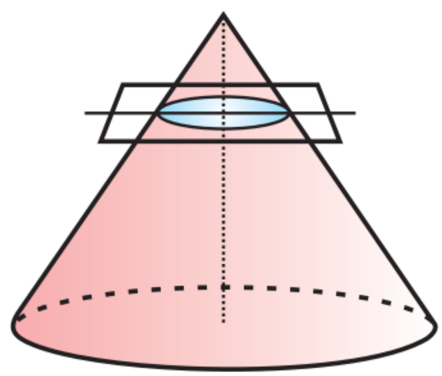

Fig 21.2

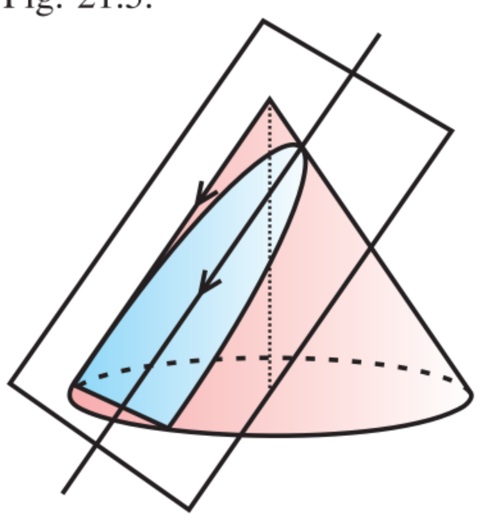

Fig 21.4

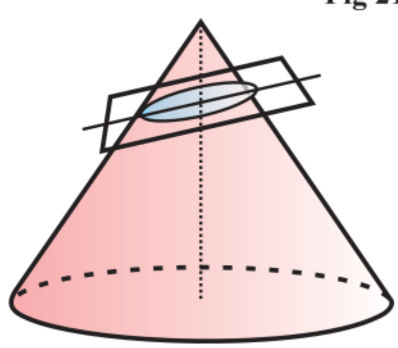

Fig 21.3

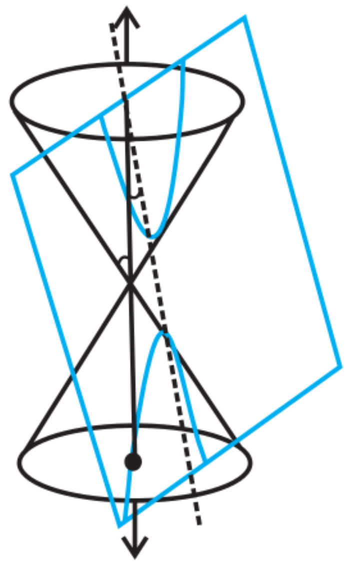

Fig 21.5

DEMONSTRATION

1. In Fig. 21.2, the transparent plane sheet cuts the cone in such a way that the sheet is parallel to the base of the cone. The section so obtained is a circle.
2. In Fig. 21.3, the plane sheet is inclined slightly to the axes of the cone. The section so obtained is an ellipse.
3. In Fig. 21.4, the plane sheet is parallel to a generator (slant height) of the cone. The section so obtained is a parabola.
4. In Fig. 21.5 the plane is parallel to the axis of the cone. The sections so obtained is a part of a hyperbola.

OBSERVATION

1. In Fig. 21.2, the transparent plane sheet is ______ to the base of the cone. The section obtained is _____.
2. In Fig. 21.3, the plane sheet is inclined to ______. The conic section obtained is __________.
3. In Fig. 21.4, the plane sheet is parallel to the __________. The conic section so obtained is ______.
4. In Fig. 21.5, the plane sheet is ________ to the axis. The conic section so obtained is a part of ______.

APPLICATION

This activity helps in understanding various types of conic sections which have wide spread applications in real life situations and modern sciences. For example, conics have interesting geometric properties that can be used for the reflection of light rays and beams of sound, i.e.

1. Circular disc reflects back the light issuing from centre to the centre again.
2. Elliptical disc reflects back the light issuing from one focus to the other focus.
3. Parabolic disc reflects back the light issuing from one focus parallel to its axis.
4. Hyperbolic disc reflects back the light issuing from one focus as if coming from other focus.

Activity 28

OBJECTIVE

To find analytically $\lim_{x \to c} f(x) = \frac{x^2 - c^2}{x - c}$

MATERIAL REQUIRED

Pencil, white paper, calculator.

METHOD OF CONSTRUCTION

1. Consider the function f given by $f(x) = \frac{x^2 - 9}{x - 3}$
2. In this case $c = 3$ and the function is not defined at $x = 3$.

DEMONSTRATION

1. Take some values of c less than $c = 3$ and some other values of c more than $c = 3$.
2. In both cases, the values to be taken have to be very close to $c = 3$.
3. Calculate the corresponding values of f at each of the values of c taken close to $c = 3$.

DEMONSTRATION : TABLE 1

1. Write the values of $f(x)$ in the following tables:

Table 1

x	2.9	2.99	2.999	2.9999	2.99999	2.999999
$f(x)$	5.9	5.99	5.999	5.9999	5.99999	5.999999

Table 2

x	3.1	3.01	3.001	3.0001	3.00001	3.000001
$f(x)$	6.1	6.01	6.001	6.0001	6.00001	6.000001

OBSERVATION

1. Values of $f(x)$ as $x \to 3$ from the left, as in Table 1 are coming closer and closer to ______.
2. Values of $f(x)$ as $x \to 3$ from the right, as in Table 2 are coming closer and closer to ______ from tables (2) and (3), $\lim_{x \to 3} f(x) = \frac{x^2 - 9}{x - 3} =$ ______.

APPLICATION

This activity can be used to demonstrate the concept of a limit $\lim_{x \to c} f(x)$ when $f(x)$ is not defined at $x = c$.

Activity 32

OBJECTIVE

To write the sample space, when a die is rolled once, twice --------

MATERIAL REQUIRED

A die, paper, pencil/pen, plastic discs, marked with 1, 2, 3, 4, 5 or 6.

METHOD OF CONSTRUCTION

1. Throw a die once. The number on its top will be 1, 2, 3, 4, 5 or 6.
2. Make a tree diagram showing its six branches with number 1,2, 3, 4, 5 or 6 (See Fig. 32.1)
3. Write the sample space of these outcomes.
4. Throw a die twice. It can fall in any of the 36 ways as shown in Fig. 32.2 by the tree diagram. Write the sample space of these outcomes.

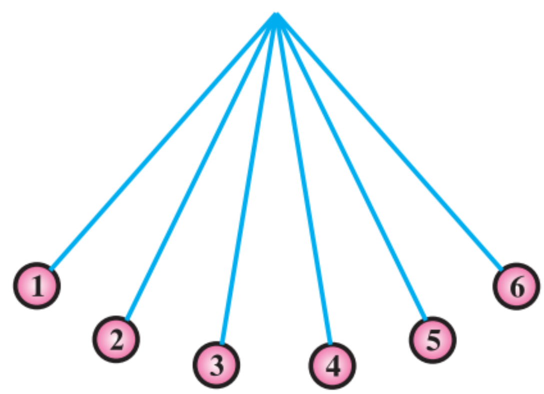

Fig 32.1

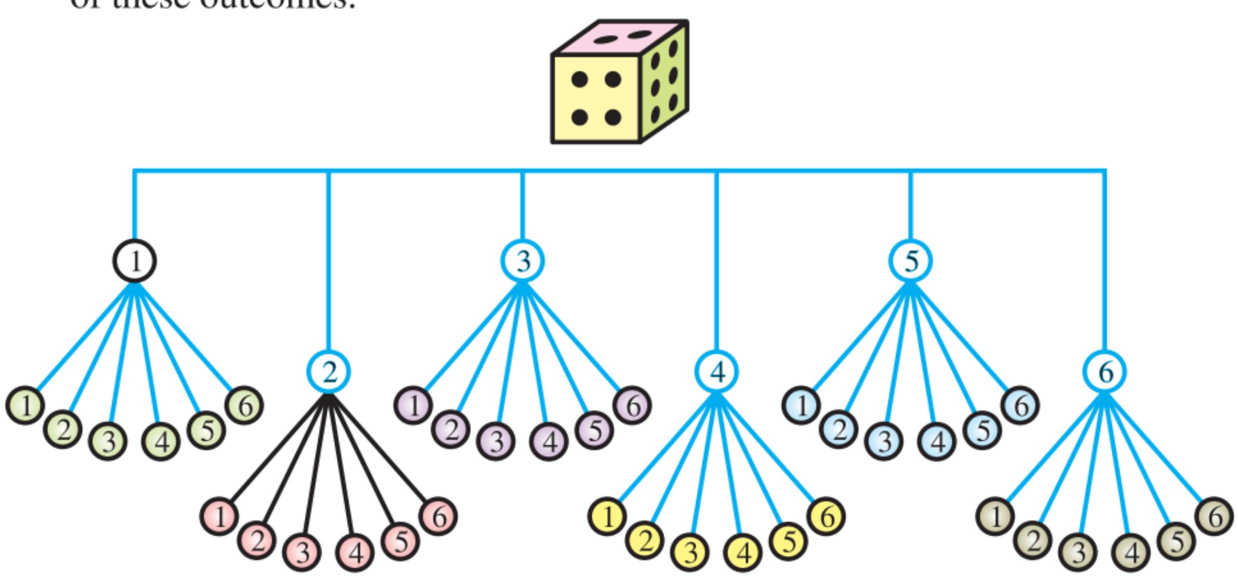

Fig 32.2

5. Repeat the experiment by throwing a die 3 times, and write the sample space of the outcomes using a tree diagram.

DEMONSTRATION

1. If a die is thrown once, the sample space is

$$S = \{1, 2, 3, 4, 5, 6\}. \text{ Number of elements in } S = 6 = 6^1$$

2. If a die is thrown twice, the sample space is

$$\text{Sample space } S = \begin{Bmatrix} (1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6) \\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6) \\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6) \end{Bmatrix}$$

The number of elements in $S = 36 = 6^2$ and so on.

OBSERVATION

Number of elements in sample space when a die is thrown

Once = __________, Thrice = ________, Four times = __________

APPLICATION

Sample space of an experiment is useful in determining the probabilities of different events associated with the sample space.