

Name: _____

Date: _____

Notes: Operations with Radicals

Do Now: Simplify each expression.

1) $7x + 9x$

$$16x$$

2) $7x^2 + 9x$

$$7x^2 + 9x$$

3) $7\sqrt{2} + 9\sqrt{2}$

$$16\sqrt{2}$$

4) $7\sqrt{3} + 9\sqrt{2}$

$$7\sqrt{3} + 9\sqrt{2}$$

When **ADDING** or **SUBTRACTING** radicals, you must have ...

LIKE TERMS

Simplify each of the following radical expressions:

A) $10\sqrt{11} + 2\sqrt{11}$

$$12\sqrt{11}$$

B) $4\sqrt{2} + 3\sqrt{2}$

$$7\sqrt{2}$$

C) $5\sqrt{13} + 3\sqrt{7}$

$$5\sqrt{13} + 3\sqrt{7}$$

D) $\sqrt{3} - 8\sqrt{3}$

$$-7\sqrt{3}$$

But what if we have ...

UNLIKE TERMS

$$5\sqrt{3} - \sqrt{4}\sqrt{3} \quad \textcircled{1}$$

$$5\sqrt{3} - 2\sqrt{3}$$
$$\boxed{3\sqrt{3}} \quad \textcircled{2}$$

$$3\sqrt{9}\sqrt{3} + 2\sqrt{4}\sqrt{3}$$
$$3(3)\sqrt{3} + 2(2)\sqrt{3}$$

$$9\sqrt{3} + 4\sqrt{3}$$
$$\boxed{13\sqrt{3}}$$

1) SIMPLIFY all radicals.

2) Combine all like terms.

Checkpoint:

Completely simplify each of the following expressions.

A) $2\sqrt{32} + 7\sqrt{98}$

$$2\sqrt{16}\sqrt{2} + 7\sqrt{49}\sqrt{2}$$
$$2(4)\sqrt{2} + 7(7)\sqrt{2}$$
$$8\sqrt{2} + 49\sqrt{2}$$
$$\boxed{57\sqrt{2}}$$

B) $10\sqrt{27} - 9\sqrt{18}$

$$10\sqrt{9}\sqrt{3} - 9\sqrt{9}\sqrt{2}$$
$$10(3)\sqrt{3} - 9(3)\sqrt{2}$$
$$\boxed{30\sqrt{3} - 27\sqrt{2}}$$

C) $\frac{2}{3}\sqrt{27} - 3\sqrt{108} + 1.2\sqrt{75}$

$$\frac{2}{3}\sqrt{9}\sqrt{3} - 3\sqrt{36}\sqrt{3} + 1.2\sqrt{25}\sqrt{3}$$
$$\frac{2}{3}(3)\sqrt{3} - 3(6)\sqrt{3} + 1.2(5)\sqrt{3}$$
$$2\sqrt{3} - 18\sqrt{3} + 6\sqrt{3}$$
$$\boxed{-10\sqrt{3}}$$

D) $\sqrt{98} - 4\sqrt{8} + 3\sqrt{128}$

$$\sqrt{2}\sqrt{49} - 4\sqrt{4}\sqrt{2} + 3\sqrt{64}\sqrt{2}$$
$$7\sqrt{2} - 4(2)\sqrt{2} + 3(8)\sqrt{2}$$
$$7\sqrt{2} - 8\sqrt{2} + 24\sqrt{2}$$
$$\boxed{23\sqrt{2}}$$

MULTIPLYING

$$\begin{array}{l} \sqrt{5}(\sqrt{10}) \\ \sqrt{50} \\ \sqrt{25}\sqrt{2} \end{array} \quad \boxed{5\sqrt{2}}$$

- 1) **MULTIPLY** coefficients.
- 2) **MULTIPLY** radicands.
- 3) If possible, simplify the result.

$$(2\sqrt{15})(3\sqrt{3})$$

$$\begin{array}{l} 6\sqrt{45} \\ 6\sqrt{9}\sqrt{5} \end{array}$$

$$\begin{array}{l} 6(3)\sqrt{5} \\ \boxed{18\sqrt{5}} \end{array}$$

Checkpoint:

Completely simplify each of the following expressions.

1) $3\sqrt{27} \cdot 5\sqrt{2}$

$$\begin{array}{l} 15\sqrt{54} \\ 15\sqrt{9}\sqrt{6} \\ 15(3)\sqrt{6} \end{array} \quad \boxed{45\sqrt{6}}$$

2) $(\frac{1}{2}\sqrt{3})^2$

$$\begin{array}{l} (\frac{1}{2}\sqrt{3})(\frac{1}{2}\sqrt{3}) \\ \frac{1}{4}\sqrt{9} \\ \frac{1}{4}(3) \\ \boxed{\frac{3}{4}} \end{array}$$

3) $-4\sqrt{a}(3\sqrt{a})$

$$\begin{array}{l} -12\sqrt{a^2} \\ \boxed{-12a} \end{array}$$

4) $(4 - \sqrt{6})(3 + \sqrt{6})$

$$\begin{array}{l} 4(3) + 4\sqrt{6} - 3\sqrt{6} - \sqrt{36} \\ 12 + \sqrt{6} - 6 \\ \boxed{6 + \sqrt{6}} \end{array}$$

Dividing

We must first RATIONALIZE THE DENOMINATOR!

You can multiply numerator and denominator by $2\sqrt{5}$, you would just need to simplify more at the end.

$$\frac{\sqrt{3} \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}}$$

$$\frac{\sqrt{21}}{\sqrt{49}}$$

$$\boxed{\frac{\sqrt{21}}{7}}$$

$$\frac{2\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}$$

$$\boxed{\frac{2\sqrt{15}}{5}}$$

$$\frac{5\sqrt{3} \cdot \sqrt{5}}{2\sqrt{5} \cdot \sqrt{5}}$$

$$\frac{5\sqrt{15}}{2(5)}$$

$$\frac{5\sqrt{15}}{10}$$

$$\boxed{\frac{\sqrt{15}}{2}}$$

$$\frac{5\sqrt{3}}{2 + \sqrt{5}} \cdot \frac{(2 - \sqrt{5})}{(2 - \sqrt{5})}$$

← Multiply numerator and denominator by the **CONJUGATE** of the denominator.

To find the conjugate of a binomial change the operation from + to - or - to +.

$$\frac{10\sqrt{3} - 5\sqrt{15}}{4 - 2\sqrt{5} + 2\sqrt{5} - \sqrt{25}}$$

middle terms should always cancel

$$\frac{10\sqrt{3} - 5\sqrt{15}}{4 - 5}$$

$$\frac{10\sqrt{3} - 5\sqrt{15}}{-1}$$

$$\boxed{-10\sqrt{3} + 5\sqrt{15}}$$

$$\frac{9 - 2\sqrt{3}}{\sqrt{3} + 6} \cdot \frac{(\sqrt{3} - 6)}{(\sqrt{3} - 6)}$$

$$\frac{9\sqrt{3} - 54 - 2\sqrt{9} + 12\sqrt{3}}{\sqrt{9} - 6\sqrt{3} + 6\sqrt{3} - 36}$$

$$\frac{9\sqrt{3} - 54 - 6 + 12\sqrt{3}}{3 - 36}$$

$$\frac{-60 + 21\sqrt{3}}{-33}$$

$$\boxed{\frac{20 - 7\sqrt{3}}{11}}$$

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Classwork: Operations with Radicals

Completely simplify each expression.

1) $\sqrt{5} + 6\sqrt{5}$

$7\sqrt{5}$

2) $-8.7\sqrt{3} - 3\sqrt{3}$

$-11.7\sqrt{3}$

3) $2\sqrt{72} + 3\sqrt{98}$

$$2\sqrt{36}\sqrt{2} + 3\sqrt{49}\sqrt{2}$$

$$2(6)\sqrt{2} + 3(7)\sqrt{2}$$

$$12\sqrt{2} + 21\sqrt{2}$$

$$\boxed{33\sqrt{2}}$$

4) $\sqrt{7}(24 + \sqrt{7})$

$$24\sqrt{7} + \sqrt{49}$$

$$\boxed{24\sqrt{7} + 7}$$

5) $3\sqrt{18}(-4\sqrt{8})$

$$(3\sqrt{9}\sqrt{2})(-4\sqrt{4}\sqrt{2})$$

$$(9\sqrt{2})(-8\sqrt{2})$$

$$-72\sqrt{4}$$

$$\boxed{-144}$$

6) $\sqrt{75} - 4\sqrt{12} + 3\sqrt{192}$

$$\sqrt{25}\sqrt{3} - 4\sqrt{4}\sqrt{3} + 3\sqrt{64}\sqrt{3}$$

$$5\sqrt{3} - 8\sqrt{3} + 24\sqrt{3}$$

$$-3\sqrt{3} + 24\sqrt{3}$$

$$\boxed{21\sqrt{3}}$$

7) $\sqrt{81x} + \sqrt{16x}$

$$\sqrt{81}\sqrt{x} + \sqrt{16}\sqrt{x}$$

$$9\sqrt{x} + 4\sqrt{x}$$

$$\boxed{13\sqrt{x}}$$

8) $(\frac{1}{3} + \sqrt{18})^2$

$$(\frac{1}{3} + \sqrt{18})(\frac{1}{3} + \sqrt{18})$$

$$\frac{1}{9} + \frac{1}{3}\sqrt{18} + \frac{1}{3}\sqrt{18} + 18$$

$$18\frac{1}{9} + \frac{2}{3}\sqrt{18}$$

$$18\frac{1}{9} + \frac{2}{3}\sqrt{9}\sqrt{2}$$

$\boxed{18\frac{1}{9} + 2\sqrt{2}}$

9) $(\sqrt{3} - \sqrt{6})(\sqrt{3} + \sqrt{6})$

$$\sqrt{9} + \sqrt{18} - \sqrt{18} - \sqrt{36}$$

$$3 - 6$$

$$\boxed{-3}$$

Completely simplify each expression.

10) $\frac{\sqrt{27}}{\sqrt{15}}$

$$\frac{\sqrt{9}\sqrt{3}}{\sqrt{5}\sqrt{3}}$$

$$\frac{3 \cdot \sqrt{3}}{\sqrt{5} \cdot \sqrt{3}}$$

$$\boxed{\frac{3\sqrt{3}}{5}}$$

11) $\frac{4-\sqrt{768}}{4}$

$$\frac{4-\sqrt{256}\sqrt{3}}{4}$$

$$\frac{4-16\sqrt{3}}{4}$$

$$\boxed{1-4\sqrt{3}}$$

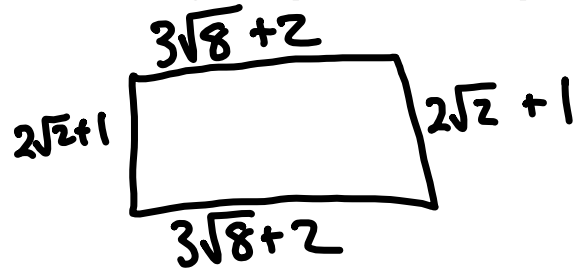
12) $\frac{(2-\sqrt{5})(-3-\sqrt{5})}{-3+\sqrt{5}}$

$$\frac{-6-2\sqrt{5}+3\sqrt{5}+\sqrt{25}}{9+3\sqrt{5}-3\sqrt{5}-\sqrt{25}}$$

$$\frac{-6+\sqrt{5}+5}{9-5}$$

$$\boxed{\frac{-1+\sqrt{5}}{4}}$$

13) The length of a rectangle is $(3\sqrt{8} + 2)$ and the width is $(2\sqrt{2} + 1)$. Express the perimeter of the rectangle in simplest radical form. Express the area of the rectangle in simplest radical form.



$$3\sqrt{8} + 2$$

$$3\sqrt{4}\sqrt{2} + 2$$

$$6\sqrt{2} + 2$$

Perimeter:

$$2(6\sqrt{2} + 2) + 2(2\sqrt{2} + 1)$$

$$12\sqrt{2} + 4 + 4\sqrt{2} + 2$$

$$\boxed{6 + 16\sqrt{2} \text{ units}}$$

Area:

$$(6\sqrt{2} + 2)(2\sqrt{2} + 1)$$

$$12\sqrt{4} + 6\sqrt{2} + 4\sqrt{2} + 2$$

$$24 + 10\sqrt{2} + 2$$

$$\boxed{26 + 10\sqrt{2} \text{ units}^2}$$

14) If $A = -3 + 8\sqrt{5}$ and $B = \sqrt{5} - 9$, then $A - B$ equals

- (1) $-12 + 7\sqrt{5}$
- (2) $6 + 7\sqrt{5}$
- (3) $-12 + 8\sqrt{5}$
- (4) $6 + 8\sqrt{5}$

$$(-3 + 8\sqrt{5}) - (\sqrt{5} - 9)$$

$$-3 + 8\sqrt{5} - \sqrt{5} + 9$$

$$6 + 7\sqrt{5}$$

15) Dominique is installing a rectangular window in his neighbor's house. If the width of the window is $\frac{8+2\sqrt{10}}{3+\sqrt{5}}$ in. and the length is $\frac{5-\sqrt{5}}{\sqrt{3}}$ in. Find the window's total area, keeping your answer in simplest radical form.

Width Simplified:

$$\frac{8+2\sqrt{10}}{3+\sqrt{5}} \cdot \frac{(3-\sqrt{5})}{(3-\sqrt{5})}$$

Length Simplified:

$$\frac{5-\sqrt{5}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{24 - 8\sqrt{15} + 6\sqrt{10} - 2\sqrt{50}}{9 - 3\sqrt{5} + 3\sqrt{5} - \sqrt{25}}$$

$$\frac{5\sqrt{3} - \sqrt{15}}{3} \cdot l$$

$$\frac{24 - 8\sqrt{15} + 6\sqrt{10} - 10\sqrt{2}}{4} : w$$

$$A = lw \left(\frac{24 - 8\sqrt{15} + 6\sqrt{10} - 10\sqrt{2}}{4} \right) \left(\frac{5\sqrt{3} - \sqrt{15}}{3} \right)$$

$$\frac{(24 - 8\sqrt{15} + 6\sqrt{10} - 10\sqrt{2})(5\sqrt{3} - \sqrt{15})}{12}$$

$$\frac{120\sqrt{3} - 24\sqrt{45} - 40\sqrt{45} + 8(15) + 30\sqrt{30} - 6\sqrt{150} - 50\sqrt{6} + 10\sqrt{30}}{12}$$

$$\frac{120\sqrt{3} - 24\sqrt{45} - 40\sqrt{45} + 120 + 30\sqrt{30} - 30\sqrt{6} - 50\sqrt{6} + 10\sqrt{30}}{12}$$

$$\frac{120\sqrt{3} - 24\sqrt{15} - 120\sqrt{5} + 120 + 40\sqrt{30} - 80\sqrt{6}}{12}$$

$$\frac{30\sqrt{3} - 6\sqrt{15} - 30\sqrt{5} + 30 + 10\sqrt{30} - 20\sqrt{6}}{3} \text{ units}^2$$