Name: _____ Date: _____

Notes: Solving Absolute Value Equations and Inequalities

Do Now: Find **ALL** possible values of x in each equation.

1) $|x| = 7$

$x = -7, 7$

2) $|x| + 2 = 7$

$-2 \quad -2$

$|x| = 5$

$x = -5, 5$

3) $|x - 2| = 7$

$x - 2 = 7$

$+2 \quad +2$

$x = 9$

BUT ALSO

$x - 2 = -7$

$+2 \quad +2$

$x = -5$

$x = -5, 9$

4) $|x| + 10 = 7$

$-10 \quad -10$

$|x| = -3$

An absolute value can't be negative

$\therefore \emptyset$

5) $\frac{2|x|}{2} = \frac{7}{2}$

$|x| = 3.5$

$x = -3.5, 3.5$

What Should I Be Able to Do?

- I can solve absolute value equations with variables on one side or both sides of the equation.

- I can explain the rationale behind the process of solving an absolute value equation.

- I can assess whether an absolute value equation has no solutions.

- I can solve absolute value inequalities.

- I can explain the rationale behind the process of solving an absolute value inequality.

- I can assess when an absolute value inequality has no solutions or has a solution set of all real numbers.

Solve each of the following equations:

1) $|3x - 1| = 28$

$3x-1=28$ $\quad +1 \quad +1$

$\frac{3x}{3}=\frac{29}{3}$

$\boxed{x=\frac{29}{3}}$

$3x-1=-28$ $\quad +1 \quad +1$

$\frac{3x}{3}=\frac{-27}{3}$

$\boxed{x=-9}$

Check:

$|3(-9)-1|=28$

$|-28|=28$

$28=28$ ✓

$|3(\frac{29}{3})-1|=28$

$|28|=28$

$28=28$ ✓

2) $-6|x - 15| - 10 = 2$

$+10 \quad +10$

$\frac{-6|x-15|}{-6}=\frac{12}{-6}$

$|x-15| = -2$

$x-15=-2$ $\quad +15 \quad +15$

$x=13$

Reject

$x-15=2$ $\quad +15 \quad +15$

$x=17$

Reject

$\boxed{\varnothing}$

Check:

$-6|13-15|-10=2$

$-6|-2|-10=2$

$-22 \neq 2$ X

$-6|17-15|-10=2$

$-6|2|-10=2$

$-22=2$ X

Vocab Corner

Extraneous Solution: A solution to a transformed version of an equation that is not a true solution to the original equation.

3) $|x - 2| = -3x + 18$

$x-2=-3x+18$

$+3x \quad +2 \quad +3x \quad +2$

$\frac{4x}{4}=\frac{20}{4}$

$\boxed{x=5}$

$x-2=-(-3x+18)$

$x-2=3x-18$

$-x \quad +18 \quad -x \quad +18$

$\frac{16}{2}=\frac{2x}{2}$

$x=8$

Reject

Check:

$|8-2|=-3(8)+18$

$|6|=-24+18$

$6 \neq -6$ X

$|5-2|=-3(5)+18$

$|3|=-15+18$

$3=3$ ✓

Checkpoint:

Solve each of the following equations:

1) $-5|2x+3|+1=-24$

$-1 \quad -1$

$\frac{-5|2x+3|}{-5}=\frac{-25}{-5}$

$|2x+3|=5$

$2x+3=5$ $\quad$ $2x+3=-5$

$-3 \quad -3$ $\quad$ $-3 \quad -3$

$\frac{2x}{2}=\frac{2}{2}$ $\quad$ $\frac{2x}{2}=\frac{-8}{2}$

$x=1$ $\quad$ $x=-4$

Check: $-5|2(1)+3|+1=-24$

$-24=-24$ ✓

$-5|2(-4)+3|+1=-24$

$-24=-24$ ✓

2) $3|x-15|+9=8$

$-9 \quad -9$

$\frac{3|x-15|}{3}=\frac{-1}{3}$

$|x-15|=-\frac{1}{3}$

$\emptyset$

3) $6x-22=|x+4|-8$

$+8 \quad +8$

$6x-14=|x+4|$

$6x-14=x+4$ $\quad$ $-6x+14=x+4$

$-x+14 \quad -x+14$ $\quad$ $+6x-4 \quad +6x-4$

$\frac{5x}{5}=\frac{18}{5}$ $\quad$ $\frac{10}{7}=\frac{7x}{7}$

$x=\frac{18}{5}$ $\quad$ ~~$x=\frac{10}{7}$~~

Check:

$6\left(\frac{18}{5}\right)-22=\left|\frac{18}{5}+4\right|-8$

$-\frac{2}{5}=-\frac{2}{5}$ ✓

$6\left(\frac{10}{7}\right)-22=\left|\frac{10}{7}+4\right|-8$

$-\frac{94}{7} \neq -\frac{18}{7}$ X

4) The equation $|3x-2|-1=17$ has 2 unique solutions that can be found using which two equations?

A. $3x-3=17$ and $-3x-3=-18$

B. $3x-2=16$ and $3x-2=18$

C. $3x-2=16$ and $-(3x-2)=16$

D. $3x-2=18$ and $-(3x-2)=18$

E. $3x-2=18$ and $-(3x-2)=16$

$|3x-2|-1=17$

$+1 \quad +1$

$|3x-2|=18$

$3x-2=18$

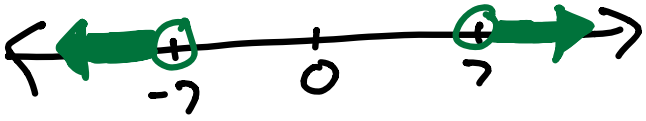
$-1(3x-2)=-18 \cdot -1$

$-(3x-2)=18$

Solve the following inequalities:

1) $|x| > 7$

$x > 7 \quad x < -7$



$x < -7$ or $x > 7$

$(-\infty, -7) \cup (7, \infty)$

2) $|x - 6| \leq 7$

$x - 6 \leq 7 \quad x - 6 \geq -7$

$+6 \quad +6 \quad +6 \quad +6$

$x \leq 13 \quad x \geq -1$

$-1 \leq x \leq 13$

$[-1, 13]$

What makes the process of solving an absolute value *inequality* different from solving an absolute value *equation*?

When we account for the negative value inside of the absolute value, we must flip the sign.
Our answer is an interval of numbers instead of one specific number.

Solve the following inequalities:

1) $|x-6|+3>7$

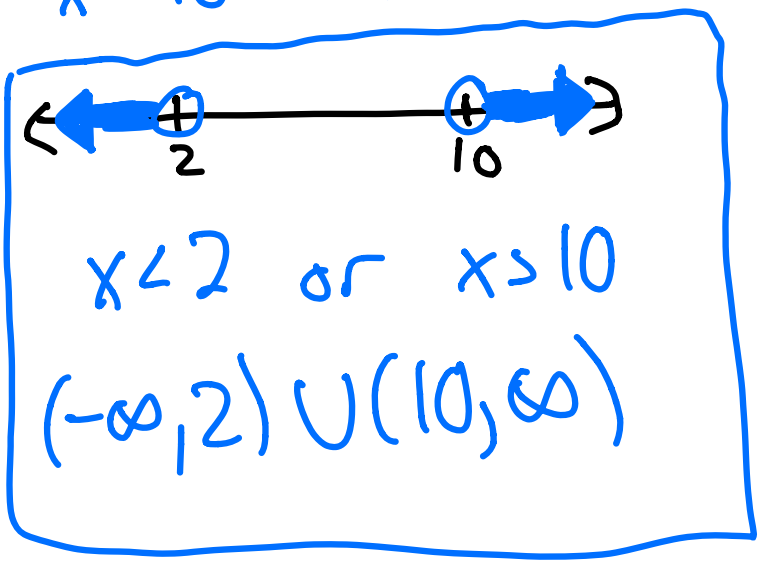
$-3\ -3$

$|x-6|>4$

$x-6>4$ $\quad$ $x-6<-4$

$+6\ +6$ $\quad$ $+6\ +6$

$x>10$ $\quad$ $x<2$



$x<2$ or $x>10$

$(-\infty,2)\cup(10,\infty)$

2) $3|2x-5|-6\leq 15$

$+6\ +6$

$\frac{3|2x-5|}{3}\leq\frac{21}{3}$

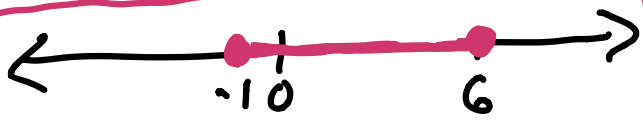
$2x-5\leq 7$

$2x-5\leq 7$ $\quad$ $2x-5\geq -7$

$+5\ +5$ $\quad$ $+5\ +5$

$\frac{2x}{2}\leq\frac{12}{2}$ $\quad$ $\frac{2x}{2}\geq\frac{-2}{2}$

$x\leq 6$ $\quad$ $x\geq -1$



$-1\leq x\leq 6$

$[-1,6]$

3) $|x-1|>-6$

An absolute value will always be greater than a negative number ∴

$(-\infty,\infty)$

All real numbers

4) $|x-1|<-6$

An absolute value will never be less than a negative number ∴

$\varnothing$

Success Criteria

- I can solve absolute value equations with variables on one side or both sides of the equation.

Solve each of the following equations.

1) $|-2x+8|-7=23$

$+7 \quad +7$

$|-2x+8|=30$

$-2x+8=30$ $\quad -2x+8=-30$

$-8 \quad -8$ $\quad -8 \quad -8$

$-2x=22$ $\quad -2x=-38$

$x=-11$ $\quad x=19$

Check:

$|-2(-11)+8|-7=23$

$23=23$ ✓

$|-2(19)+18|-7=23$

$23=23$ ✓

2) $|x-3|+9=3x-10$

$-9 \quad -9$

$|x-3|=3x-19$

$x-3=3x-19$ $\quad -x+3=3x-19$

$-x+19 \quad -x+19$ $\quad +x+19 \quad +x+19$

$16=2x$ $\quad 22=4x$

$x=8$ $\quad x=5.5$

Check:

$|8-3|+9=3(8)-10$

$14=14$ ✓

$|5.5-3|+9=3(5.5)-10$

$14.5 \neq 6.5$ ✗

- I can explain the rationale behind the process of solving an absolute value equation.

Because absolute value makes the value inside positive, we must account for the value inside the absolute value to be positive of negative.

- I can assess whether an absolute value equation has no solutions.

Explain when an absolute value equation has no solutions. Then, give an example of an absolute value equation that has no solutions.

When the absolute value is equal to a negative number because an absolute value can't be negative.

$|x-5|=-100$

- I can solve absolute value inequalities.

Solve each of the following inequalities.

1) $|5x + 27| - 12 \leq 10$

$+12 \quad +12$

$|5x+27| \leq 22$

$5x+27 \leq 22 \qquad 5x+27 \geq -22$

$-27 \quad -27 \qquad -27 \quad -27$

$\frac{5x}{5} \leq \frac{-5}{5} \qquad \frac{5x}{5} \geq \frac{-49}{5}$

$x \leq -1 \qquad x \geq -9.8$

$-9.8 \leq x \leq -1$

$[-9.8, -1]$

2) $-2|x - 12| - 6 < 8$

$+6 \quad +6$

$\frac{-2|x-12|}{-2} < \frac{14}{-2}$

$|x-12| > -7$

All real numbers

- I can explain the rationale behind the process of solving an absolute value inequality.

When we account for the negative value inside of the absolute value, we must flip the sign when we write the second inequality.

- I can assess when an absolute value inequality has no solutions or has a solution set of all real numbers.

Explain when an absolute value inequality has no solutions. Then, give an example of an absolute value inequality that has no solutions.

When the absolute value is less than zero or a negative number because an absolute value is always positive and can't be negative. $|x-5| < -12$

Explain when an absolute value inequality has a solution set of all real numbers. Then, give an example of an absolute value inequality that has a solution set of all real numbers.

When an absolute value is greater than or equal to zero or greater than a negative number because absolute value is always positive. $|x-3| > -12$

Name: _____ Date: _____

Classwork: Solving Absolute Value Equations and Inequalities

Solve each of the following equations and inequalities.

1) $\left|\frac{3x-9}{7}\right| = 10$

$\frac{3x-9}{7} = 10$ $\quad$ $\frac{3x-9}{7} = -10$

$3x-9=70$ $\quad$ $3x-9=-70$

$3x=79$ $\quad$ $3x=-61$

$x=\frac{79}{3}$ $\quad$ $x=\frac{-61}{3}$

Check

$\left|\frac{3(\frac{79}{3})-9}{7}\right|=10$ $\quad$ $\left|\frac{3(\frac{-61}{3})-9}{7}\right|=10$

$10=10$ ✓ $\quad$ $10=10$ ✓

2) $4|5x| - 8 \leq 9$

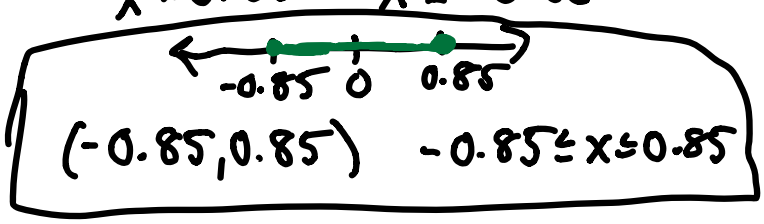
$+8 \quad +8$

$4|5x| \leq 17$

$|5x| \leq \frac{17}{4}$

$5x \leq 4.25$ $\quad$ $5x \geq -4.25$

$x \leq 0.85$ $\quad$ $x \geq -0.85$



$(-0.85, 0.85)$ $\quad$ $-0.85 \leq x \leq 0.85$

3) $\left|\frac{7}{13}x - 4\right| + 20 \geq 15$

$-20 \quad -20$

$\left|\frac{7}{13}x-4\right| \geq -5$

Infinite solutions

4) $\left|\frac{4}{5}x - 18\right| + 12 = 18$

$-12 \quad -12$

$\left|\frac{4}{5}x-18\right|=6$

$\frac{4}{5}x-18=6$ $\quad$ $\frac{4}{5}x-18=-6$

$\frac{4}{5}x=24$ $\quad$ $\frac{4}{5}x=12$

$x=30$ $\quad$ $x=15$

Check:

$\left|\frac{4}{5}(30)-18\right|+12=18$ $\quad$ $\left|\frac{4}{5}(15)-18\right|+12=18$

$18=18$ ✓ $\quad$ $18=18$ ✓

5) $-2|x+1| + 9 = 4x - 7$

$-9 \quad -9$

$\frac{-2|x+1|}{-2} = \frac{4x-16}{-2}$

$|x+1| = -2x+8$

$x+1=-2x+8$ $\quad$ $x+1=2x-8$

$3x=7$ $\quad$ $9=x$

$x=7/3$

$-2|9+1|+9=4(9)-7$

$-11=29$ X

Check:

$-2|\frac{7}{3}+1|+9=4(\frac{7}{3})-7$ ✓ $7/3=7/3$

6) $-2|6x - 16| + 54 > -18$

$-54 \quad -54$

$-2|6x-16| > -72$

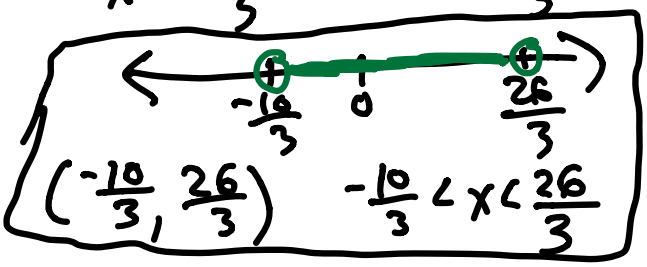
$|6x-16| < 36$

$6x-16<36$ $\quad$ $6x-16>-36$

$+16 \quad +16$ $\quad$ $+16 \quad +16$

$6x<52$ $\quad$ $6x>-20$

$x<\frac{26}{3}$ $\quad$ $x>\frac{-10}{3}$



$\left(\frac{-10}{3}, \frac{26}{3}\right)$ $\quad$ $\frac{-10}{3} < x < \frac{26}{3}$

7) The inequality $|3.75F - 24| \leq 70$ represents the range of monthly average temperatures, F, in degrees Fahrenheit, for Fairbanks, Alaska. Solve for F.

$|3.75F - 24| \leq 70$

$3.75F - 24 \leq 70$
$+24 \quad +24$

$\frac{3.75F}{3.75} \leq \frac{94}{3.75}$

$F \leq 25.0\overline{6}$

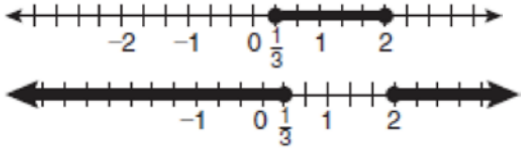
$3.75F - 24 \geq -70$
$+24 \quad +24$

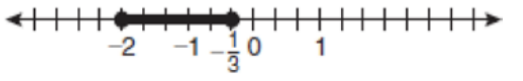
$\frac{3.75F}{3.75} \geq \frac{-46}{3.75}$

$F \geq -12.2\overline{6}$

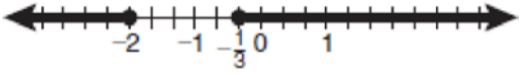
$[-12.2\overline{6}, 25.0\overline{6}]$

8)
Which graph represents the solution set of $|6x - 7| \leq 5$?

1)


2)


3)

4)


$6x - 7 \leq 5$
$+7 \quad +7$

$\frac{6x}{6} \leq \frac{12}{6}$

$x \leq 2$

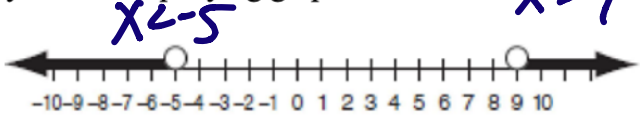
$6x - 7 \geq -5$
$+7 \quad +7$

$\frac{6x}{6} \geq \frac{2}{6}$

$x \geq \frac{1}{3}$

9)
The solution set of which inequality is represented by the accompanying graph?

$x < -5$ $\quad$ $x > 9$



1) $|x - 2| > 7$
2) $|x - 2| < 7$
3) $|2 - x| > -7$
4) $|2 - x| < -7$

$x - 2 > 7$ $\quad$ $x - 2 < -7$
$+2 \quad +2$ $\quad$ $+2 \quad +2$

$x > 9$ $\quad$ $x < -5$