Name: negative exponent ∴ flip the base Date:__________

Notes: Introduction to Logarithms

Do Now: ***Completely simplify the following expressions.***

1) $\left(\frac{4}{5}\right)^{-2}$

$\left(\frac{5}{4}\right)^2 = \boxed{\frac{25}{16}}$

2) 2^{-4}

$\left(\frac{2}{1}\right)^{-4} = \left(\frac{1}{2}\right)^4 = \boxed{\frac{1}{16}}$

3) $4^{1/2} = \sqrt{4} = \boxed{2}$

Algebraically solve the following.

4) Find the *x* value for the equation $64 = (4)^x$.

$4^3 = 64$ $\boxed{x=3}$

5) Find the *x* value for the equation $2{,}560 = 5(8)^x$.

$\frac{2{,}560}{5} = \frac{5(8)^x}{5}$

$512 = 8^x$

$8(8)(8) = 512$

$8^3 = 512$

$\boxed{x=3}$

6) Find or estimate the *x* value for the equation $80 = 2(3)^x$.

$\frac{80}{2} = \frac{2(3)^x}{2}$

$40 = 3^x$

$3^3 = 27 \ldots 3^4 = 81 \ldots ?$

x must be a number in between 3 and 4, but how do we figure out the exact number?

What Should I Be Able to Do?

- I can explain how to perform logarithmic expression.

- I can to calculate or estimate logarithms without using a calculator.

- I can convert equations between logarithmic and exponential form.

- I can find the inverse of an exponential equation

- I can graph a logarithmic equation.

- I can show and explain how a logarithmic function is the inverse of an exponential function.

"How many of one number do we multiply to get another number?"

How many times do I multiply 8 to get 64?

$\underbrace{8 \cdot 8}_{\text{2 times}} = 64$

$8^2 = 64$ (8: base, 2: exponent, 64: argument)

$\log_8 64 = 2$

How many times do I multiply 3 to get 81?

$3 \cdot 3 \cdot 3 \cdot 3 = 81$ $\boxed{\text{4 times}}$

$3^4 = 81$ (3: base, 4: exponent, 81: argument)

$\log_3 81 = 4$

How many times do I multiply 2 to get 256?

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 256$

$2^8 = 256$

$\boxed{8}$ $\log_2 256 = 8$

How many times do I multiply 4 to get 1048576?

$4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1048576$

$4^{10} = 1048576$

$\boxed{10}$ $\log_4 1048576 = 10$

How many times do I multiple $\frac{1}{3}$ to get $\frac{1}{27}$?

$\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{27}$

$\left(\frac{1}{3}\right)^3 = \frac{1}{27}$

$\boxed{3}$ $\log_{\frac{1}{3}} \frac{1}{27} = 3$

How many times do I multiply 7 to get 45?

$7^1 = 7$ $7^2 = 49$

In between 1 and 2.

Estimate: $\boxed{1.85}$

$7^x = 45$

$\log_7 45 = x$

$\boxed{x \approx 1.96}$

Logarithm with Base b

Suppose $b > 0$ and $b \neq 1$. For $x > 0$, there is a number y such that

$$\log_b x = y \text{ if and only if } b^y = x$$

(b: base, y: exponent, x: argument)

Logarithms are the inverse property of an exponential. Therefore logarithms get the exponent alone on one side of the equation!

How do I say $\log_b x = y$? "Log base b of x equals y."

Convert each of the six questions above into an exponential equation and a logarithmic equation.

Use your calculator to evaluate the logarithms to check your solutions.

TI-nspire: $\boxed{\text{ctrl}} \rightarrow \boxed{10^x}$

TI-84: $\boxed{\text{MATH}} \rightarrow$ Scroll down to A:logBASE(

$\log_b x = y$ $\quad$ $b^y = x$

Checkpoint:

Convert each exponential equation into its equivalent logarithmic equation.

1) $3^4 = x$ $\quad$ $\log_3 x = 4$

2) $a^7 = 539$ $\quad$ $\log_a 539 = 7$

3) $e^y = 12$ $\quad$ $\log_e 12 = y$

Convert each logarithmic equation into its equivalent exponential equation.

4) $\log_2 8 = y$ $\quad$ $2^y = 8$

5) $\log_e 28 = x$ $\quad$ $e^x = 28$

6) $\log_{\frac{1}{4}} \frac{1}{16} = a$ $\quad$ $\left(\frac{1}{4}\right)^a = \frac{1}{16}$

7) $\log_b 36 = y$ $\quad$ $b^y = 36$

Evaluate or estimate each logarithm without a calculator.

8) $\log_7 49 = x$

$7^x = 49$

$\boxed{x = 2}$

9) $\log_{\frac{1}{5}} \frac{1}{125} = x$

$\left(\frac{1}{5}\right)^x = \frac{1}{125}$

$5 \cdot 5 \cdot 5 = 125$

$\left(\frac{1}{5}\right)^3 = \frac{1}{125}$ $\quad$ $\boxed{x = 3}$

10) $\log_2 64 = x$

$2^x = 64$

$\boxed{x = 6}$

11) $\log_6 \frac{1}{36} = x$

$6^x = \frac{1}{36}$

must be negative because base is flipped

$6^{-2} = \frac{1}{36}$ $\quad$ $\boxed{x = -2}$

12) $\log_{49} 7 = x$

$49^x = 7$

$x < 1$ because 49 becomes smaller

$\sqrt{49} = 7$

$49^{1/2} = 7$

$\boxed{x = \frac{1}{2}}$

13) $\log_5 60 = x$

$5^x = 60$

$5^2 = 25$ $\quad$ $\boxed{x \approx 2.5}$

$5^3 = 125$

Calculator Check:

$\log_5 60 = 2.5439593 1063$

14) $\log_{64} 4 = x$

$64^x = 4$

$4^3 = 64 \therefore \sqrt[3]{64} = 4$

$64^{1/3} = 4$

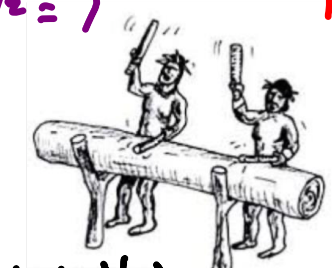
$\boxed{x = 1/3}$

15) $\log_{11} \sqrt[6]{11} = x$

$11^x = \sqrt[6]{11}$

$11^x = 11^{1/6}$

$\boxed{x = \frac{1}{6}}$



Hey look it is my favorite band, the Logarhythms!

Why do we use logarithms?

- inverse operation of exponential
- exponential growth/decay
- interest (banking)
- science

Do Now: Find the inverse of the following function.

*When finding the inverse of a function, swap the x and y, then solve for y.

1) $y = 7^x$

$x = 7^y$

$\boxed{\log_7 x = y}$

What is the inverse of $y = 2^x$?

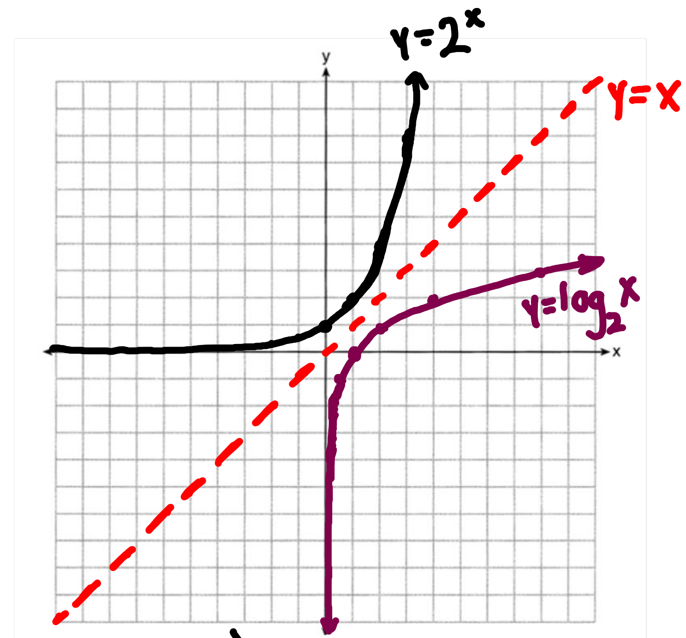
$x = 2^y$

$\boxed{\log_2 x = y}$

On the set of axes, graph $y = 2^x$ and its inverse.

x	y
0	1
1	2
2	4
3	8
4	16

x	y
1	0
2	1
4	2
8	3
16	4



For $y = 2^x$, domain: $(-\infty, \infty)$ range: $(0, \infty)$ asymptote: $y = 0$

For $y = \log_2 x$, domain: $(0, \infty)$ range: $(-\infty, \infty)$ asymptote: $x = 0$

Explain how the x-y tables prove that the functions are inverses.

For each corresponding point, the x and y are flipped.

Explain how the graph of each function prove that the functions are inverses.

The curve of $y = \log_2 x$ is the reflection of $y = 2^x$ over the line $y = x$.

Success Criteria

- I can explain how to simplify a logarithmic expression.

For $\log_a b$ you want to figure out how many times to multiply a to get b. For example, $\log_5 25$ is equal to 2 because $5 \cdot 5$ or $5^2 = 25$. You can also think of it (logarithm) as finding the exponent of the base in order to obtain the arguement $\rightarrow 5^x = 25 \Rightarrow x = 2$.

- I can to calculate or estimate logarithms without using a calculator.

1) $\log_{81} 9 = x$

$81^x = 9$ $\boxed{x = \frac{1}{2}}$

2) $\log_2 16 = x$

$2^x = 16$ $\boxed{x = 4}$

3) $\log_{24} 1 = x$

$24^x = 1$ $\boxed{x = 0}$

4) $\log_4 90 = x$

$4^x = 90$ $\boxed{x \approx 3.2}$

$4^3 = 64$

$4^4 = 256$

- I can convert equations between logarithmic and exponential form.

1) $\log_b 81 = 2.5$

$b^{2.5} = 81$

2) $\log_{10} 1000 = y$

$10^y = 1000$

3) $y = e^9$

$\log_e y = 9$

4) $256 = 2^x$

$\log_2 256 = x$

- I can find the inverse of an exponential equation

1) $y = 4^x$

$x = 4^y$

$\log_4 x = y$

2) $y = \frac{1}{2}^x$

$x = \left(\frac{1}{2}\right)^y$

$\log_{1/2} x = y$

- I can graph a logarithmic equation.

$3^y = x \leftarrow$ $y = \log_3 x$

x	y
1/9	-2
1/3	-1
1	0
3	1
9	2

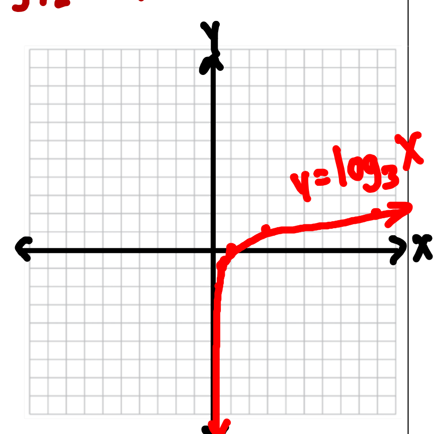
You could jump right into graphing $y = \log_3 x$ or graph it's inverse first.

OR $x = \log_3 y \rightarrow 3^x = y$

x	y
0	1
1	3
2	9
3	27

$\therefore$ $y = \log_3 x \rightarrow$

x	y
1	0
3	1
9	2
27	3



- I can show and explain how a logarithmic function is the inverse of an exponential function.

SHOW: $y = 4^x \xrightarrow{\text{inverse}} x = 4^y \rightarrow \log_4 x = y$

EXPLAIN: When you perform a logarithmic operation, the exponent gets isolated in the equation therefore making it the inverse operation. (Can also mention reflection over $y = x$)

Name:____________________ Date:__________

Classwork: Introduction to Logarithms

Write each equation in logarithmic form.

1) $3^4 = 81$

$\log_3 81 = 4$

2) $4^2 = 16$

$\log_4 16 = 2$

3) $b^3 = 729$

$\log_b 729 = 3$

4) $64^{\frac{1}{3}} = 4$

$\log_{64} 4 = \frac{1}{3}$

5) $12^{-1} = \frac{1}{12}$

$\log_{12} \frac{1}{12} = -1$

6) $e^{2/3} = y$

$\log_e y = \frac{2}{3}$

Write each equation in exponential form.

7) $\log_9 81 = 2$

$9^2 = 81$

8) $\log_e 2 = y$

$e^y = 2$

9) $\log_5 1 = 0$

$5^0 = 1$

10) $\log_{25} 125 = \frac{3}{2}$

$(25)^{3/2} = 125$

Evaluate or estimate each expression.

11) $\log_3 27 = x$

$3^x = 27$

$3^3 = 27$

$\boxed{3}$

12) $\log_6 \frac{1}{216} = x$

$6^x = \frac{1}{216}$

$6^3 = 216$

$6^{-3} = \frac{1}{216}$

$\boxed{-3}$

13) $\log_4 1{,}024 = x$

$4^x = 1024$

$4^5 = 1024$

$\boxed{5}$

14) $\log_{\frac{1}{3}} \frac{1}{9} = x$

$\left(\frac{1}{3}\right)^x = \frac{1}{9}$

$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$\boxed{2}$

15) $\log_5 130 = x$

$5^x = 130$

$5^3 = 125$

$5^4 = 625$

$\boxed{x \approx 3.02}$

16) $\log_{100} 10 = x$

$00^x = 10$

$\sqrt{100} = 10$

$100^{1/2} = 10$

$\boxed{\frac{1}{2}}$

17) $\log_{64} 2 = x$

$64^x = 2$

$2^6 = 64 \therefore \sqrt[6]{64} = 2$

$\sqrt[6]{64} = 64^{1/6}$

$\boxed{1/6}$

18) $\log_9 \sqrt[4]{9} = x$

$9^x = \sqrt[4]{9}$

$9^x = 9^{1/4}$

$\boxed{1/4}$

19) Between which to consecutive integers must $\log_3 50$ lie?

(1) 1 and 2
(2) 2 and 3
(3) 3 and 4
(4) 4 and 5

$3^0 = 1$ $3^1 = 3$ $3^2 = 9$ $\boxed{3^3 = 27 \quad 3^4 = 81}$

20) Which of the following is equivalent to $y = \log_8 x$?

$8^y = x$

(1) $y = x^8$
(2) $x = y^8$
(3) $x = 8^y$
(4) $y = x^{1/8}$

21) A local pizza parlor is trying to spend money on advertising in order to increase their revenue. The revenue of the pizza parlor, in thousands of dollars, can be modeled by the equation $R(m) = 5 + 8\log_5(m + 2)$, where m is the amount of money spent on advertising in thousands, when $m \geq 0$.

a) Find the value of:

i) $R(0)$ $R(0) = 5 + 8\log_5(0+2)$

$= 5 + 8\log_5(2)$ → plug into calculator

$\boxed{R(0) = 8.445412464 59 \text{ thousands of dollars}}$

ii) $R(6)$ $R(6) = 5 + 8\log_5(6+2)$

$= 5 + 8\log_5(8)$

$\boxed{R(6) = 15.3362373938 \text{ thousands of dollars}}$

22) Elisa and Matthew are evaluating $\log_2 \frac{1}{32}$. Is either of them correct? Explain your reasoning WITHOUT USING A CALCULATOR.

Elisa	Me	Matthew
$\log_2 \frac{1}{32} = y$	$\log_2 \frac{1}{32} = y$	$\log_2 \frac{1}{32} = y$
$(2)^y = \frac{1}{32}$	$2^y = \frac{1}{32}$	$\frac{1}{32}^y = 2$
$(2)^y = 32^{-1}$	$2^5 = 32$	$(32^{-1})^y = 2$
$2^y = (2^5)^{-1}$		$32^{-y} = 2$
$2^y = 2^{-5}$	$2^{-5} = \frac{1}{32}$	$-y = 2$
$y = -5$	$\boxed{y = -5}$	$y = -2$

C'mon Matt! 2 is the base not $\frac{1}{32}$. Elisa converted into exponential form correctly and accurately solved for y.

23)

a) What is the inverse of $y = 4^x$?

$x = 4^y$

$\boxed{\log_4 x = y}$

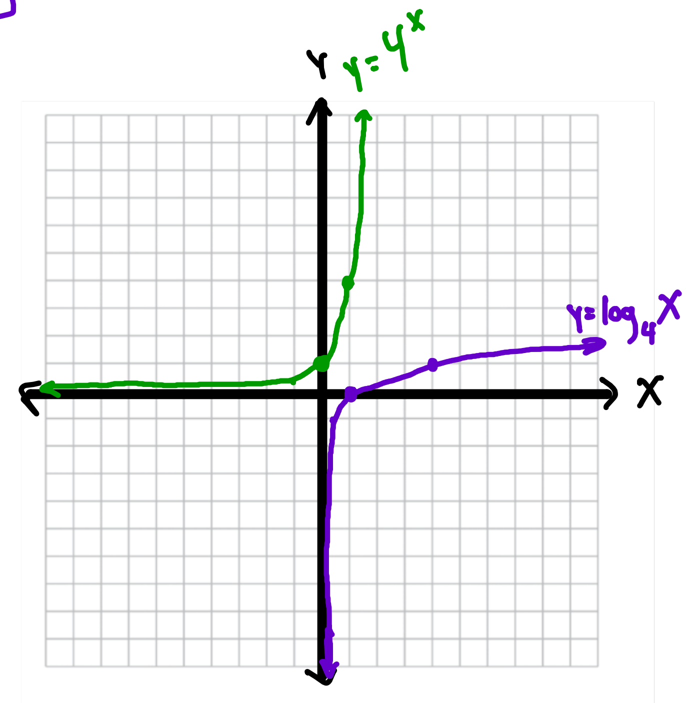
b) On the set of axes, graph $y = 4^x$ and its inverse.

$y = 4^x$

x	y
-2	1/16
-1	1/4
0	1
1	4
2	16
3	64
4	256

$\log_4 x = y$

x	y
1/16	-2
1/4	-1
1	0
4	1
16	2
64	3
256	4



c) For $y = 4^x$, find each of the following:

domain: $(-\infty, \infty)$ range: $(0, \infty)$ asymptote: $y = 0$ y-intercept: $(0, 1)$

d) For the inverse of $y = 4^x$, find each of the following:

domain: $(0, \infty)$ range: $(-\infty, \infty)$ asymptote: $x = 0$ x-intercept: $(1, 0)$