Name: ______________________ Date: ________

Notes: The Complex Plane

Do Now: 1) Simplify the following expression in $a + bi$ form.

$$\frac{1-i}{2+i} + \frac{1+i}{1-2i}$$

$$\frac{(1-i)(2-i)}{(2+i)(2-i)}$$

$$\frac{2-i-2i+i^2}{4-2i+2i-i^2}$$

$$\frac{1-3i}{5}$$

$$\frac{(1+i)(1+2i)}{(1-2i)(1+2i)}$$

$$\frac{1+2i+i+2i^2}{1+2i-2i-4i^2}$$

$$\frac{-1+3i}{5}$$

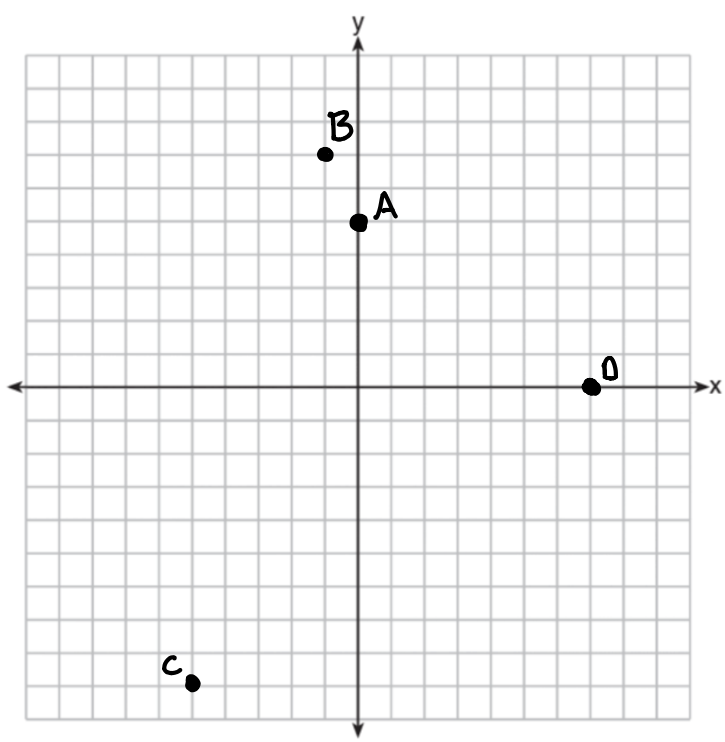
$$\frac{1-3i}{5} + \frac{-1+3i}{5}$$

$$\frac{1-3i+(-1)+3i}{5}$$

$$\frac{0}{5} = \boxed{0}$$

2) Graph and label each of the following points.

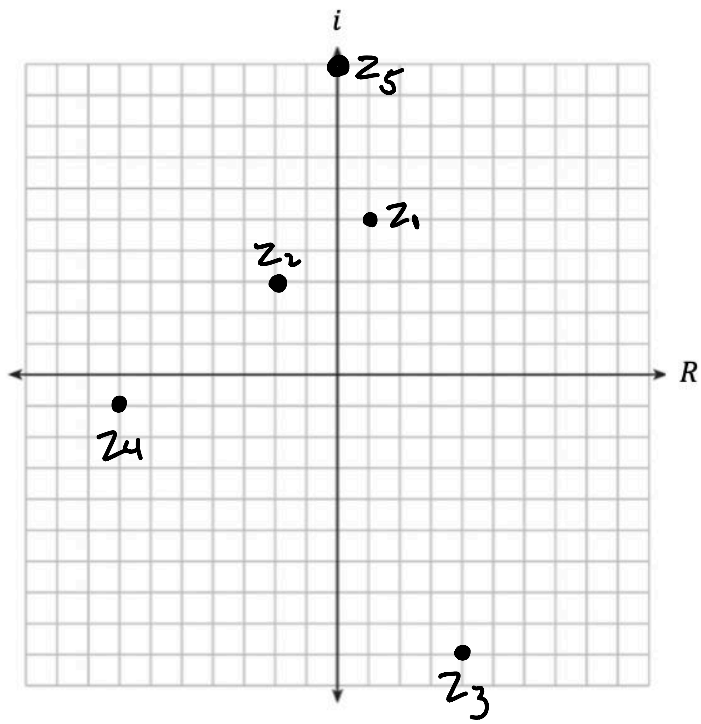
A: $(0,5)$ B: $(-1,7)$ C: $(-5,-9)$ D: $(7,0)$



What Should I Be Able to Do?

- I can graph complex numbers on a complex plane.

The Complex Plane:



What do you notice is different about the complex plane than a coordinate plane?

The x-axis is now the R-axis for real.
The y-axis is now the i-axis for imaginary.

Graph each complex number on the complex plane.

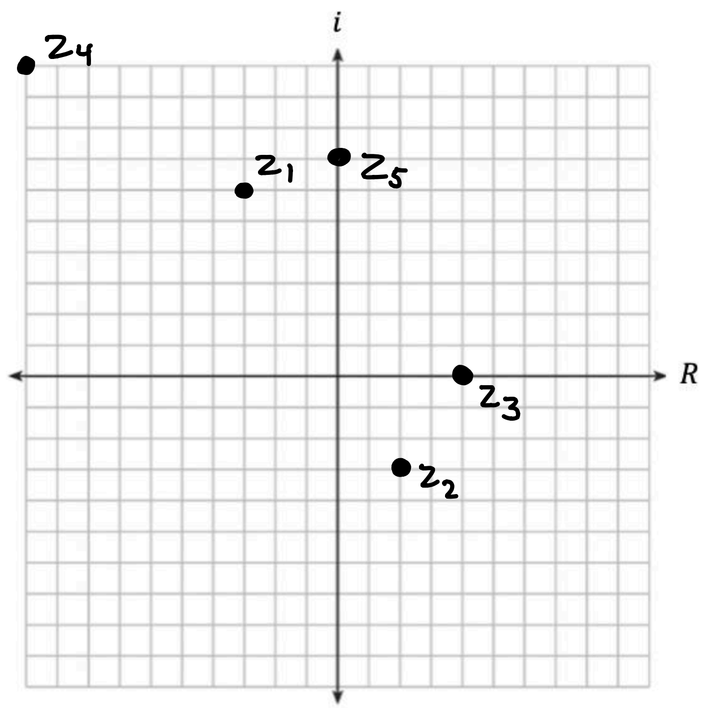
$z_1 = 1 + 5i$ $\qquad z_2 = -2 + 3i$ $\qquad z_3 = 4 - 9i$ $\qquad z_4 = -7 - i$ $\qquad z_5 = 10i$

Success Criteria

- I can graph complex numbers on a complex plane.

1) Graph each complex number on the complex plane below.

$z_1 = -3 + 6i$ $z_2 = 2 - 3i$ $z_3 = 4$ $z_4 = -10 + 10i$ $z_5 = 7i$



2) Given a and b are positive real numbers, describe how you would plot $a - bi$ on a complex plane.

I would start at 0+0i and plot a point that is a units to the right and b units down.

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Classwork: The Complex Plane

1) Write $-\frac{1}{2}i^3(\sqrt{-9}-4)-3i^2$ in simplest $a+bi$.

$-\frac{1}{2}(-i)(3i-4)-3(-1)$

$\frac{1}{2}i(3i-4)+3$

$1.5i-2+3$

$\boxed{1+1.5i}$

2)

The expression $6-(3x-2i)^2$ is equivalent to

(1) $-9x^2+12xi+10$ (3) $-9x^2+10$

~~(2)~~ $9x^2-12xi+2$ ~~(4)~~ $-9x^2+12xi-4i+6$

$6-(3x-2i)(3x-2i)$

$6-(9x^2-6xi-6xi+4i^2)$

$6-(9x^2-12xi-4)$

$6-9x^2+12xi+4$

$-9x^2+12xi+10$

3) What is the product of the complex numbers $(2-6i^3)$ and $(-9+2i^{81})$?

$(2-6(-i))(-9+2i)$

$(2+6i)(-9+2i)$

$-18+4i-54i+12i^2$

$-18-50i+(12)(-1)$

$\boxed{-30-50i}$

4) What is the sum of the complex numbers $(12-3i^{49})$, $(-1-i^{1,028,253})$, and $(-11+i^4)$?

$(12-3i)$, $(-1-i)$, and (-11)

$12-3i$

$-1-i$

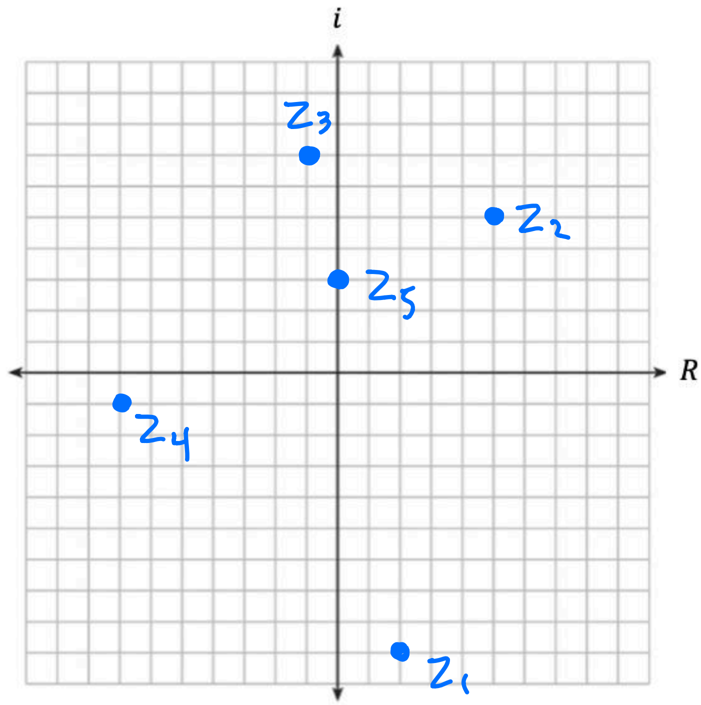
$+\ -11$

$-4i$

$\boxed{-4i}$

5) Graph each complex number on the complex plane below.

$z_1 = 2 - 9i$ $\quad$ $z_2 = 5 + 5i$ $\quad$ $z_3 = -1 + 7i$ $\quad$ $z_4 = -7 - i$ $\quad$ $z_5 = 3i$



6) Given n is a positive integer and i is the imaginary unit, $i^2 = -1$, such that $i^n = i$, which of the following statements about n must be true?

A. When n is divided by 4, the remainder is 0.
B. When n is divided by 4, the remainder is 1.
C. When n is divided by 4, the remainder is 2.
D. When n is divided by 4, the remainder is 3.
E. Cannot be determined from the given information.

7) What is the multiplicative inverse of $-5i$?

$$\frac{-5i(x)}{-5i} = \frac{1}{-5i}$$

$$x = \frac{-1}{5i}$$

$$\frac{-1}{5i} \cdot \frac{i}{i}$$

$$\frac{-i}{5i^2} = \frac{-i}{-5} = \boxed{\frac{i}{5}}$$

Simplify each of the following expressions in $a + bi$ form.

8) $\left(12 - \sqrt{-112}\right) - (45 + \sqrt{-28})$

$(12 - i\sqrt{16}\sqrt{7}) - (45 + i\sqrt{4}\sqrt{7})$

$(12 - 4i\sqrt{7}) - (45 + 2i\sqrt{7})$

$12 - 4i\sqrt{7} - 45 - 2i\sqrt{7}$

$\boxed{-33 - 6i\sqrt{7}}$

9) $(x + 3i)^2 - (2x - 3i)^2$

$(x+3i)(x+3i) - (2x-3i)(2x-3i)$

$(x^2 + 3xi + 3xi + 9i^2) - (4x^2 - 6xi - 6xi + 9i^2)$

$(x^2 + 6xi - 9) - (4x^2 - 12xi - 9)$

$x^2 + 6xi - 9 - 4x^2 + 12xi + 9$

$\boxed{-3x^2 + 18xi}$

10) $\frac{x+yi}{x-yi} \cdot \frac{(x+yi)}{(x+yi)}$

$\frac{x^2 + xyi + xyi + y^2i^2}{x^2 + xyi - xyi - y^2i^2}$

$\frac{x^2 + 2xyi + y^2(-1)}{x^2 - y^2(-1)}$

$\boxed{\frac{x^2 + 2xyi - y^2}{x^2 + y^2}}$

not a+bi but ok

$\boxed{\frac{x^2 - y^2}{x^2 + y^2} + \frac{2xy}{x^2+y^2}i}$

11) $(2 - i)^{-3}$

$\left(\frac{1}{2-i}\right)\left(\frac{1}{2-i}\right)\left(\frac{1}{2-i}\right)$

$\left(\frac{1}{4 - 2i - 2i + i^2}\right)\left(\frac{1}{2-i}\right)$

$\left(\frac{1}{3-4i}\right)\left(\frac{1}{2-i}\right)$

$\frac{1}{6 - 3i - 8i + 4i^2}$

$\frac{1}{2 - 11i}$

$\frac{1}{2-11i} \cdot \frac{(2+11i)}{(2+11i)}$

$\frac{2+11i}{4 + 22i - 22i - 121i^2}$

$\frac{2+11i}{4+121}$

$\frac{2+11i}{125}$

$\boxed{\frac{2}{125} + \frac{11}{125}i}$

12) Completely simplify the expression $(x - 4i)^3$.

$(x-4i)(x-4i)(x-4i)$

$(x^2 - 4xi - 4xi + 16i^2)(x - 4i)$

$(x^2 - 8xi - 16)(x - 4i)$

$x^3 - 4x^2i - 8x^2i + 32xi^2 - 16x + 64i$

$x^3 - 4x^2i - 8x^2i - 32x - 16x + 64i$

not a+bi form but ok →

$\boxed{x^3 - 12x^2i - 48x + 64i}$

$\boxed{x^3 - 48x - (12x^2 - 64)i}$

13) What is the multiplicative inverse of $i - 2$?

$\frac{1}{i-2} \cdot \frac{(i+2)}{(i+2)} = \frac{i+2}{i^2 + 2i - 2i - 4} = \frac{i+2}{-1-4} = \frac{i+2}{-5} = \boxed{\frac{-i-2}{5}}$

14)

Expressed in simplest $a + bi$ form, $(7 - 3i) + (x - 2i)^2 - (4i + 2x^2)$ is

(1) $(3 - x^2) - (4x + 7)i$ (3) $(3 - x^2) - 7i$

(2) $(3 + 3x^2) - (4x + 7)i$ (4) $(3 + 3x^2) - 7i$

$(x-2i)(x-2i)$

$x^2-2xi-2xi+4i^2$

$x^2-4xi-4$

$(7-3i)+(x^2-4xi-4)-(4i+2x^2)$

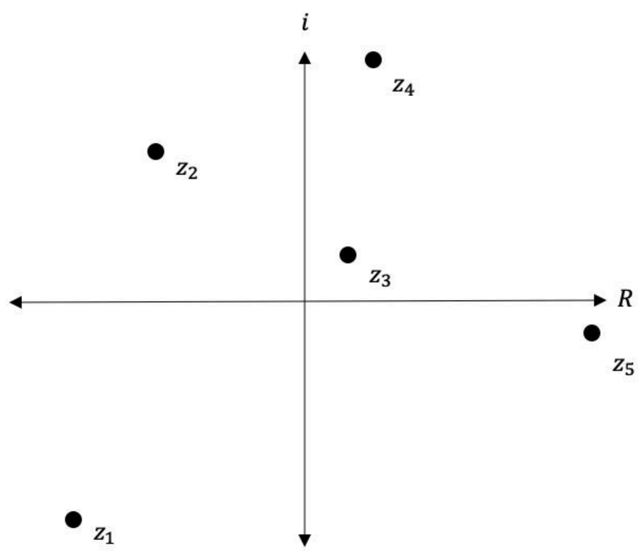
$7-3i\ +x^2-4xi-4-4i-2x^2$

$-x^2+3-4xi-7i$

$-x^2+3-(4x+7)i$

15) The modulus of the complex number $a + bi$ is given by $\sqrt{a^2 + b^2}$. Which of the complex numbers z_1, z_2, z_3, z_4, and z_5 below has the greatest modulus?

A. z_1
B. z_2
C. z_3
D. z_4
E. z_5



16) Given i is the imaginary unit, find $(2 - yi)^2$ in simplest form.

$(2-yi)(2-yi)$

$4-2yi-2yi+y^2i^2$

$4-4yi-y^2$

$\boxed{-y^2+4-4yi}$

17) If $A = -2 + 7i, B = -3 - 5i$ and $C = 1 + 8i$ where i is the imaginary unit, find $A - BC$ in $a + bi$ form.

$(-2+7i)-(-3-5i)(1+8i)$

$(-2+7i)-(-3-24i-5i-40i^2)$

$(-2+7i)-(-3-29i+40)$

$(-2+7i)+(+37+29i)$

$\boxed{35+36i}$

18) If $x = 4i, y = 2i,$ and $z = m + i$, find the expression x^3y^3z in $a + bi$ form.

$(4i)^3(2i)^3(m+i)$

$64i^3(8i^3)(m+i)$

$512i^6(m+i)$

$512(-1)(m+i)$

$-512(m+i)$

$\boxed{-512m-512i}$

19) Write two complex numbers with a product 18.

$(6i)(-3i)$

$-18i^2$

18

Solve each of the following equations.

20) $\sqrt{-4x + 16} - \sqrt{3x - 12} = 0$

$+\sqrt{3x-12} \quad +\sqrt{3x-12}$

$\sqrt{-4x+16}^2 = \sqrt{3x-12}^2$

$-4x+16 = 3x-12$

$+4x \; +12 \quad +4x \; +12$

$\frac{28}{7} = \frac{7x}{7}$

$\boxed{x=4}$

Check:

$\sqrt{-4(4)+16} - \sqrt{3(4)-12} = 0$

$0 - 0 = 0$

$0 = 0$ ✓

21) $\frac{4}{5}(2x - 15)^{5/3} - 12 = 807.2$

$+12 \quad +12$

$\left(\frac{5}{4}\right) \frac{4}{5}(2x-15)^{5/3} = 819.2 \left(\frac{5}{4}\right)$

$\left[(2x-15)^{5/3}\right]^{3/5} = (1024)^{3/5}$

$2x-15 = 64$

$+15 \quad +15$

$\frac{2x}{2} = \frac{79}{2}$

$\boxed{x=39.5}$

Check:

$\frac{4}{5}(2(39.5)-15)^{5/3} - 12 = 807.2$

$807.2 = 807.2$

22) Is the sum of two irrational numbers always irrational? Justify your answer.

No

$\pi + (-\pi) = 0$

23) Completely simplify the following expression:

$$\frac{(-8)^{2/3} + (-100)^{1/2}}{\left(\frac{1}{4}\right)^{-3/2} - (-2)^{5/2}}$$

$(\sqrt[3]{-8})^2 = (-2)^2 = 4$

$\sqrt{-100} = 10i$

$4^{3/2} = (\sqrt{4})^3 = 2^3 = 8$

$\sqrt{(-2)^5} = \sqrt{-32} = i\sqrt{16}\sqrt{2}$

$4i\sqrt{2}$

$$\frac{(4 + 10i)}{(8 - 4i\sqrt{2})} \frac{(8 + 4i\sqrt{2})}{(8 + 4i\sqrt{2})}$$

$$\frac{32 + 16i\sqrt{2} + 80i + 40i^2\sqrt{2}}{64 + 32i\sqrt{2} - 32i\sqrt{2} - 16i^2(2)}$$

$$\frac{32 + 16i\sqrt{2} + 80i + 40(-1)\sqrt{2}}{64 - 16(-1)(2)}$$

$$\frac{32 - 40\sqrt{2} + 80i + 16i\sqrt{2}}{96}$$

$$\frac{4 - 5\sqrt{2} + 10i + 2i\sqrt{2}}{12}$$

$$\rightarrow \frac{4 - 5\sqrt{2}}{12} + \frac{10i + 2i\sqrt{2}}{12}$$

$$\frac{4 - 5\sqrt{2}}{12} + \frac{2(5i + i\sqrt{2})}{12}$$

$$\frac{4 - 5\sqrt{2}}{12} + \frac{5i + i\sqrt{2}}{6} = \boxed{\frac{4 - 5\sqrt{2}}{12} + \frac{5 + \sqrt{2}}{6}i}$$