Name:____________________ Date:__________

Notes: Common Logs and Natural Logs

Do Now: Evaluate each logarithm without a calculator.

1) $\log_2 8 = x$

$2^x = 8$

$2^3 = 8$

$\boxed{3}$

2) $\log_8 2 = x$

$8^x = 2$

$\sqrt[3]{8} = 2$

$8^{1/3} = 2$

$\boxed{\frac{1}{3}}$

3) $\log_2 \frac{1}{8} = x$

$2^x = \frac{1}{8}$

$2^3 = 8$

$2^{-3} = \frac{1}{8}$

$\boxed{-3}$

Think for a minute, describe when you have used each of the mathematical operations below

4) 2.4×10^9

This notation is called scientific notation, and is used to represent very large or small numbers.

5) $A = Pe^{rt}$

This equation is used to find continuous compounded interest. (You may not have used this equation before, but you will use it a lot in the future!)

What Should I Be Able to Do?

- I can explain what a common logarithm is and why we use shorthand to write a common logarithm.

- I can explain what a natural logarithm is and why we use shorthand to write a natural logarithm.

- I can generalize a rule to simplify $\log_a 1$ and explain/justify why $\log_a 1$ simplifies to that given value.

- I can generalize a rule to simplify $\log_a a$ and explain/justify why $\log_a a$ simplifies to that given value.

- I can generalize a rule to simplify $\log_a a^x$ and explain/justify why $\log_a a^x$ simplifies to that given value.

- I can generalize a rule to simplify $a^{\log_a x}$ and explain/justify why $a^{\log_a x}$ simplifies to that given value.

Since we frequently use 10 or e as a base for exponential expressions and equations, we give the logarithms with a base of 10 or e a special name.

Common Logarithm:

The logarithm with base 10 is called the **common logarithm** and is written by excluding the base:

$$\log_{10} x = \log x$$

Evaluate or estimate the following logarithms:

1) $\log_{10} 100 = x$

$10^x = 100$

10^2

$\boxed{2}$

2) $\log_{10} 350 = x$

$10^x = 350$

$10^2 = 100$

$10^3 = 1000$

$\boxed{x \approx 2.5}$

Check:
log350 : 2.54406804935

Scientific calculators of a LOG button you can press to evaluate any common log. Check each of you answers using the calculator.

Natural Logarithm:

The logarithm with base e is called the **natural logarithm** and is written by **ln**:

$$\log_e x = \ln x$$

Scientific calculators of a LN button you can press to evaluate any natural log. Use your calculator to evaluate the following:

1) $\ln 9$

$\ln 9 = 2.19722457734$

2) $\ln 64$

$\ln 64 = 4.15888308336$

Simplify the following logarithmic expressions:

1) $\log_8 1 = x$

$8^x = 1$

$8^0 = 1$

$\boxed{0}$

2) $\log_{36} 1 = x$

$36^x = 1$

$36^0 = 1$

$\boxed{0}$

3) $\log_x 1 = y$

$x^y = 1$

$x^0 = 1$

$\boxed{0}$

Generalize your findings:

$$\log_a 1 = 0$$

Explain why this is true, using correct and effective mathematical vocabulary.

When a number raised to an unknown exponent has a solution of 1, the exponent has to be 0. (Any number raised to the power of 0 is equal to 1. Example: $5^0 = 1$)

Simplify the following logarithmic expressions:

1) $\log_2 2 = x$

$2^x = 2$

$2^1 = 2$

$\boxed{1}$

2) $\log_{12} 12 = x$

$12^x = 12$

$12^1 = 12$

$\boxed{1}$

3) $\log_x x = y$

$x^y = x$

$x^1 = x$

$\boxed{1}$

Generalize your findings:

$$\log_a a = 1$$

Explain why this is true, using correct and effective mathematical vocabulary.

When a number is raised to the first power, the result is always that same number (the base). Example: $12^1 = 12$

Simplify the following logarithmic expressions:

1) $\log_4 4^5 = x$

$4^x = 4^5$

$\boxed{5}$

2) $\log_9 9^3 = x$

$9^x = 9^3$

$\boxed{3}$

3) $\log_c c^d = x$

$c^x = c^d$

$\boxed{d}$

Generalize your findings:

$$\log_a a^x = x$$

Explain why this is true, using correct and effective mathematical vocabulary.

If the base of the logarithm is the same as the base of the argument, then the logarithm is equal to the exponent of the argument. Also, $\log_a$ _ and a^x are inverse operations, so they would cancel each other out, thus leaving just the exponent, x.

Simplify the following logarithmic expressions:

1) $3^{\log_3 7} = x$

exponent

base

argument

$\log_3 x = \log_3 7$

$x = 7$ $\boxed{7}$

2) $9^{\log_9 20} = x$

$\log_9 x = \log_9 20$

$x = 20$

$\boxed{20}$

3) $c^{\log_c d} = x$

$\log_c x = \log_c d$

$x = d$

$\boxed{d}$

Convert each expression out of exponential form!

Generalize your findings:

$$a^{\log_a x} = x$$

Explain why this is true, using correct and effective mathematical vocabulary.

If a number is getting raised to a power that is a logarithm with the same base, then the expression is equal to the argument of the logarithm. Also, a^x and $\log_a x$ are inverse operations, so they would cancel each other out, just leaving just the argument.

Putting Everything Together

Simplify the following logarithmic expressions:

1) $\log 1 = x$

$10^x = 1$

$\boxed{0}$

2) $\ln 1 = x$

$e^x = 1$

$\boxed{0}$

3) $\log 10 = x$

$0^x = 10$

$\boxed{1}$

4) $\ln e = x$

$e^x = e$

$\boxed{1}$

5) $\log 10^x = y$

$10^y = 10^x$

$\boxed{x}$

6) $\ln e^x = y$

$e^y = e^x$

$\boxed{x}$

7) $\log 10^4 = x$

$10^x = 10^4$

$\boxed{4}$

8) $10^{\log x} = y$

$\log_{10} y = \log x$

$\log y = \log x$

$\boxed{x}$

9) $\cancel{e}^{\ln x}$

$\boxed{x}$

10) $\cancel{e}^{\ln 5}$

$\boxed{5}$

Success Criteria

- I can explain what a common logarithm is and why we use shorthand to write a common logarithm.

A common log is a logarithm with the base of 10, $\log_{10} x$ which is commonly written $\log x$. We use shorthand because 10 is a very common base.

- I can explain what a natural logarithm is and why we use shorthand to write a natural logarithm.

A natural logarithm is a log with the base of e, $\log_e x$ which can be written as $\ln x$. We use shorthand because e is a base that is used very often.

- I can generalize a rule to simplify $\log_a 1$ and explain/justify why $\log_a 1$ simplifies to that given value.

When a number raised to an unknown exponent has a solution of 0, the exponent has to be 0. Any number raised to the power of 0 is equal to 1. (Example: $5^0 = 1$)

- I can generalize a rule to simplify $\log_a a$ and explain/justify why $\log_a a$ simplifies to that given value.

When a number is raised to the first power, the result is always that same number (the base). Example: $12^1 = 12$

- I can generalize a rule to simplify $\log_a a^x$ and explain/justify why $\log_a a^x$ simplifies to that given value.

Since $\log_a x$ and a^x are inverse operations, they would cancel out, thus leaving just the exponent, x.

- I can generalize a rule to simplify $a^{\log_a x}$ and explain/justify why $a^{\log_a x}$ simplifies to that given value.

Since $\log_a x$ and a^x are inverse operations, they would cancel out, thus leaving just the argument, x.

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Logarithm Practice

Convert each exponential equation into its equivalent logarithmic equation.

1) $e^4 = x$

$\ln x = 4$

2) $10^3 = 1000$

$\log 1000 = 3$

3) $2^y = 14$

$\log_2 14 = y$

4) $a^{10} = 76$

$\log_a 76 = 10$

5) $10^4 = x$

$\log x = 4$

6) $\frac{1}{6}^3 = b$

$\log_{1/6} b = 3$

7) $e^x = 9$

$\ln 9 = x$

8) $10^y = 76$

$\log 76 = y$

Convert each logarithmic equation into its equivalent exponential equation.

9) $\log 9 = x$

$10^x = 9$

10) $\ln x = 14.5$

$e^{14.5} = x$

11) $\log_{2.5} x = 7$

$2.5^7 = x$

12) $\ln \frac{1}{2} = y$

$e^y = \frac{1}{2}$

Evaluate each logarithm without a calculator.

13) $\log_{10} 10 = x$

$10^x = 10$

$\boxed{x = 1}$

14) $\cancel{\ln e}^9$

$\boxed{9}$

15) $\ln e = \boxed{1}$

work:

$e^x = e$

$x = 1$

16) $\cancel{10^{\log}}{}^{30}$

$\boxed{30}$

17) $\cancel{e}^{\ln x^2}$

$\boxed{x^2}$

18) $\log_{10} 1 = x$

$10^x = 1$

$x = 0$

$\boxed{0}$

19) $\log_{10} \frac{1}{100} = x$

$10^x = \frac{1}{100}$

$x = -2$

$\boxed{-2}$

20) $\ln \frac{1}{e^8} = x$

$e^x = \frac{1}{e^8}$

$e^x = e^{-8}$

$\boxed{-8}$

21) $3(\log 100)$

$3(2)$

$\boxed{6}$

22) $\cancel{10^{\log}}{}^{\sqrt{x}}$

$\boxed{\sqrt{x}}$

23) $\cancel{\ln e}^{x-7}$

$\boxed{x-7}$

24) $\cancel{e}^{\ln 8x}$

$\boxed{8x}$

25) Is the following equation true or false? Justify your answer.

$$\frac{\log_2 16}{\log_2 4} = 4$$

$\log_2 16 = x$

$2^x = 16$

$\log_2 16 = 4$

$\log_2 4$

$2^x = 4$

$\log_2 4 = 2$

$\frac{4}{2} = 2 \therefore$ FALSE

26) Is the following equation true or false? Justify your answer.

$$\log(-100) = -2$$

$10^{-2} = -100$

$\frac{1}{100} \neq -100$

$\therefore$ FALSE

27) Is the following equation true or false? Justify your answer.

$$2(\ln e^3) = 6$$

$2(\cancel{\ln e}^3) = 6$

$2(3) = 6$

$6 = 6$

$\therefore$ TRUE

Evaluate each of the following expressions without a calculator.

28) $\log 1000 + \ln e$

$10^x = 1000$ 1

3

$3+1=\boxed{4}$

29) $\cancel{e^{\ln}}{}^{11} + \cancel{\ln e}{}^8$

$11+8$

$\boxed{19}$

30) $\log_4(\log_5 5)$ $\to 5^x = 5$

$\log_5 5 = 1$

$\log_4 1 = \boxed{0}$

31) $\cancel{10^{\log}}\,x^2 - \cancel{\ln e}\,{}^x$

$\boxed{x^2 - x}$

$\to 10^x = 10000$

32) $\log_2(\log 10000)$

$\log_2(4) \to 2^x = 4$

$\boxed{2}$

33) $\ln(\log 10)$

$\ln(1) \to e^x = 1$

$\boxed{0}$

Using you calculator, round each expression to the nearest thousandth.

34) $\log 341$

2.533

35) $\ln 4.5$

1.504

36) $\log 87 + \ln 87$

6.405

37) A group of scientists are studying the division of amebas to better understand how to better treat patients. The ameba the scientists are studying divides itself into two amebas every hour. The scientists use the equation $t = \log_2 A$ where t, is the number of hours it takes to produce A number of amebas. Find, to the nearest hundredth of an hour, how long it takes to produce 25,000 amebas if the scientists start with one ameba.

$t = \log_2 25{,}000$

$t = 14.6096404744$

$\boxed{14.61 \text{ hours}}$

Evaluate the following expression without a calculator.

38) $\frac{\log_{\sqrt{2}} 1 - \log 0.1}{\ln e^4 - \log_3 27}$

$\sqrt{2}^x = 1$

$\log_{\sqrt{2}} 1 = 0$

$10^x = 0.1$

$10^{-1} = \frac{1}{10}$

$\frac{1}{10} = \frac{1}{10}$ $\therefore \log 0.1 = -1$

$\ln e^4 = 4$

$\log_3 27$

$3^x = 27$

$\log_3 27 = 3$

$\frac{0 - (-1)}{1 - 3} = \frac{1}{1} = \boxed{1}$

Looking Ahead...

Describe the transformation being done from the graph of $f(x)$ to obtain the graph of $g(x)$.

39) $f(x) = \ln x$

$g(x) = \ln(x - 4)$

g(x) is being translated right 4 units from f(x).

40) $f(x) = \log x$

$g(x) = \log x + 9$

g(x) is being translated up 9 units from f(x).