Name:______________________ Date:__________

Notes: Graphing Logarithmic Functions

Do Now: What is the inverse of $y = 2^x$?

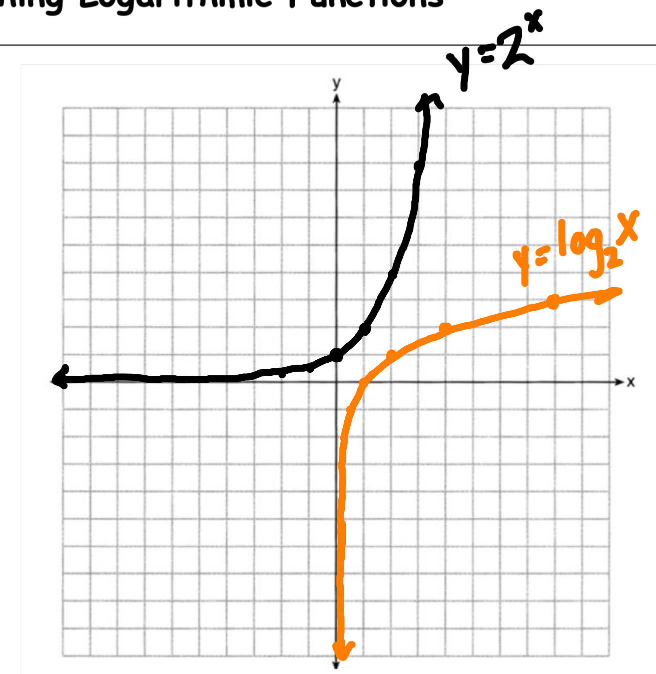
$x = 2^y$

$y = \log_2 x$

On the set of axes, graph $y = 2^x$ and its inverse.

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16

x	y
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3
16	4



What Should I Be Able to Do?

- I can describe how a logarithmic equation is being transformed.

- I can graph a logarithmic equation that is undergoing multiple transformations.

- I can determine the asymptote of a logarithmic equation.

- I can determine the domain and range of a logarithmic equation.

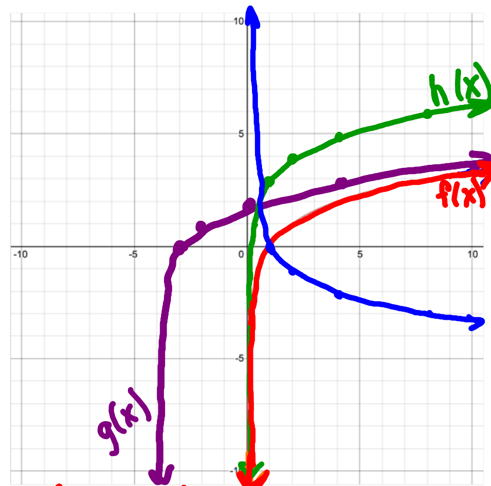
- I can determine the x-intercept of a logarithmic equation.

- I can determine the end behavior of a logarithmic equation.

- I can graph a logarithmic equation with a base such that $0 < b < 1$.

Let's look deeper into the graph of $f(x) = \log_2 x$. k(x)

x	$f(x)$
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2
8	3



$g(x) = \log_2(x+4)$

x	y
-3	0
-2	1
0	2
4	3
12	4

$k(x) = -\log_2 x$

x	y
1	0
2	-1
4	-2
8	-3

$h(x) = \log_2(x)+3$

x	y
1/4	1
1/2	2
1	3
2	4
4	5
8	6

Domain: $(0,\infty)$ Range: $(-\infty,\infty)$ Asymptote: $x=0$ X-Intercept: $(1,0)$

"when $y=0$"

End Behavior:

Left-end Behavior -

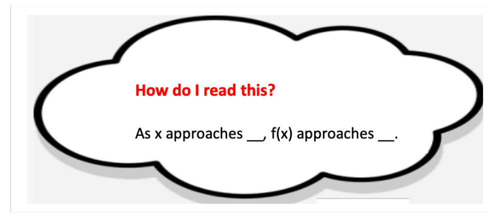
As $x \longrightarrow$ 0

$f(x) \longrightarrow$ $-\infty$

Right-end Behavior-

As $x \longrightarrow$ ∞

$f(x) \longrightarrow$ ∞



Now let's graph the following three equations and see how the graph of $f(x) = \log_2 x$ is changed:

$g(x) = \log_2(x+4)$ $h(x) = \log_2 x + 3$ $k(x) = -\log_2 x$

Describe each transformation:

$g(x)$: 4 units left

Domain: $(-4,\infty)$

Range: $(-\infty,\infty)$

Asymptote: $x=-4$

X-Intercept: $(-3,0)$

$0=\log_2(x+4)$
$2^0 = x+4$
$1 = x+4$
$-3 = x$

Left-end Behavior:
as $x \to -4$
$g(x) \to -\infty$

Right-end Behavior:
as $x \to \infty$
$g(x) \to \infty$

$h(x)$: 3 units up

Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

Asymptote: $x=0$

X-Intercept: $(\frac{1}{8},0)$

$0=\log_2 x+3$
$-3=\log_2 x$
$2^{-3}=x$
$x=\frac{1}{8}$

Left-end Behavior:
as $x \to 0$
$h(x) \to -\infty$

Right-end Behavior:
as $x \to \infty$
$h(x) \to \infty$

$k(x)$: reflection over x-axis

Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

Asymptote: $x=0$

X-Intercept: $(1,0)$

Left-end Behavior:
as $x \to 0$
$k(x) \to \infty$

Right-end Behavior:
as $x \to \infty$
$k(x) \to -\infty$

Describe how $f(x) = \log x$ changes to form each of the following equations:

1) $g(x) = \log(x - 6)$

translated 6 units right

2) $h(x) = \log x - 9$

translated 9 units down

3) $j(x) = \log(x + 1) + 10$

translated 1 unit left and 10 units up

4) $k(x) = -\log x$

reflection over x-axis

5) $m(x) = \log(x - 14) - 21$

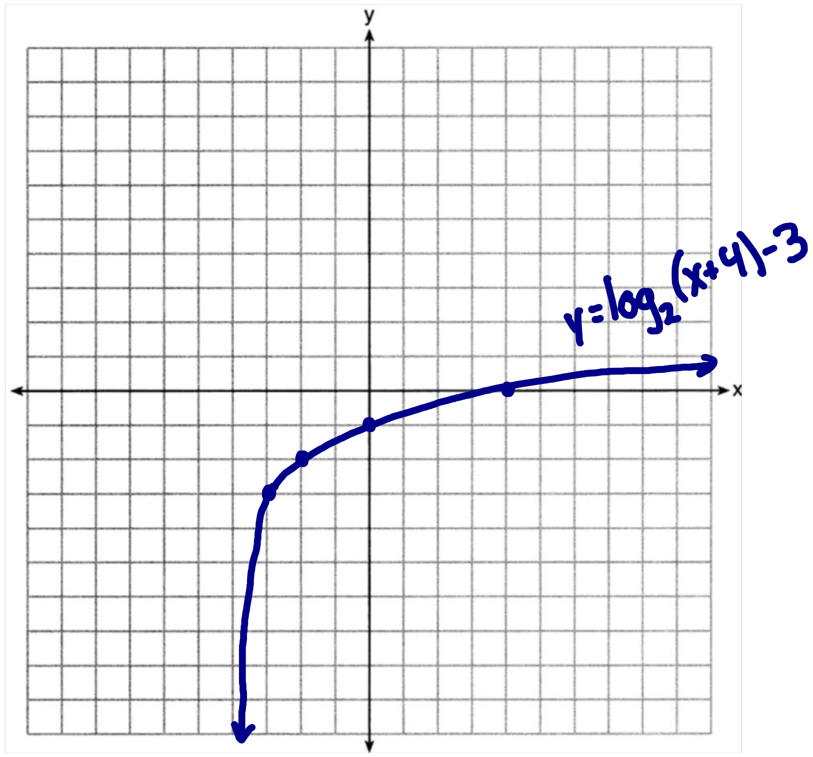
translated 14 units right and 21 units down

6) $n(x) = -\log(x - 1) + 2$

reflection over x-axis then translated 1 unit right and 2 units up

Graph $y = \log_2(x + 4) - 3$ on the set of axes below. Use an appropriate scale to include *both* intercepts.

x	y
-3	-3
-2	-2
0	-1
4	0
12	1



Describe the behavior of the given function as x approaches -4 and as x approaches positive infinity.

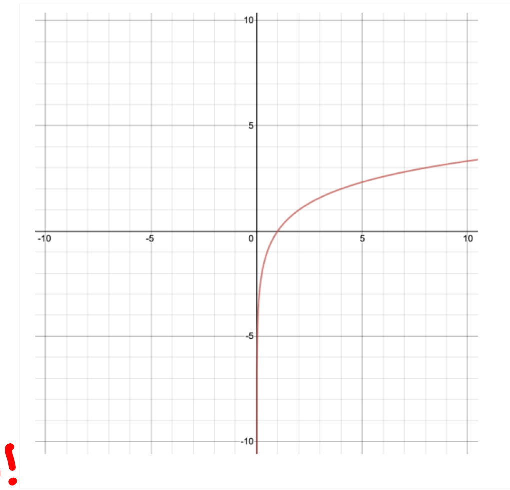
As $x \to -4$
$y \to -\infty$

As $x \to \infty$
$y \to \infty$

Let's take a look at when the bases of logarithm equations ($f(x) = \log_b x$) are different...

Base Greater Than 1 ($b > 1$)

$f(x) = \log_2 x$



Why does the ends of the graph behave like this?

Left-End: $\log_2 x = y \rightarrow 2^y = x$

Think... as y gets smaller

$2^{-1} = \frac{1}{2}$ $\quad 2^{-2} = \frac{1}{4}$ $\quad 2^{-5} = \frac{1}{32}$ $\quad 2^{-10} = \frac{1}{1024}$

the x value gets closer and closer to 0 but never hits 0!

Right-End: $2^y = x$

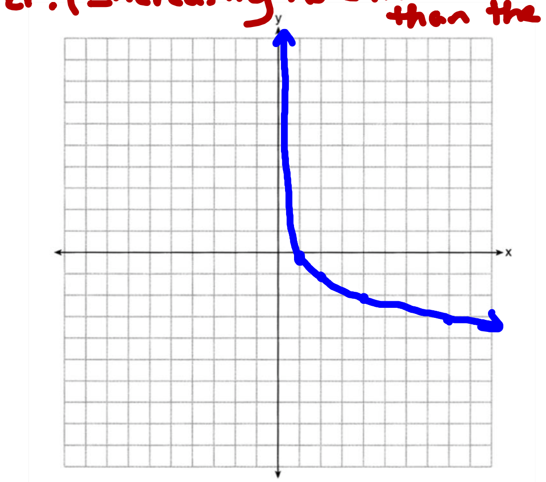
$2^1 = 2$ $\quad 2^3 = 8$ $\quad 2^{10} = 1024$ $\quad 2^{20} = 1048576$

As the exponent (y) gets larger, the product, x, gets exponentially larger! (Increasing more in the x than the y)

Base Between 0 and 1 ($0 < b < 1$)

$f(x) = \log_{\frac{1}{2}} x$

x	y
1	0
2	-1
4	-2
8	-3
16	-4



Why does the ends of the graph behave like this?

Left-End: $y = \log_{\frac{1}{2}} x \rightarrow \left(\frac{1}{2}\right)^y = x$

Think... as x approaches 0

$\left(\frac{1}{2}\right)^0 = 1$ $\quad \left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$, $\left(\frac{1}{2}\right)^{10} = \frac{1}{1024}$

As the product (x) decreases, the exponent (y) increases. That is because the base is less than 1. When you multiply by more numbers that are less than one, the product decreases.

Right-End:

Think... as x approaches ∞.

$\left(\frac{1}{2}\right)^0 = 1$, $\left(\frac{1}{2}\right)^{-1} = 2$, $\left(\frac{1}{2}\right)^{-3} = 8$, $\left(\frac{1}{2}\right)^{-10} = 1024$

As the value of x increases, the exponent (y) decreases. That is because when the exponent is negative, the base flips!

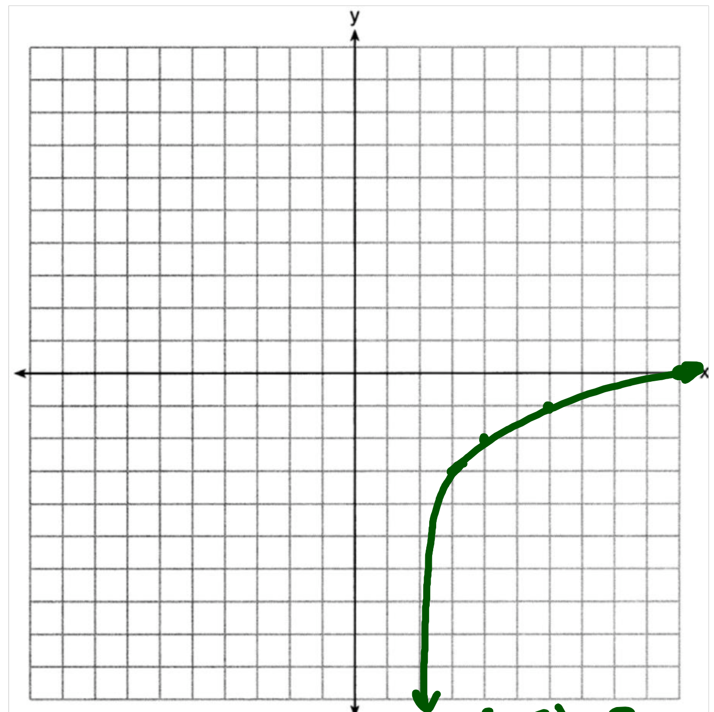
Success Criteria

- I can describe how a logarithmic equation is being transformed.

- I can graph a logarithmic equation that is undergoing multiple transformations.

- I can determine the asymptote, domain, range, x-intercept, and end behavior of a logarithmic equation.

Graph $y = \log_2(x-2) - 3$ on the set of axes below.



x	y
3	-3
4	-2
6	-1
10	0
18	1

$y = \log_2(x-2) - 3$

Describe the transformation from the parent function, $y = \log_2 x$:

Translated 2 units right and 3 units down

Domain: $(2, \infty)$ Range: $(-\infty, \infty)$ Asymptote: $x = 2$ X-Intercept: $(10, 0)$

End Behavior:

Left-end Behavior -

As $x \longrightarrow 2$

$f(x) \longrightarrow -\infty$

Right-end Behavior-

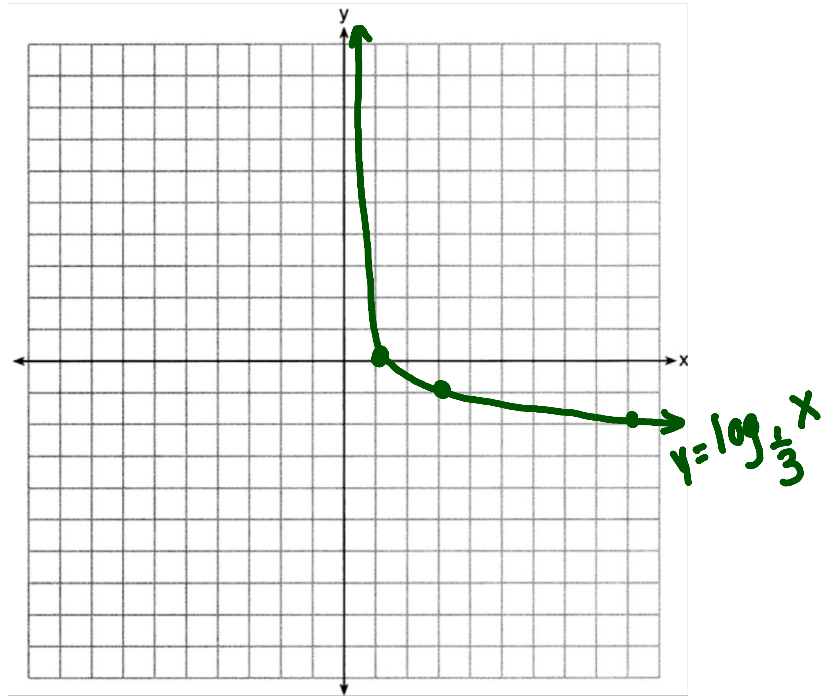
As $x \longrightarrow \infty$

$f(x) \longrightarrow \infty$

- I can graph a logarithmic equation with a base such that $0 < b < 1$.

Graph $y = \log_{\frac{1}{3}}(x)$ on the set of axes below.

x	y
1	0
3	-1
9	-2
27	-3
81	-4



End Behavior:

Left-end Behavior -

As $x \longrightarrow$ 0

$f(x) \longrightarrow$ ∞

Right-end Behavior-

As $x \longrightarrow$ ∞

$f(x) \longrightarrow$ $-\infty$

Describe why each end behavior behaves the way it does.

As the product (x) decreases, the exponent (y) increases. That is because the base is less than 1. When you multiply by ore numbers that are less than one, the product decreases.

As the value of x increases, the exponent (y) decreases. That is because when the exponent is negative, the base flips!

Name:______________________ Date:__________

Classwork: Graphing Logarithmic Functions

1) Using $f(x) = \log x$ as the parent function, fill in the following for each of the functions below:

$a(x) = \log x - 3$

Describe each transformation: 3 units down

Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 0$

X-Intercept: $(1000, 0)$

$0 = \log x - 3$
$3 = \log x$
$10^3 = x$
$1000 = x$

Left-end Behavior: as $x \to 0$, $a(x) \to -\infty$

Right-end Behavior: as $x \to \infty$, $a(x) \to \infty$

$b(x) = \log(x-3)$

3 units right

Domain: $(3, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = 3$

X-Intercept: $(4, 0)$

$0 = \log(x-3)$
$10^0 = (x-3)$
$1 = x - 3$
$x = 4$

Left-end Behavior: as $x \to 3$, $b(x) \to -\infty$

Right-end Behavior: as $x \to \infty$, $b(x) \to \infty$

$c(x) = \log(x+2) - 4$

2 units left, 4 units down

Domain: $(-2, \infty)$

Range: $(-\infty, \infty)$

Asymptote: $x = -2$

X-Intercept: $(9998, 0)$

$0 = \log(x+2) - 4$
$4 = \log(x+2)$
$10^4 = x+2$
$10^4 - 2 = x$
$10000 - 2 = x$
$9998 = x$

Left-end Behavior: as $x \to -2$, $c(x) \to -\infty$

Right-end Behavior: as $x \to \infty$, $c(x) \to \infty$

2) Find the inverse of the following functions.

a) $y = 5^x$.

$x = 5^y$

$\boxed{\log_5 x = y}$

b) $y = 10^{3x-5}$.

$x = 10^{3y-5}$
$\log x = 3y - 5$
$\log(x) + 5 = 3y$

$\boxed{\frac{\log(x)+5}{3} = y}$

3) Evaluate the following logarithmic expressions without using a calculator.

a) $\log_2 \frac{1}{32} = x$

$2^x = \frac{1}{32}$
$2^5 = 32$
$2^{-5} = \frac{1}{32}$

$\boxed{-5}$

b) $4 \cdot 7$

$4 \cdot 7$

$\boxed{28}$

c) $\log_{81} 3 = x$

$81^x = 3$
$\sqrt[4]{81} = 3$
$81^{1/4} = 3$

$\boxed{\frac{1}{4}}$

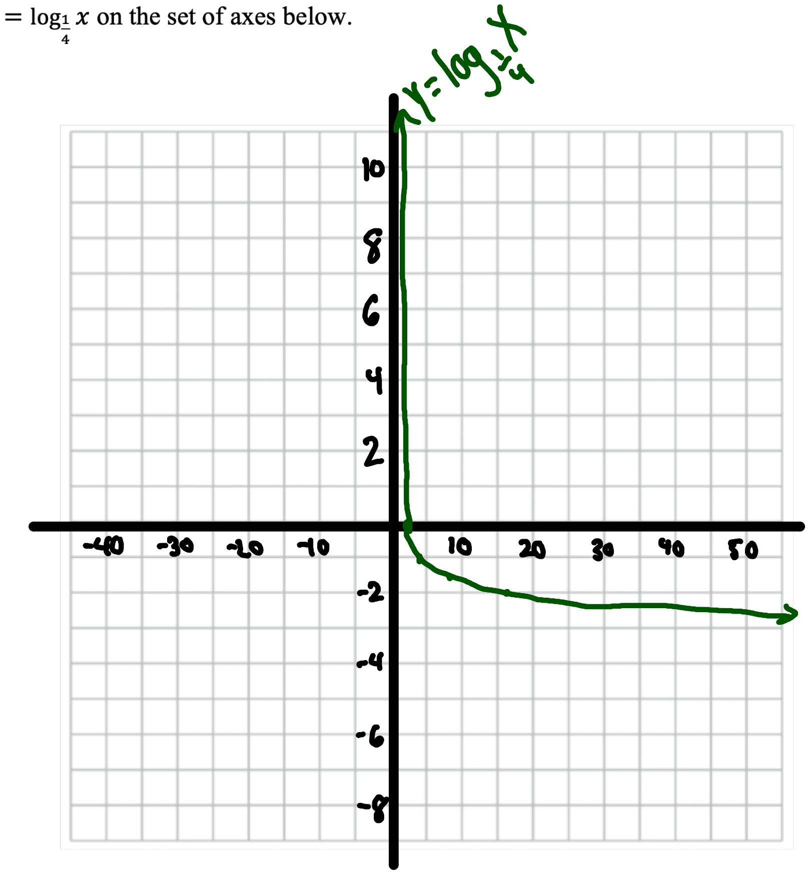
4) $\log \sqrt{10} = x$

$10^x = \sqrt{10}$
$10^x = 10^{1/2}$

$\boxed{1/2}$

4) Graph $y = \log_{\frac{1}{4}} x$ on the set of axes below.

x	y
1	0
2	-0.5
4	-1
8	-1.5
16	-2
64	-4



Domain: $(0, \infty)$ Range: $(-\infty, \infty)$ Asymptote: $x = 0$ X-Intercept: $(1, 0)$

End Behavior:

Left-end Behavior -

As $x \longrightarrow$ 0

$f(x) \longrightarrow$ ∞

Right-end Behavior-

As $x \longrightarrow$ ∞

$f(x) \longrightarrow$ $-\infty$