Name:______________________ Date:__________

Notes: Laws of Exponents

Do Now: Simplify each expression in the left column, then explain in words the exponent law in the middle column. Lastly, in the right column, write a general rule using mathematical symbols.

Simplify:	**Explain:**	**Generalize:**
$\frac{x^9}{x^3} = x^{9-3}$ $\boxed{x^6}$	When dividing terms with the same base, subtract the exponents.	$\frac{x^a}{x^b} = x^{a-b}$
$w^7(w^8) = w^{7+8}$ $\boxed{w^{15}}$	When multiplying terms with the same base, add the exponents.	$x^a(x^b) = x^{a+b}$
$(h^3)^4 = h^{3(4)}$ $\boxed{h^{12}}$	To raise a power to a power, multiply the exponents.	$(x^a)^b = x^{ab}$
$(mn)^7 = m^7n^7$	To raise a product to a power, raise each factor to the power.	$(xy)^a$ $x^a y^a$
$\left(\frac{c}{d}\right)^3 = \frac{c^3}{d^3}$	To raise a fraction to a power, raise the numerator and denominator to the power	$\left(\frac{x}{y}\right)^a$ $\frac{x^a}{y^a}$

What Should I Be Able to Do?

- I can simplify any expression that raises a base to the power of zero.

- I can explain the reasoning behind rule for simplifying any base that is raised to the power of zero.

- I can simplify any expression with negative exponents.

- I can explain the reasoning behind the rule for simplifying an expression with negative exponents.

- I can simplify expressions that raises a base to a rational exponent.

- I can explain how to simplify expressions that involve raising a base to a rational exponent.

Zero Exponent:

How can I simplify 3^0 ? Hmmmmmmm...

Let's simplify $\frac{3^2}{3^2}$ two different methods....

a) Simplify $\frac{3^2}{3^2}$ by using the laws of exponents.

$3^{2-2} = 3^0$

3^0

b) Simplify $\frac{3^2}{3^2}$ by writing the expression out using all of its factors.

$\frac{\cancel{3}^1 \cdot \cancel{3}^1}{\cancel{3}_1 \cdot \cancel{3}_1} = \frac{1 \cdot 1}{1 \cdot 1} = \frac{1}{1} = 1$

Then, $3^0 =$ __1__.

Let's try that with a different value... simplify $\frac{m^6}{m^6}$ using two different methods.

Write $\frac{m^6}{m^6}$ in simplest exponential form and in standard form.

a) Simplify $\frac{m^6}{m^6}$ by using the laws of exponents.

m^{6-6}

m^0

b) Simplify $\frac{m^6}{m^6}$ by writing the expression out using all of its factors.

$\frac{\cancel{m}^1 \cdot \cancel{m}^1 \cdot \cancel{m}^1 \cdot \cancel{m}^1 \cdot \cancel{m}^1 \cdot \cancel{m}^1}{\cancel{m}_1 \cdot \cancel{m}_1 \cdot \cancel{m}_1 \cdot \cancel{m}_1 \cdot \cancel{m}_1 \cdot \cancel{m}_1}$

$\frac{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1} = \frac{1}{1} = 1$

Then, $m^0 =$ __1__.

Generalized Rule: For any number nonzero real number x , $x^0 =$ __1__.

Negative Exponent:

How can I simplify 5^{-2} into a positive exponent? Hmmmmmmm...

Let's simplify $\frac{5^4}{5^6}$ two different methods....

a) Simplify $\frac{5^4}{5^6}$ by using the laws of exponents.

5^{4-6}

5^{-2}

b) Simplify $\frac{5^4}{5^6}$ by writing the expression out using all of its factors.

$$\frac{\cancel{5}\cdot\cancel{5}\cdot\cancel{5}\cdot\cancel{5}}{\cancel{5}\cdot\cancel{5}\cdot\cancel{5}\cdot\cancel{5}\cdot 5\cdot 5} = \frac{1}{5\cdot 5} = \frac{1}{5^2}$$

Then, $5^{-2} = \underline{\frac{1}{5^2}}$.

Let's try that with a different value... simplify $\frac{n^{10}}{n^{14}}$ using two different methods.

Write $\frac{n^{10}}{n^{14}}$ in simplest exponential form and in standard form.

a) Simplify $\frac{n^{10}}{n^{14}}$ by using the laws of exponents.

$n^{10-14} = n^{-4}$

b) Simplify $\frac{n^{10}}{n^{14}}$ by writing the expression out using all of its factors.

$$\frac{\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}}{\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot\cancel{n}\cdot n\cdot n\cdot n\cdot n}$$

$$= \frac{1}{n\cdot n\cdot n\cdot n} = \frac{1}{n^4}$$

Then, $n^{-4} = \underline{\frac{1}{n^4}}$.

Generalized Rule: For any number nonzero real number x, if n is a natural number, then $x^{-n} = \underline{\frac{1}{x^n}}$.

Simplify the following expressions using only positive exponents.

or $\rightarrow \dfrac{24x^8y^5z^5}{8x^7y^7}$

1) 4^{-1}

$\boxed{\dfrac{1}{4}}$

2) $\frac{2}{3}^{-2}$

$\left(\frac{3}{2}\right)^2$

$\frac{3^2}{2^2}$

$\boxed{\frac{9}{4}}$

3) $(-10)^0$

$\boxed{1}$

4) $\frac{24x^{-7}y^5}{8x^{-8}y^7z^{-5}}$

$\frac{24}{8} \cdot \frac{x^{-7}}{x^{-8}} \cdot \frac{y^5}{y^7} \cdot \frac{1}{z^{-5}}$

$3 \cdot x \cdot y^{-2} \cdot z^5$

$\boxed{\dfrac{3xz^5}{y^2}}$

Checkpoint:

Simplify the following expressions using only positive exponents.

1) $\frac{18a^9b^{-7}}{27a^{11}b^{-3}}$

$-7+(+3)=-4$

$\frac{2}{3}a^{-2}b^{-4}$

$\boxed{\dfrac{2}{3a^2b^4}}$

2) $(3x^3y^{-2}z^7)^2$

$3^2x^6y^{-4}z^{14}$

$\boxed{\dfrac{9x^6z^{14}}{y^4}}$

3) $\left(\frac{-40ab^2c^{13}}{8ab^{-5}c^{12}}\right)^{-3}$

$\left(\frac{8ab^{-5}c^{12}}{40ab^2c^{13}}\right)^3$

$\left(\frac{1}{5} \cdot b^{-7} \cdot c^{-1}\right)^3$

$\left(\frac{1}{5b^7c}\right)^3 = \boxed{\dfrac{1}{125b^{21}c^3}}$

4) $(2d^{-24}f^{16}g^0)^{-3}\left(\frac{1}{4}d\right)^{-2}$

$\left(2^{-3}d^{72}f^{-48}\right)\left(\frac{1}{4}^{-2}d^{-2}\right)$

$\left(\frac{1}{8}d^{72}f^{-48}\right)\left(16d^{-2}\right)$

$2d^{70}f^{-48}$

$\boxed{\dfrac{2d^{70}}{f^{48}}}$

Do Now: Simplify each of the following expressions without using a calculator.

1) 2^3

$2 \times 2 \times 2$

$\boxed{8}$

2) 14^2

$$\begin{array}{r} {}^{1}14 \\ \times 14 \\ \hline 56 \\ +140 \\ \hline \boxed{196} \end{array}$$

3) $9^{\frac{1}{2}}$

$\sqrt{9}$

$\boxed{3}$

4) $4^{\frac{3}{2}}$

$(\sqrt{4})^3$

2^3

$\boxed{8}$

But why does this make sense?

What does $(\sqrt{5})^2$ equal to?

$\sqrt{5} \cdot \sqrt{5} = 5$

What does $\left(5^{\frac{1}{2}}\right)^2$ equal to?

$5^{\frac{1}{2}(2)} = 5^1 = 5$

What does this tell you about $\sqrt{5}$ and $5^{\frac{1}{2}}$?

$\sqrt{5}$ and $5^{1/2}$ are equal

What does $(\sqrt[3]{8})^3$ equal to?

$\sqrt[3]{8} \cdot \sqrt[3]{8} \cdot \sqrt[3]{8} = 8$

What does $\left(8^{\frac{1}{3}}\right)^3$ equal to?

$8^{\frac{1}{3}(3)} = 8^1 = 8$

What does this tell you about $\sqrt[3]{8}$ and $8^{\frac{1}{3}}$?

$\sqrt[3]{8}$ and $8^{\frac{1}{3}}$ are equal

Rational Exponents: For any rational exponent $\frac{m}{n}$, where m and n are integers and $n \geq 2$,

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}$$

equivalently written

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

1) Explain how $(27)^{4/3}$ can be evaluated using properties of rational exponents to result in an integer answer.

Rewrite $(27)^{4/3}$ in radical form where the numerator is the power and the denominator is the index on the radical, $(\sqrt[3]{27})^4$ or $\sqrt[3]{27^4}$. First evaluate $\sqrt[3]{27} = 3$. Then evalvate $3^4 = 81$.

2) Explain how $\left(8^{\frac{1}{7}}\right)^2$ can be written as the equivalent radical expression $\sqrt[7]{64}$.

First multiply the exponents to obtain $8^{2/7}$. Then rewrite $8^{2/7}$ into radical form $\sqrt[7]{8^2}$. Evaluate 8^2 to get 64. Thus $(8^{1/7})^2 = \sqrt[7]{64}$.

3)Write each in simplest radical form. Then completely simplify, without a calculator.

a) $64^{\frac{1}{3}}$

$\sqrt[3]{64}$

$\boxed{4}$

b) $625^{\frac{1}{4}}$

$\sqrt[4]{625}$

$\boxed{5}$

c) $(-8)^{\frac{2}{3}}$

$\sqrt[3]{(-8)}^{\,2}$

$(-2)^2 = \boxed{4}$

d) $36^{-\frac{1}{2}}$

$\frac{1}{\sqrt{36}} = \boxed{\frac{1}{6}}$

Success Criteria

- I can simplify any expression that raises a base to the power of zero.

1) 12^0 2) $(-2abc)^0$ 3) $[(-3a^{45}b^{-5}c^7)^0]^4$

1

1

$1^4 = \boxed{1}$

- I can explain the reasoning behind rule for simplifying any nonzero real number **raised to the power of zero.**

Any nonzero real number raised to the zero power is equal to 1.

- I can simplify any expression with negative exponents.

1) $\left(\frac{1}{9}a^{-101}b^{41}c^{-1}\right)^{-2}(3c^4)^{-3}$

$\left(\left(\frac{1}{9}\right)^{-2}a^{202}b^{-81}c^2\right)\left(3^{-3}c^{-12}\right)$

$\left(81a^{202}b^{-81}c^2\right)\left(\frac{1}{27}c^{-12}\right)$

$3a^{202}b^{-81}c^{-10}$

$\boxed{\frac{3a^{202}}{b^{81}c^{10}}}$

2) $\left(\frac{-48xy^{-12}z^{21}}{-36x^{-2}z^{-5}}\right)^{-3}$

$\left(\frac{4}{3}x^3y^{-12}z^{26}\right)^{-3}$

$\left(\frac{4}{3}\right)^{-3}x^{-9}y^{36}z^{-78}$

$\boxed{\frac{27y^{36}}{64x^9z^{78}}}$

- I can explain the reasoning behind the rule for simplifying an expression with negative exponents.

For any base raised to a negative exponent, $\left(\frac{a}{b}\right)^{-c}$, flip the base and make the exponent positive, $\left(\frac{b}{a}\right)^{c}$.

- I can simplify expressions that raises a base to a rational exponent.

1) $144^{\frac{1}{2}}$

$\sqrt{144}$

$\boxed{12}$

2) $32^{\frac{4}{5}}$

$\left(\sqrt[5]{32}\right)^4$

$(2)^4$

$\boxed{16}$

3) $\left(-\frac{27}{8}\right)^{-\frac{2}{3}}$

$\left(-\frac{8}{27}\right)^{2/3}$

$\left(\sqrt[3]{\frac{-8}{27}}\right)^2$

$\left(\frac{-2}{3}\right)^2$

$\boxed{\frac{4}{9}}$

- I can explain how to simplify expressions that involve raising a base to a rational exponent.

Explain how $(81)^{3/4}$ can be evaluated using properties of rational exponents to result in an integer answer.

First rewrite $(81)^{3/4}$ in radical form, $\left(\sqrt[4]{81}\right)^3$. Then evaluate $\sqrt[4]{81}$ to get $(3)^3$. Finally, $(3)^3 = 27$.

Name:______________________ Date:__________

Classwork: Laws of Exponents

Simplify each expression.

$-7+(+9)=2$

1) $\frac{17x^6y^{-7}}{68x^9y^{-9}}$

$\frac{1}{4}x^{-3}y^{2}$

$\boxed{\frac{y^2}{4x^3}}$

2) $\left(x^{\frac{1}{3}}y^{\frac{3}{4}}\right)^{-12}$

$x^{-\frac{12}{3}}y^{-\frac{36}{4}}$

$x^{-4}y^{-9}$

$\boxed{\frac{1}{x^4y^9}}$

3) $\left(3x^{\frac{3}{2}}y^{-\frac{7}{4}}z^2\right)^0$

$\boxed{1}$

4) $\left(\frac{-46xy^{-12}z^{50}}{12x^4y^{-9}z^{21}}\right)^{-2}$

$\left(\frac{-23}{6}x^{-3}y^{-3}z^{29}\right)^{-2}$

$\left(\frac{-23}{6}\right)^{-2}x^{6}y^{6}z^{-48}$

$\boxed{\frac{36x^6y^6}{529z^{48}}}$

5) $\left(\frac{81x^8}{y^{-44}}\right)^{1/4}$

$\frac{3x^2}{y^{-11}}$

$\boxed{3x^2y^{11}}$

6) $\left(\frac{4}{3}c^{-43}d^{43}\right)^{-2}\left(\frac{4}{5}c^{-43}d^{20}\right)^{2}$

$\left(\frac{9}{16}c^{86}d^{-86}\right)\left(\frac{16}{25}c^{-86}d^{40}\right)$

$\frac{9}{25}d^{-46}$

$\boxed{\frac{9}{25d^{46}}}$

7) Explain how $(25)^{5/2}$ can be evaluated using properties of rational exponents to result in an integer answer.

First rewrite $(25)^{5/2}$ into radical form, $(\sqrt{25})^5$.
Then evaluate $\sqrt{25}$ to get $(5)^5$. Finally $(5)^5 = 3125$.

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Write each in simplest radical form. Then completely simplify, without a calculator.

8) $(-216)^{\frac{1}{3}}$

$\sqrt[3]{-216}$

$\boxed{-6}$

9) $(-32)^{\frac{2}{5}}$

$\left(\sqrt[5]{-32}\right)^2$

$(-2)^2$

$\boxed{4}$

10) $\left(\frac{16}{625}\right)^{\frac{3}{4}}$

$\left(\sqrt[4]{\frac{16}{625}}\right)^3$

$\left(\frac{2}{5}\right)^3 = \boxed{\frac{8}{125}}$

11) $\left(-\frac{64}{27}\right)^{-\frac{5}{3}}$

$\left(-\frac{27}{64}\right)^{5/3}$

$\left(\sqrt[3]{\frac{-27}{64}}\right)^5 = \left(\frac{-3}{4}\right)^5$

$\boxed{\frac{-243}{1024}}$

12) If $\left(n^{\frac{3}{4}}\right)^x = \frac{1}{n^3}$, what is the value of x? $\leftarrow n^{-3}$

$n^{\frac{3}{4}x} = n^{-3}$

$\left(\frac{4}{3}\right)\frac{3}{4}x = -3\left(\frac{4}{3}\right)$

$x = -4$

(1) -2

(2) 2

(3) -4

(4) 4

13) Which equation is equivalent to $y = 11^x$?

(1) $y = 11^{-x}$

(2) $y = \left(\frac{1}{11}\right)^x$

(3) $y = -11^x$

(4) $y = \left(\frac{1}{11}\right)^{-x}$

14) Write $\sqrt[3]{x} \cdot \sqrt[4]{x^3}$ as a single term with a rational exponent.

$x^{\frac{1}{3}} \cdot x^{3/4} = x^{\frac{1}{3} + 3/4}$

$\frac{1}{3} + \frac{3}{4} = \frac{4}{12} + \frac{9}{12} = \frac{13}{12}$

$\boxed{x^{13/12}}$

15)Which number is the largest?

(1) $\left(\frac{1}{5}\right)^{-1} = 5$

(2) $\left(\frac{1}{5}\right)^0 = 1$

(3) $\left(\frac{1}{5}\right)^{1/2} = \sqrt{\frac{1}{5}}$

(4) $\left(\frac{1}{5}\right)^3 = \frac{1}{125}$

16) Explain how $\left(3^{\frac{2}{5}}\right)^2$ can be written as the equivalent radical expression $\sqrt[5]{81}$.

First multiply the exponents to obtain $3^{4/5}$. Then rewrite $3^{4/5}$ into radical form to obtain $\sqrt[5]{3^4}$. Then evaluate 3^4 to obtain $\sqrt[5]{81}$.