Name:______________________ Date:__________

Notes: Expanding and Condensing Logarithmic Expressions

Do Now:

Simplify each of the following expressions.

1) $x^3(x^7)$

x^{3+7}

$\boxed{x^{10}}$

2) $\frac{x^{30}}{x^{10}}$

x^{30-10}

$\boxed{x^{20}}$

3) $(x^4)^{12}$

$x^{4(12)}$

$\boxed{x^{48}}$

Solve each of the following equations.

4) $\log_4(x + 15) = 3$

$4^3 = x+15$

$64 = x+15$

$\boxed{49 = x}$

5) $\log(x - 5) = 2$

$10^2 = x-5$

$100 = x-5$

$\boxed{x = 105}$

6) $\log(x + 7) - \log 6 = \log(2x + 3)$

UMMMMM... HOW?!?

What we will be learning today will help us solve this equation!

What Should I Be Able to Do?

- I can completely expand logarithmic expressions.

- I can condense logarithmic expressions into a single logarithm.

How would you expand x^{8+3} into product with two bases? How about a^{b+c}?

$x^8 \cdot x^3$ $a^b \cdot a^c$

Remember logarithms are the inverse operations to exponentials. Therefore you would do the opposite of what you are doing above!

Expand each of the following expressions into an expression with multiple logarithms.

$$\log(12x)$$

$$\log 12 + \log x$$

1) $\log(6 \cdot 7)$

$\log 6 + \log 7$

2) $\log_3(9a)$

$\log_3 9 + \log_3 a$

$\boxed{2 + \log_3 a}$

3) $\ln(7cd)$

$\ln 7 + \ln c + \ln d$

How would you expand x^{8-3} into a quotient with two bases? How about a^{b-c}?

$\frac{x^8}{x^3}$ $\frac{a^b}{a^c}$

Remember logarithms are the inverse operations to exponentials. Therefore you would do the opposite of what you are doing above!

Expand each of the following expressions into an expression with multiple logarithms.

$$\log \frac{x}{5}$$

$$\log x - \log 5$$

1) $\log_9(\frac{4}{x})$

$\log_9 4 - \log_9 x$

2) $\ln\left(\frac{g}{10}\right)$

$\ln g - \ln 10$

3) $\log\left(\frac{g}{10}\right)$

$\log(g) - \log 10$

$\boxed{\log(g) - 1}$

How would you simplify $(x^9)^2$?

$x^{9(2)} = x^{18}$

How about $(a^b)^c$?

a^{bc}

You can reference this property to understand the power rule for logarithms!

Completely expand each of the following logarithmic expressions.

$\log x^2$

$2\log x$

1) $\ln w^7$

$7\ln w$

2) $\log_7 \sqrt{x}$

$\log_7 (x)^{1/2}$

$\boxed{\frac{1}{2}\log_7 x}$

3) $\log(9x)^2$

$2\log(9x)$

$2(\log 9 + \log x)$

$\boxed{2\log 9 + 2\log x}$

Putting All the Rules Together!

Completely expand each of the following logarithmic expressions.

1) $\log \frac{x^2}{y}$

$\log x^2 - \log y$

$\boxed{2\log x - \log y}$

2) $\log_b \left(\frac{2h}{5k^9}\right)$

$\log_b (2h) - \log_b (5k^9)$

$(\log_b 2 + \log_b h) - (\log_b 5 + \log_b k^9)$

$\boxed{(\log_b 2 + \log_b h) - (\log_b 5 + 9\log_b k)}$

$\log_b(3\sqrt{x}y)^2$

$2\log_b(3\sqrt{x}\,y)$

$2(\log_b 3 + \log_b x^{1/2} + \log_b y)$

$2(\log_b 3 + \frac{1}{2}\log_b x + \log_b y)$

$\boxed{2\log_b 3 + \log_b x + 2\log_b y}$

Checkpoint:

Completely expand each of the following logarithmic expressions.

1) $\log(8e^3)$

$\log 8 + \log e^3$

$\boxed{\log 8 + 3\log e}$

2) $\log\left(\frac{x}{1000}\right)$

$\log x - \log 1000$

$\boxed{\log x - 3}$

3) $\ln(3x^9\sqrt[3]{y})$

$\ln 3 + \ln x^9 + \ln y^{1/3}$

$\boxed{\ln 3 + 9\ln x + \frac{1}{3}\ln y}$

4) $\log_{11}\left(\frac{4f^2}{m}\right)^2$

$2\log_{11}\left(\frac{4f^2}{m}\right)$

$2(\log_{11} 4 + \log_{11} f^2 - \log_{11} m)$

$2(\log_{11} 4 + 2\log_{11} f - \log_{11} m)$

$\boxed{2\log_{11} 4 + 4\log_{11} f - \log_{11} m}$

5) $\log_b \sqrt[4]{\frac{x^2}{16y}} = \log_b\left(\frac{x^2}{16y}\right)^{1/4}$

$\frac{1}{4}\log_b\left(\frac{x^2}{16y}\right)$

$\frac{1}{4}(\log_b x^2 - \log_b 16y)$

$\frac{1}{4}[\log_b x^2 - (\log_b 16 + \log_b y)]$

$\frac{1}{4}[2\log_b x - (\log_b 16 + \log_b y)]$

$\boxed{\frac{1}{2}\log_b x - (\frac{1}{4}\log_b 16 + \frac{1}{4}\log_b y)}$

Now that you can **expand** logarithms, in order to **condense** logarithms, you just go in **reverse**!

Simplify each expression into one logarithm.

$\log 5 + \log p$

$\log(5p)$

$\log_4 48 - \log_4 3$

$\log_4\left(\frac{48}{3}\right)$

$\log_4 16$

$\boxed{2}$

$\log 2x + \log(x - 4)$

$\log[2x(x-4)]$

$2\ln x - 3\ln y$

$\ln x^2 - \ln y^3$

$\boxed{\ln\left(\frac{x^2}{y^3}\right)}$

$\frac{1}{2}(\log_3 a + \log_3 b) - 3\log_3(c + d)$

$\frac{1}{2}\log_3 ab - 3\log_3(c+d)$

$\log_3\sqrt{ab} - \log_3(c+d)^3$

$\boxed{\log_3\left(\frac{\sqrt{ab}}{(c+d)^3}\right)}$

Success Criteria

- I can completely expand logarithmic expressions.

1) $\log 15m$

$\log 15 + \log m$

2) $\ln \frac{r}{t}$

$\ln r - \ln t$

3) $\ln 8e^5$

$\ln 8 + \ln e^5$

$\boxed{\ln 8 + 5}$

4) $\log_9 \left(\frac{81}{\sqrt{y+3}}\right)$

$\log_9 81 - \log_9 (y+3)^{1/2}$

$\boxed{2 - \frac{1}{2}\log_9 (y+3)}$

5) $\log \left(\frac{5x^4\sqrt{x+1}}{(x-8)^3}\right)$

$\log(5x^4\sqrt{x+1}) - \log(x-8)^3$

$\log 5 + \log x^4 + \log(x+1)^{1/2} - 3\log(x-8)$

$\boxed{\log 5 + 4\log x + \frac{1}{2}\log(x+1) - 3\log(x-8)}$

*Why don't we need more parentheses here?
PEMDAS!

- I can condense logarithmic expressions into a single logarithm.

1) $\ln a + \ln b$

$\ln(ab)$

2) $\log_m 25 - \log_m 3$

$\log_m\left(\frac{25}{3}\right)$

3) $\frac{1}{2}\log 6 + \frac{1}{3}\log y$

$\log\sqrt{6} + \log\sqrt[3]{y}$

$\boxed{\log \sqrt{6}\sqrt[3]{y}}$

4) $4\ln x + 9\ln y - 2\ln z$

$\ln x^4 + \ln y^9 - \ln z^2$

$\boxed{\ln\left(\frac{x^4y^9}{z^2}\right)}$

5) $\frac{1}{3}(\log x - 5\log y) + 2\log(x+3)$

$\left(\frac{1}{3}\log x - \frac{5}{3}\log y\right) + \log(x+3)^2$

$\log\sqrt[3]{x} - \log\sqrt[3]{y^5}) + \log(x+3)^2$

$\log\left(\frac{\sqrt[3]{x}}{\sqrt[3]{y^5}} \cdot (x+3)^2\right)$

$\boxed{\log\left(\frac{(x+3)^2\sqrt[3]{x}}{\sqrt[3]{y^5}}\right)}$