Name: ______________________ Date: ________

Notes: Operations with Radicals

Do Now: Simplify each expression.

1) $\sqrt{72}$

$\sqrt{36}\sqrt{2}$

$\boxed{6\sqrt{2}}$

2) $\sqrt{x^{11}}$

$\sqrt{x^{10}}\sqrt{x}$

$\boxed{x^5\sqrt{x}}$

3) $\sqrt{243x^8}$

$\sqrt{81}\sqrt{3}\sqrt{x^8}$

$\boxed{9x^4\sqrt{3}}$

4) $\sqrt{1728a^{15}bc^{29}}$

$\sqrt{576}\sqrt{3}\sqrt{a^{14}}\sqrt{a}\sqrt{b}\sqrt{c^{28}}\sqrt{c}$

$\boxed{24a^7c^{14}\sqrt{3abc}}$

Step 1: Find the largest perfect square that divides the radicand.
Step 2: Rewrite the square root as the product of the square root of the perfect square and the other factor.
Step 3: Find the square root of the perfect square.

Remember how to simplify radicals...

1) $5x + 8x$

$13x$

2) $5x^2 + 8x$

$5x^2 + 8x$

3) $5\sqrt{2} + 8\sqrt{2}$

$13\sqrt{2}$

4) $5\sqrt{3} + 8\sqrt{2}$

$5\sqrt{3} + 8\sqrt{2}$

What Should I Be Able to Do?

- I can completely simplify radical expressions with both numbers and variables.

- I can explain the process of completely simplifying a radical.

- I can add and subtract radical expressions.

- I can multiply radical expressions.

- I can divide radical expressions.

- I can explain the process of dividing radical expressions

Simplifying Radicals

Simplify each of the following radical expressions.

$\sqrt{108x^9y^{13}}$

$\sqrt{36}\sqrt{3}\sqrt{x^8}\sqrt{x}\sqrt{y^{12}}\sqrt{y}$

$6\sqrt{3}\,x^4\sqrt{x}\,y^6\sqrt{y}$

$\boxed{6x^4y^6\sqrt{3xy}}$

$\sqrt[3]{108x^9y^{13}}$

$\sqrt[3]{27}\,\sqrt[3]{4}\,\sqrt[3]{x^9}\,\sqrt[3]{y^{12}}\,\sqrt[3]{y}$

$3\sqrt[3]{4}\,x^3\,y^4\,\sqrt[3]{y}$

$\boxed{3x^3y^4\sqrt[3]{4y}}$

$\sqrt{160a^5b^6c^7d^8}$

$\sqrt{16}\sqrt{10}\sqrt{a^4}\sqrt{a}\sqrt{b^6}\sqrt{c^6}\sqrt{c}\sqrt{d^8}$

$4\sqrt{10}a^2\sqrt{a}\,b^3c^3\sqrt{c}\,d^4$

$\boxed{4a^2b^3c^3d^4\sqrt{10ac}}$

$\sqrt[3]{160a^5b^6c^7d^8}$

$\sqrt[3]{8}\,\sqrt[3]{20}\,\sqrt[3]{a^3}\,\sqrt[3]{a^2}\,\sqrt[3]{b^6}\,\sqrt[3]{c^6}\,\sqrt[3]{c}\,\sqrt[3]{d^6}\,\sqrt[3]{d^2}$

$2\sqrt[3]{20}\,a\sqrt[3]{a^2}\,b^2c^2\sqrt[3]{c}\,d^2\sqrt[3]{d^2}$

$\boxed{2ab^2c^2d^2\sqrt[3]{20a^2cd^2}}$

$\sqrt[4]{160a^5b^6c^7d^8}$

$\sqrt[4]{16}\,\sqrt[4]{10}\,\sqrt[4]{a^4}\,\sqrt[4]{a}\,\sqrt[4]{b^4}\,\sqrt[4]{b^2}\,\sqrt[4]{c^4}\,\sqrt[4]{c^3}\,\sqrt[4]{d^8}$

$2\sqrt[4]{10}\,a\sqrt[4]{a}\,b\sqrt[4]{b^2}\,c\sqrt[4]{c^3}\,d^2$

$\boxed{2abcd^2\sqrt[4]{10ab^2c^3}}$

$\sqrt[5]{160a^5b^6c^7d^8}$

$\sqrt[5]{32}\,\sqrt[5]{5}\,\sqrt[5]{a^5}\,\sqrt[5]{b^5}\,\sqrt[5]{b}\,\sqrt[5]{c^5}\,\sqrt[5]{c^2}\,\sqrt[5]{d^5}\,\sqrt[5]{d^3}$

$2\sqrt[5]{5}\,ab\sqrt[5]{5}\,c\sqrt[5]{c^2}\,d\sqrt[5]{d^3}$

$\boxed{2abcd\sqrt[5]{5bc^2d^3}}$

When **ADDING** or **SUBTRACTING** radicals, you must have ....

LIKE TERMS

Simplify each of the following radical expressions:

A) $7\sqrt{6} + 2\sqrt{6}$

$9\sqrt{6}$

B) $4\sqrt{5} + 3\sqrt{10}$

$4\sqrt{5} + 3\sqrt{10}$

C) $\sqrt{14} - 3\sqrt{14}$

$\boxed{-2\sqrt{14}}$

But what if we have ...

UNLIKE TERMS

$3\sqrt{24} - 9\sqrt{6}$

$3\sqrt{4}\sqrt{6} - 9\sqrt{6}$

$3(2)\sqrt{6} - 9\sqrt{6}$

$6\sqrt{6} - 9\sqrt{6}$

$\boxed{-3\sqrt{6}}$

1) **SIMPLIFY** all radicals.

2) Combine all like terms.

$-\sqrt{27} + 2\sqrt{12}$

$-\sqrt{9}\sqrt{3} + 2\sqrt{4}\sqrt{3}$

$-3\sqrt{3} + 2(2)\sqrt{3}$

$-3\sqrt{3} + 4\sqrt{3}$

$\boxed{\sqrt{3}}$

Multiplying Radicals

Step 1: Simplify any radicals possible.

Step 2: **MULTIPLY** coefficients

MULTIPLY radicands.

Step 3: If possible, simplify the product.

$\sqrt{5}(\sqrt{10})$

$\sqrt{50}$

$\sqrt{25}\sqrt{2}$

$\boxed{5\sqrt{2}}$

$3\sqrt{192}\cdot 5\sqrt{2}$

$3\sqrt{64}\sqrt{3}\cdot 5\sqrt{2}$

$3(8)\sqrt{3}\cdot 5\sqrt{2}$

$24\sqrt{3}\cdot 5\sqrt{2}$

$\boxed{120\sqrt{6}}$

$\left(\frac{1}{5}\sqrt{5}\right)^2$

$\left(\frac{1}{5}\sqrt{5}\right)\left(\frac{1}{5}\sqrt{5}\right)$

$\frac{1}{25}\sqrt{25}$

$\frac{1}{25}(5) = \frac{5}{25}$

$\boxed{\frac{1}{5}}$

$-4\sqrt{98a^7}(3\sqrt{54a^8})$

$-4\sqrt{49}\sqrt{2}\sqrt{a^6}\sqrt{a}(3\sqrt{9}\sqrt{6}\sqrt{a^8})$

$-4(7)a^3\sqrt{2a}(3(3)a^4\sqrt{6})$

$(-28a^3\sqrt{2a})(9a^4\sqrt{6})$

$\boxed{-252a^7\sqrt{12a}}$

$(2-\sqrt{10})(7+\sqrt{10})$

$14+2\sqrt{10}-7\sqrt{10}-\sqrt{100}$

$14-5\sqrt{10}-10$

$\boxed{4-5\sqrt{10}}$

Dividing Radicals

$\frac{\sqrt{15}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{45}}{3}$

$\frac{\sqrt{9}\sqrt{5}}{3}$

$\frac{3\sqrt{5}}{3} = \boxed{\sqrt{5}}$

$\frac{3\sqrt{6x}}{\sqrt{5x}} \cdot \frac{\sqrt{5x}}{\sqrt{5x}}$

$\frac{3\sqrt{30x^2}}{5x}$

$\frac{3x\sqrt{30}}{5x} = \boxed{\frac{3\sqrt{30}}{5}}$

$\sqrt{9}\sqrt{5}$

$3\sqrt{5}$

$2\sqrt{4}\sqrt{5}$

$2 \cdot 2\sqrt{5}$

$4\sqrt{5}$

$\frac{\sqrt{45}}{2\sqrt{20}}$

$\frac{3\sqrt{5}}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\frac{3 \cdot 5}{4 \cdot 5} = \frac{15}{20} = \boxed{\frac{3}{4}}$

$\frac{5\sqrt{3}}{(6+\sqrt{7})} \cdot \frac{(6-\sqrt{7})}{(6-\sqrt{7})}$

$\frac{30\sqrt{3} - 5\sqrt{21}}{36 \cancel{-6\sqrt{7}} \cancel{+6\sqrt{7}} - 7}$

$\boxed{\frac{30\sqrt{3} - 5\sqrt{21}}{29}}$

Vocab Corner

Conjugate: A conjugate of a binomial is made by changing the operation with + to – or from – to +.

$\frac{4-3\sqrt{8}}{\sqrt{5}-9}$

$4-3\sqrt{4}\sqrt{2}$

$4-6\sqrt{2}$

$\frac{(4-6\sqrt{2})}{(\sqrt{5}-9)} \cdot \frac{(\sqrt{5}+9)}{(\sqrt{5}+9)}$

$\frac{4\sqrt{5}+36-6\sqrt{10}-54\sqrt{2}}{5 \cancel{+9\sqrt{5}} \cancel{-9\sqrt{5}} - 81}$

$\frac{\overset{2}{\cancel{4}}\sqrt{5}+\overset{18}{\cancel{36}}-\overset{3}{\cancel{6}}\sqrt{10}-\overset{27}{\cancel{54}}\sqrt{2}}{-\underset{38}{\cancel{76}}}$

$\frac{2\sqrt{5}+18-3\sqrt{10}-54\sqrt{2}}{-38}$

$\boxed{\frac{-2\sqrt{5}-18+3\sqrt{10}+54\sqrt{2}}{38}}$

Success Criteria

- I can completely simplify radical expressions with both numbers and variables.

Completely simplify the following radical expressions.

1) $\sqrt{507}$

$\sqrt{169}\sqrt{3}$

$\boxed{13\sqrt{3}}$

2) $\sqrt[3]{384x^7y^{32}z^{66}}$

$\sqrt[3]{64}\sqrt[3]{6}\sqrt[3]{x^6}\sqrt[3]{x}\sqrt[3]{y^{30}}\sqrt[3]{y^2}\sqrt[3]{z^{66}}$

$4\sqrt[3]{6}x^2\sqrt[3]{x}y^{10}\sqrt[3]{y^2}z^{22}$

$\boxed{4x^2y^{10}z^{22}\sqrt[3]{6xy^2}}$

3) $\sqrt{3920m^{107}n}$

$\sqrt{784}\sqrt{m^{106}}\sqrt{m}\sqrt{n}$

$28m^{103}\sqrt{mn}$

- I can explain the process of completely simplifying a radical.

Explain each step to completely simplifying the following radical expression.

$$\sqrt{108x^5}$$

First write 108 and x^5 as factors with one factor being the highest perfect square that divides each.

$\sqrt{36}\sqrt{3}\sqrt{x^4}\sqrt{x}$

Next, evaluate each square root. $6\sqrt{3}x^2\sqrt{x}$

Lastly, rewrite $6x^2\sqrt{3x}$

- I can add and subtract radical expressions.

Completely simplify the following radical expressions.

1) $7\sqrt{8} + 2\sqrt{8}$

$7\sqrt{4}\sqrt{2} + 2\sqrt{4}\sqrt{2}$

$14\sqrt{2} + 4\sqrt{2}$

$\boxed{18\sqrt{2}}$

2) $\sqrt{2} - 18\sqrt{72}$

$\sqrt{2} - 18\sqrt{36}\sqrt{2}$

$\sqrt{2} - 108\sqrt{2}$

$\boxed{-107\sqrt{2}}$

3) $\sqrt{56x^{11}} - 3\sqrt{60x^7} + \sqrt{20x^8}$

$\sqrt{4}\sqrt{14}\sqrt{x^{10}}\sqrt{x} - 3\sqrt{4}\sqrt{15}\sqrt{x^6}\sqrt{x} + \sqrt{4}\sqrt{5}\sqrt{x^8}$

$\boxed{2x^5\sqrt{14x} - 6x^3\sqrt{15x} + 2x^4\sqrt{5}}$

- I can multiply radical expressions.

Completely simplify the following radical expressions.

1) $5\sqrt{150} \cdot 2\sqrt{128}$

$5\sqrt{25}\sqrt{6} \cdot 2\sqrt{64}\sqrt{2}$

$25\sqrt{6} \cdot 16\sqrt{2}$

$\boxed{400\sqrt{12}}$

2) $4\sqrt{6h^{27}}(-3\sqrt{24h^{13}})$

$-12\sqrt{144h^{40}}$

$-12(12h^{20})$

$\boxed{-144h^{20}}$

3) $(4 - \sqrt{30})^2$

$(4-\sqrt{30})(4-\sqrt{30})$

$16 - 4\sqrt{30} - 4\sqrt{30} + 30$

$\boxed{46 - 8\sqrt{30}}$

- I can divide radical expressions.
Completely simplify the following radical expressions.

1) $\dfrac{(3-\sqrt{3})}{(3\sqrt{3})} \cdot \dfrac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{3\sqrt{3}-3}{3(3)}$

$\dfrac{\cancel{3}\sqrt{3}-\cancel{3}}{\cancel{9}}$ 1 1 3

$\boxed{\dfrac{\sqrt{3}-1}{3}}$

2) $\dfrac{(12-\sqrt{10})}{(\sqrt{2}+8\sqrt{5})} \dfrac{(\sqrt{2}-8\sqrt{5})}{(\sqrt{2}-8\sqrt{5})}$

$\sqrt{4}\sqrt{5}$
$2\sqrt{5}$

$8\sqrt{25}\sqrt{2}$
$40\sqrt{2}$

$\dfrac{12\sqrt{2}-96\sqrt{5}-\sqrt{20}+8\sqrt{50}}{4-8\sqrt{10}+8\sqrt{10}-64(5)}$

$\dfrac{12\sqrt{2}-96\sqrt{5}-2\sqrt{5}+40\sqrt{2}}{-316}$

$\dfrac{-98\sqrt{5}+52\sqrt{2}}{-316}$

$\boxed{\dfrac{49\sqrt{5}-26\sqrt{2}}{158}}$

- I can explain the process of dividing radical expressions.
Explain each step to completely simplifying the following radical expression.

$$\frac{a}{b-\sqrt{c}}$$

First multiply the numerator and denominator by the conjugate of the denominator. $\dfrac{(a)}{(b-\sqrt{c})} \cdot \dfrac{(b+\sqrt{c})}{(b+\sqrt{c})}$

$= \dfrac{ab+a\sqrt{c}}{b^2+b\sqrt{c}-b\sqrt{c}-\sqrt{c}^2}$

Next, simplify and combine any like terms.

$= \dfrac{ab+a\sqrt{c}}{b^2-c^2}$

As we cannot simplify any further, this is our answer.

Name: ______________________ Date: ________

Classwork: Operations with Radicals

Completely simplify each radical expression.

1) $\sqrt{108h^{57}j^{29}k^{10}}$

$\sqrt{36}\sqrt{3}\sqrt{h^{56}}\sqrt{h}\sqrt{j^{28}}\sqrt{j}\sqrt{k^{10}}$

$6\sqrt{3}h^{28}\sqrt{h}j^{14}\sqrt{j}k^{5}$

$\boxed{6h^{28}j^{14}k^{5}\sqrt{3hj}}$

2) $\frac{-3\sqrt{3}}{8\sqrt{27}}\cdot\frac{\sqrt{27}}{\sqrt{27}}$

$\frac{-3\sqrt{81}}{8(27)}$

$\frac{-27}{216} = \boxed{-\frac{1}{8}}$

3) $\sqrt{216m^{22}} + 3\sqrt{96m^{23}} + \sqrt{24m^{22}}$

$\sqrt{36}\sqrt{6}\sqrt{m^{22}} + 3\sqrt{16}\sqrt{6}\sqrt{m^{22}}\sqrt{m} + \sqrt{4}\sqrt{6}\sqrt{m^{22}}$

$6m^{11}\sqrt{6} + 12m^{11}\sqrt{6m} + 2m^{11}\sqrt{6}$

$\boxed{8m^{11}\sqrt{6} + 12m^{11}\sqrt{6m}}$

4) $-4\sqrt{72}(-3\sqrt{128})$

$(-4\sqrt{36}\sqrt{2})(-3\sqrt{64}\sqrt{2})$

$(-4(6)\sqrt{2})(-3(8)\sqrt{2})$

$(-24\sqrt{2})(-24\sqrt{2})$

$576\sqrt{4}$

$\boxed{1152}$

5) $\frac{9}{-4-\sqrt{11}}\cdot\frac{(-4+\sqrt{11})}{(-4+\sqrt{11})}$

$\frac{-36+9\sqrt{11}}{16-4\sqrt{11}+4\sqrt{11}-11}$

$\boxed{\frac{-36+9\sqrt{11}}{5}}$

6) $\sqrt{1445} - \sqrt{1280} - \sqrt{3125}$

$\sqrt{289}\sqrt{5} - \sqrt{256}\sqrt{5} - \sqrt{625}\sqrt{5}$

$17\sqrt{5} - 16\sqrt{5} - 25\sqrt{5}$

$\boxed{24\sqrt{5}}$

7) $(13 - 2\sqrt{288})(\sqrt{160} - 3\sqrt{98})$

$13 - 2\sqrt{144}\sqrt{2}$

$13 - 24\sqrt{2}$

$\sqrt{16}\sqrt{10} - 3\sqrt{49}\sqrt{2}$

$4\sqrt{10} - 21\sqrt{2}$

$(13-24\sqrt{2})(4\sqrt{10}-21\sqrt{2})$

$52\sqrt{10} - 273\sqrt{2} - 96\sqrt{20} + 504\sqrt{4}$

$52\sqrt{10} - 273\sqrt{2} - 96\sqrt{4}\sqrt{5} + 1008$

$\boxed{52\sqrt{10} - 273\sqrt{2} - 192\sqrt{5} + 1008}$

8) $\frac{\sqrt{18}+4}{\sqrt{14}+3}$

$\sqrt{9}\sqrt{2} + 4$

$3\sqrt{2} + 4$

$\frac{(3\sqrt{2}+4)}{(\sqrt{14}+3)}\cdot\frac{(\sqrt{14}-3)}{(\sqrt{14}-3)}$

$3\sqrt{7}\sqrt{4}$

$6\sqrt{7}$

$\frac{3\sqrt{28} - 9\sqrt{2} + 4\sqrt{14} - 12}{14 - 3\sqrt{14} + 3\sqrt{14} - 9}$

$\boxed{\frac{6\sqrt{7} - 9\sqrt{2} + 4\sqrt{14} - 12}{5}}$

9) Determine whether the following statement is true or false. Explain your reasoning.

The cube root of -42 is not a real number.

$\sqrt[3]{-42}$ is a real number because you can take the cube root of a negative real number and stay within the set of real numbers.

$$\sqrt[3]{-42} \approx -3.476026645$$

10) Given $b \geq 0$, completely simplify the product of $a - \sqrt{b}$ and its conjugate.

$$(a-\sqrt{b})(a+\sqrt{b})$$

$$a^2 + a\sqrt{b} - a\sqrt{b} - \sqrt{b^2}$$

$$\boxed{a^2 - b}$$

11) Completely simplify the following expression:

$$\frac{(9)^{-3/2} + (5)^{3/2}}{(32)^{2/5} - (3)^{1/2}}$$

$$\left(\frac{1}{9}\right)^{-3/2} = \left(\sqrt{\frac{1}{9}}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$\sqrt{5^3} = \sqrt{125} = \sqrt{25}\sqrt{5} = 5\sqrt{5}$$

$$\left(\sqrt[5]{32}\right)^2 = (2)^2 = 4$$

$$\sqrt{3}$$

$$\frac{\left(\frac{1}{27} + 5\sqrt{5}\right)(4+\sqrt{3})}{(4-\sqrt{3})(4+\sqrt{3})}$$

$$\frac{\frac{4}{27} + \frac{\sqrt{3}}{27} + 20\sqrt{5} + 5\sqrt{15}}{4 + \cancel{4\sqrt{3}} - \cancel{4\sqrt{3}} - 3}$$

$$\boxed{\frac{4}{27} + \frac{\sqrt{3}}{27} + 20\sqrt{5} + 5\sqrt{15}}$$