Name:_____ Date:_____

Notes: Sequences

Do Now:

1) Consider the sequence below.

+3 +3 +3 +3 +3

4, 7, 10, 13, 16, 19, …

What are the next 4 terms?

22, 25, 28, 31

What is the pattern of the sequence?

Add 3 to the previous number to obtain the next number

2) At Mount Morris Elementary School, the auditorium has thirty-five rows of seats. The front row has 12 seats and each subsequent row has two more seats than the previous row. How many seats are in the fourteenth row?

38 seats

12	14	16	18	20	22	24	26	28	30	32	34	36	38
1	2	3	4	5	6	7	8	9	10	11	12	13	14

What Should I Be Able to Do?

- I can define sequence.
- I can create a sequence of my own.
- I can write a recursive formula given a context.
- I can interpret a recursive formula and state any term in the sequence.
- I can graph a sequence.
- I can define an arithmetic sequence.
- I can decipher whether a sequence is arithmetic or not arithmetic and provide reasoning.

Vocab Breakdown

Sequence: An ordered list of numbers that often has a pattern.

The first term is denoted a_1, second term a_2, third term a_3, etc...

$$a_1, a_2, a_3, a_4, a_5, a_6, a_7, \ldots, a_n, \ldots.$$

(Other common notations are $a(1)$ or $f(1)$ for the first term, $a(2)$ or $f(2)$ for the second term, etc...)

Example:

$a_1\ a_2\ a_3\ a_4\ a_5\ a_6\ a_7\ a_8\ a_9\ a_{10}$

$3, 9, 15, 21, 27, 33, 39, \ldots 45\ 51\ 57$

1) What is the pattern in the sequence above?

Add 6 to the previous term

2) What is a_5?

27

3) What is $a(1)$?

3

4) What is $f(10)$?

57

Create your own sequence!!!

Step 1: State the first term. $a_1 =$

Step 2: State the pattern of your sequence.

add 10 to the previous term

Step 3: State the first 5 terms of your sequence.

5, 15, 25, 35, 45

The sequence I created started with a first term of 5 and then added 10 to the previous term to get the next term. Let's list as many terms a will fit on the page.

5, 15, 25, 35, 45, 55, 65, 75, 85, 95, 105, 115, 125, 135, 145, 155, 165, 175, 185, 195, 205, 215, 225, ...

This sequence would go on forever! There is a better way of representing sequences and that is by using a **RECURSIVE FORMULA.**

Vocab Breakdown

Recursive Formula: A way of writing sequences that relates each term of the sequence to the previous term.

*** You must always state the first term to specify where the sequence starts!**

Let's look at how I would write my sequence in a recursive formula:

$$a_1 = 5$$

$$a_n = a_{n-1} + 10$$

Put this recursive formula into words.

The first term is 5. To find any term, take the previous term and add 10.

How else could you write this recursive formula using different notation?

$a(1) = 5$

$a(n) = a(n-1) + 10$

$f(1) = 5$

$f(n) = f(n-1) + 10$

Interpret each recursive formula. Then find the first 5 terms of each sequence.

1) $a_1 = 3$

$a_n = a_{n-1} - 2$

The first term is 3. To find the n^{th} term, take the previous term and subtract 2.

3, 1, −1, −3, −5

2) $a_1 = -4$

$a_n = 2a_{n-1}$

The first term is −4. To find the n^{th} term, take the previous term and multiply by 2.

−4, −8, −16, −32, −64

3) $a(1) = 1$

$a(n) = -3a(n-1) - 2$

The first term is 1. To find the n^{th} term, take the previous term, multiply by −3 then subtract 2.

$a(1) = 1$

$a(2) = -3(1) - 2 = -5$

$a(3) = -3(-5) - 2 = 13$

$a(4) = -3(13) - 2 = -41$

$a(5) = -3(-41) - 2 = 121$

1, −5, 13, −41, 121

Simon is saving up for a new pair of soccer cleats. He has $15 dollars saved from his first week and then saves $10 dollars each week from mowing lawns.

a) Write a recursive formula to represent the how much money Simon has saved in n weeks.

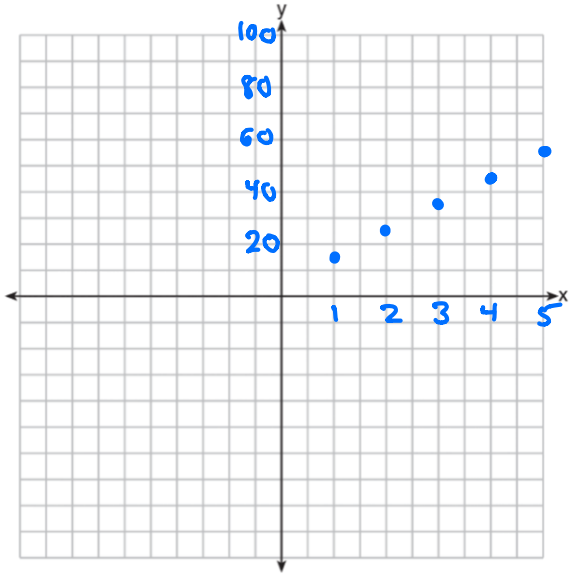
$a_1 = 15$

$a_n = a_{n-1} + 10$

b) Fill in the following table for the first five terms.

n	$a(n)$
1	15
2	25
3	35
4	45
5	55

c) Graph the sequence below.



What quadrant(s) should not be included in our graph? Explain your reasoning.

Quadrants II, III, and IV because you cannot have negative time or in this scenario negative money as Simon is only making money, not losing money.

How does the graph of a sequence differ from graphs of other equations?

You do not connect the points because there is not a 2.5th term in a sequence. A sequence only has term numbers that are whole numbers.

Checkpoint:

$\frac{1}{2}, 0, -2, -10, -42$

1) $a_1 = \frac{1}{2}$

$a_n = 4a_{n-1} - 2$

a) Interpret the recursive formula.

The first term is $\frac{1}{2}$. To find the n^{th} term, take the previous term, multiply by 4 then subtract 2.

b) Find the first 5 terms of each sequence.

$a_1 = \frac{1}{2}$

$a_2 = 4(\frac{1}{2}) - 2 = 0$

$a_3 = 4(0) - 2 = -2$

$a_4 = 4(-2) - 2 = -10$

$a_5 = 4(-10) - 2 = -42$

2) $a(1) = 2$

$a(n) = a(n-1) + n$

a) Interpret the recursive formula.

The first term is 2. To find the n^{th} term, take the previous term and add the term number.

b) Find the first 5 terms of each sequence.

$a(1) = 2$

$a(2) = 2 + 2 = 4$

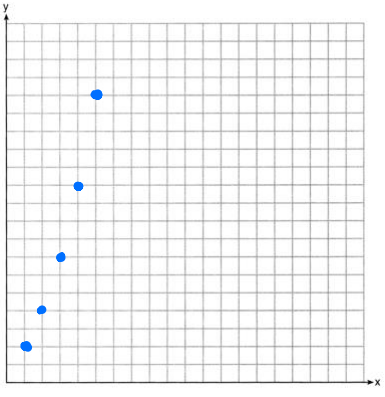
$a(3) = 4 + 3 = 7$

$a(4) = 7 + 4 = 11$

$a(5) = 11 + 5 = 16$

2, 4, 7, 11, 16

c) Graph the first 5 terms of the sequence



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Vocab Breakdown

Arithmetic Sequence: A sequence that has a common difference between terms.

Examples:

$3, 9, 15, 21, 27, 33, 39, \ldots$ (+6, +6, +6; +6, +6, +6)

$d = 6$

$4, 0, -4, -8, -12, -16, -20, \ldots$ (-4, -4, -4; -4, -4, -4)

$d = -4$

General Recursive Formula for Arithmetic Sequence:

$$a_1 = first\ term$$

$$a_n = a_{n-1} + d$$

Write the recursive formula for each of arithmetic sequence above.

$a_1 = 3$

$a_n = a_{n-1} + 6$

$a_1 = 4$

$a_n = a_{n-1} - 4$

Determine whether each of the following sequences is arithmetic. Explain your reasoning. Then, find the sequence's recursive formula or equation.

1) $-1, 6, 13, 20, 27, \ldots$ (+7, +7; +7, +7)

Arithmetic because the sequence has a common difference.

$a_1 = -1$

$a_n = a_{n-1} + 7$

2) $2, 4, 8, 16, 32, 64, \ldots$ (+2, +8; +4)

Not arithmetic because the sequence does NOT have a common difference.

$a_1 = 2$

$a_n = 2^n$

not recursive

3) $10, 2, -6, -14, \ldots$ (-8, -8; -8)

Arithmetic because the sequence has a common difference

$a_1 = 10$

$a_n = a_{n-1} - 8$

Success Criteria

- I can define sequence.

Define sequence.

An ordered list of numbers that often has a pattern.

- I can create a sequence of my own.

Create your own sequence. Write the recursive formula of your sequence and find the first five terms.

$-1, 4, 9, 14, 19$

$a_1 = -1$

$a_n = a_{n-1} + 5$

- I can write a recursive formula given a context.

Find the recursive formula for each scenario.

1) $-8, -28, -48, -68, -88, \ldots$

$a_1 = -8$

$a_n = a_{n-1} - 20$

2) A store sells packages of apples that have a dozen apples in each package. How many apples are sold after n packages are sold?

$a_1 = 12$

$a_n = a_{n-1} + 12$

- I can interpret a recursive formula and state any term in the sequence.

Interpret each recursive formula. Then find the first 5 terms of each sequence.

1) $a_1 = 7$

$a_n = a_{n-1} + 9$

$7, 16, 25, 34, 43$

The first term is 7. To get the n^{th} term, take the previous term and add 9.

2) $f(1) = 10$

$f(n) = 10f(n-1) - 100$

$10, 0, -100, -1100, -11100$

The first term is 10. To get the n^{th} term, take the previous term, multiply by 10, then subtract 100.

$a(2) = 10(10) - 100 = 0$ $a(3) = 10(0) - 100 = -100$ $a(4) = 10(-100) - 100 = -1100$

$a(5) = 10(-1100) - 100 = -11100$

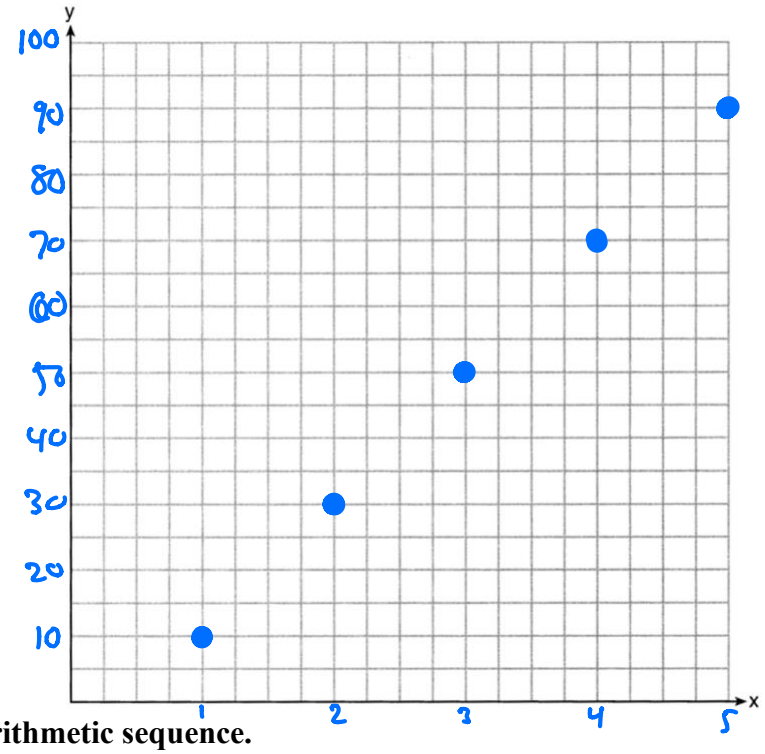
- I can graph a sequence.

Graph the first seven terms of the following sequence.

$$a(1) = 10$$

$$a(n) = a(n-1) + 20$$

a(1) = 10
a(2) = 30
a(3) = 50
a(4) = 70
a(5) = 90



- I can define an arithmetic sequence.

Define arithmetic sequence.

A sequence that has a common difference between terms.

- I can decipher whether a sequence is arithmetic or not arithmetic and provide reasoning.

Determine whether each of the following sequences is arithmetic. Explain your reasoning.

+2 +4 +6

1) $1, 3, 7, 13, \ldots$

Not Arithmetic
- no common difference

-3 -3 -3

2) $-15, -18, -21, -24, \ldots$

Arithmetic
- has a common difference of -3

+4/3

3) $\frac{1}{3}, \frac{5}{3}, 3, \frac{13}{3} \ldots$

+4/3 +4/3

Arithmetic
- has a common difference of $\frac{4}{3}$

Name:_____ Date:_____

Classwork: Sequences

Interpret each recursive formula. Then find the first 5 terms of each sequence.

1) $a_1 = -8$

$a_n = a_{n-1} - 7$

-8, -15, -22, -29, -36

The first term is -8. To find the n^{th} term, take the previous term and subtract 7.

2) $f(1) = -1$

$f(n) = -2f(n-1) + n$

-1, 4, -5, 14, -23

The first term is -1. To find the n^{th} term, take the previous term, multiply by -2 then add the term number.

$f(2) = -2(-1) + 2 = 4$ $f(3) = -2(4) + 3 = -5$ $f(4) = -2(-5) + 4 = 14$

$f(5) = -2(14) + 5 = -23$

3) What is the common difference of the arithmetic sequence $5, 8, 11, 14$?

+3 +3 +3

3

4) What is the common difference of the arithmetic sequence $-7x, -4x, -x, 2x, 5x$?

+3x +3x +3x +3x

3x

5) In a sequence, the first term is 7 and the common difference is 6. What is the sixth term of the sequence?

$a_2: 7 + 6 = 13$

$a_3: 13 + 6 = 19$

$a_4: 19 + 6 = 25$

$a_5: 25 + 6 = 31$

$a_6: 31 + 6 = 37$

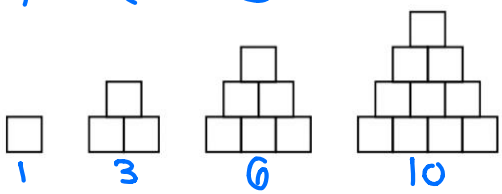
$a_6 = 37$

6) In 2014, the cost to mail a letter was 49 cents for up to one ounce. Every additional ounce cost 21 cents. Write a recursive formula that can be used to determine the cost of an n-ounce letter, in cents.

$a_1 = 49$

$a_n = a_{n-1} + 21$

7) A sequence of blocks is shown in the diagram below



1 2 3 4 5 6 7

1 3 6 10 15 21 28

This sequence can be defined by the recursive function $a_1 = 1$ and $a_n = a_{n-1} + n$. Assuming the pattern continues, how many blocks will there be when $n = 7$?

$a_5 = 10 + 5 = 15$

$a_6 = 15 + 6 = 21$

$a_7 = 21 + 7 = 28$

28 blocks

8)

Given the following three sequences:

+2 +2 +2 +2

I. 2, 4, 6, 8, 10...

~~II.~~ 2, 4, 8, 16, 32...

+2 +4

III. $a, a + 2, a + 4, a + 6, a + 8...$

+2 +2 +2 +2

Which ones are arithmetic sequences?

(1) I and II, only

(2) I and III, only

(3) II and III, only

(4) I, II, and III

9)

Given the recursive formula:

$$a_1 = 3$$
$$a_n = 2(a_{n-1} + 1)$$

State the values of a_2, a_3, and a_4 for the given recursive formula.

$a_2 = 2(3 + 1) = 8$

$a_3 = 2(8 + 1) = 18$

$a_4 = 2(18 + 1) = 38$

$a_2 = 8$

$a_3 = 18$

$a_4 = 38$

10) Write a sequence using a recursive formula that has a sixth term of 3.

$a_1 = 0.5$

$a_n = a_{n-1} + 0.5$

11) Given the sequence

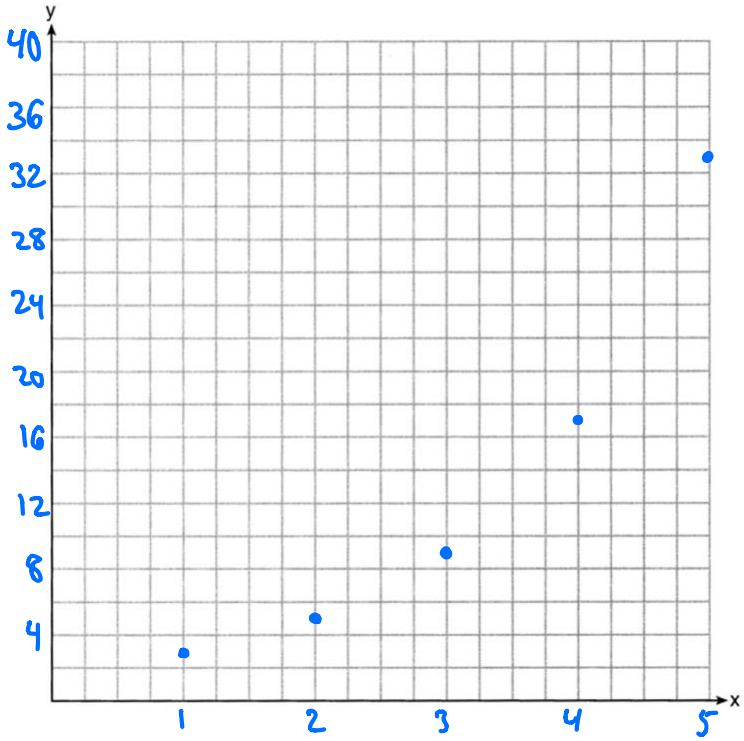
$$a(1) = 3$$

$$a(n) = 2a(n-1) - 1$$

a) Fill in the following table for the first five terms of the sequence.

n	$a(n)$
1	3
2	5
3	9
4	17
5	33

b) Graph the sequence on the interval $1 \leq n \leq 5$.



c) Kent states the domain of the sequence graphed is [1,5] and the range of the sequence is [3,33]. Is Kent correct or incorrect? Explain your reasoning.

C'mon Kent, you can't connect those points! Kent is incorrect because the domain and range is not an interval, D: {1, 2, 3, 4, 5} R: {3, 5, 9, 17, 33}.