Name:______________________ Date:__________

Notes: *Change of Base Formula and Solving Exponential Equations*

Do Now: Solve each of the following equations.

1) $3^{2x-1} = 27^{4x-7}$

$3^{2x-1} = (3^3)^{(4x-7)}$

$3^{2x-1} = 3^{12x-21}$

$2x-1 = 12x-21$

$20 = 10x$

$\boxed{x=2}$

2) $125^{8x+9} = 25^{7x+16}$

$(5^3)^{8x+9} = (5^2)^{7x+16}$

$5^{24x+27} = 5^{14x+32}$

$24x+27 = 14x+32$

$10x = 5$

$\boxed{x=0.5}$

3) $3^x = 7$

$\boxed{\log_3 7 = x}$

4) $4^{x-20} = 18$

$\log_4 18 = x-20$

$\boxed{\log_4(18)+20 = x}$

What Should I Be Able to Do?

- I can use the change of base formula to evaluate any logarithm.

- I can mathematically show how to obtain the change of base formula for any logarithm.

- I can solve exponential equations without getting common bases.

Solve:

$$20(4)^{0.1x} + 2 = 18$$

$$-2 \quad -2$$

$$20(4)^{0.1x} = 16$$

$$(4)^{0.1x} = 0.8$$

$$\log_4 0.8 = 0.1x$$

$$\frac{\log_4 0.8}{0.1} = x$$

$$\boxed{10\log_4 0.8 = x}$$

Solve, rounding your answer to the nearest thousandth:

$$35e^{8x} - 11 = 25$$

$$+11 \quad +11$$

$$35e^{8x} = 36$$

$$e^{8x} = \frac{36}{35}$$

$$\ln\frac{36}{35} = 8x$$

$$x = \frac{\ln\frac{36}{35}}{8}$$

$$\boxed{x \approx 0.004}$$

Solve:

$$14^x = 29$$

(Hint: Try to do the inverse operation of an exponential to both sides of the equation)

$\log 14^x = \log 29$

What does taking the log of both sides allow us to do with x?

$x \log 14 = \log 29$

$\boxed{x = \dfrac{\log 29}{\log 14}}$

$\boxed{\log_{14} 29 = x}$

Change of Base Formula:

$$\mathbf{\log_b x = \frac{\log_a x}{\log_a b}}$$

If you are using common logarithms for the change of base formula:

$$\log_b x = \frac{\log x}{\log b}$$

Solve each of the following:

1) $5^x = 4$

$\log 5^x = \log 4$

$x \log 5 = \log 4$

$\boxed{x = \dfrac{\log 4}{\log 5}}$

$\boxed{\log_5 4 = x}$

EQUIVALENT ANSWERS!

2) $44^x = 21$

$\log 44^x = \log 21$

$x \log 44 = \log 21$

$\boxed{x = \dfrac{\log 21}{\log 44}}$

$\boxed{\log_{44} 21 = x}$

3) $e^x = 1024$

$\ln e^x = \ln 1024$

$\boxed{x = \ln 1024}$

$\boxed{\ln 1024 = x}$

Rewrite each of the following logarithms using the Change of Base Formula, then round to the nearest thousandth.

4) $\log_2 6$

$\dfrac{\log 6}{\log 2}$

≈ 2.585

5) $\log_{\frac{1}{2}} 12$

$\dfrac{\log 12}{\log 0.5}$

≈ -3.585

6) $\log_{106} 23$

$\dfrac{\log 23}{\log 106}$

≈ 0.672

Solve:

$$3^{2x+9} = 4^{3x-1}$$

$$\log 3^{2x+9} = \log 4^{3x-1}$$

$$(2x+9)\log 3 = (3x-1)\log 4$$

$$2x\log 3 + 9\log 3 = 3x\log 4 - \log 4$$

Just as we would do when solving any equation, get the variable alone on one side!

$$2x\log 3 - 3x\log 4 = -\log 4 - 9\log 3$$

$$x(2\log 3 - 3\log 4) = -\log 4 - 9\log 3$$

$$\boxed{x = \frac{-\log 4 - 9\log 3}{2\log 3 - 3\log 4}} \quad x \approx 5.747$$

Solve the following exponential equations:

1) $5^x = 8^{3x+10}$

$$\log 5^x = \log 8^{3x+10}$$

$$x\log 5 = (3x+10)\log 8$$

$$x\log 5 = 3x\log 8 + 10\log 8$$

$$x\log 5 - 3x\log 8 = 10\log 8$$

$$x(\log 5 - 3\log 8) = 10\log 8$$

$$\boxed{x = \frac{10\log 8}{\log 5 - 3\log 8}}$$

2) $12^{2x+11} = 7^{5x-19}$

$$\log 12^{2x+11} = 7^{5x-19}$$

$$(2x+11)\log 12 = (5x-19)\log 7$$

$$2x\log 12 + 11\log 12 = 5x\log 7 - 19\log 7$$

$$2x\log 12 - 5x\log 7 = -19\log 7 - 11\log 12$$

$$x(2\log 12 - 5\log 7) = -19\log 7 - 11\log 12$$

$$\boxed{x = \frac{-19\log 7 - 11\log 12}{2\log 12 - 5\log 7}}$$

Success Criteria

- I can use the change of base formula to evaluate any logarithm.

Rewrite each of the following logarithms using the Change of Base Formula, then round to the nearest hundredth.

1) $\log_{\frac{1}{4}} 9$

$\frac{\log 9}{\log 0.25}$

-1.58

2) $\log_3 15$

$\frac{\log 15}{\log 3}$

2.46

3) $\log_{87} 31$

$\frac{\log 31}{\log 87}$

0.77

- I can mathematically show how to obtain the change of base formula for any logarithm.

Explain how you can solve $6^x = 19$ to prove the change of base formula.

The equivalent logarithmic form of $6^x = 19$ is $\log_6 19 = x$. But we can also solve the equation by taking the log of both sides of the equation,

$\log 6^x = \log 19$

$x \log 6 = \log 19$

$x = \frac{\log 19}{\log 6}$.

Therefore $\log_6 19 = \frac{\log 19}{\log 6}$.

- I can solve exponential equations without getting common bases.

1) $3^{x-1} = 2^{x+1}$

$\log 3^{x-1} = \log 2^{x+1}$

$(x-1)\log 3 = (x+1)\log 2$

$x\log 3 - \log 3 = x\log 2 + \log 2$

$x\log 3 - x\log 2 = \log 2 + \log 3$

$x(\log 3 - \log 2) = \log 2 + \log 3$

$\boxed{x = \frac{\log 2 + \log 3}{\log 3 - \log 2}}$

2) $14^{3x-3} = 17^{8x-13}$

$\log 14^{3x-3} = \log 17^{8x-13}$

$(3x-3)\log 14 = (8x-13)\log 17$

$3x\log 14 - 3\log 14 = 8x\log 17 - 13\log 17$

$3x\log 14 - 8x\log 17 = -13\log 17 + 3\log 14$

$x(3\log 14 - 8\log 17) = -13\log 17 + 3\log 14$

$\boxed{x = \frac{-13\log 17 + 3\log 14}{3\log 14 - 8\log 17}}$

By taking the log of both sides of the equation, how does that help us solve an exponential equation?

This allows us to make the exponent a coefficient, thus allowing us to get the variable by itself.

Name:_______________________ Date:__________

Classwork: Change of Base Formula and Solving Exponential Equations

Rewrite each of the following logarithms using the Change of Base Formula, then round to the nearest hundredth.

1) $\log_5 2$

$\frac{\log 2}{\log 5}$

≈ 0.43

2) $\log_{44.5} 18$

$\frac{\log 18}{\log 44.5}$

≈ 0.76

3) $\log_{0.3} 0.95$

$\frac{\log 0.95}{\log 0.3}$

≈ 0.4

Solve each of the following exponential equations.

4) $2(6)^{4x} - 17 = 65$

$+17 \quad +17$

$2(6)^{4x} = 82$

$6^{4x} = 41$ or

$\log 6^{4x} = \log 41$

$4x \log 6 = \log 41$

$\boxed{x = \frac{\log 41}{4 \log 6}}$

EQUIVALENT ANSWERS

$2(6)^{4x} = 82$

$6^{4x} = 41$

$\log_6 41 = 4x$

$\boxed{\frac{\log_6 41}{4} = x}$

5) $e^{x-4} = 4^{5x-1}$

$\ln e^{x-4} = \ln 4^{5x-1}$

$x - 4 = (5x-1)\ln 4$

$x - 4 = 5x\ln 4 - \ln 4$

$x - 5x\ln 4 = -\ln(4) + 4$

$x(1 - 5\ln 4) = -\ln(4) + 4$

$\boxed{x = \frac{-\ln(4)+4}{1-5\ln 4}}$

6) $6e^{3x-1} + 14 = 35$

$6e^{3x-1} = 21$

$e^{3x-1} = \frac{7}{2}$

$\ln e^{3x-1} = \ln 3.5$

$3x - 1 = \ln 3.5$

$3x = \ln(3.5) + 1$

$\boxed{x = \frac{\ln(3.5)+1}{3}}$

7) $8^{2x-5} = 13^{x+1}$

$\log 8^{2x-5} = \log 13^{x+1}$

$(2x-5)\log 8 = (x+1)\log 13$

$2x\log 8 - 5\log 8 = x\log 13 + \log 13$

$2x\log 8 - x\log 13 = \log 13 + 5\log 8$

$x(2\log 8 - \log 13) = \log 13 + 5\log 8$

$\boxed{x = \frac{\log 13 + 5\log 8}{2\log 8 - \log 13}}$

8) Solve for t in the equation $A = B + Ce^{-kt}$.

$$-B \quad -B$$

$$\frac{A-B}{C} = \frac{Ce^{-kt}}{C}$$

$$\frac{A-B}{C} = e^{-kt}$$

$$\ln\left(\frac{A-B}{C}\right) = \ln e^{-kt}$$

$$\frac{\ln\left(\frac{A-B}{C}\right)}{-k} = \frac{-kt}{-k}$$

$$\boxed{t = \frac{-\ln\left(\frac{A-B}{C}\right)}{k}}$$

9) Solve the following equation:

$$a^{1/\log a} = 8$$

$$\log a^{1/\log a} = \log 8$$

$$\left(\frac{1}{\log a}\right) \log a = \log 8$$

$$1 \neq \log 8$$

$$\boxed{\varnothing}$$

Explain why your solution is true.

When solving the equation, the variable, a, was canceled out. The equation simplified to 1 = log8 which is NOT true ∴ there is no solution.