Name:_____ Date:_____

Notes: Solving Logarithmic Equations

Do Now: Find the solution to each equation.

1) $\log_5(2x-7)=3$

$5^3=2x-7$

$125=2x-7$

$132=2x$

$x=66$

Check:

$\log_5(2(66)-7)=3$ ✓

2) $\log 8+\log x=2$

$\log 8x=2$

$10^2=8x$

$100=8x$

$x=12.5$

Check:

$\log 8+\log 12.5=2$ ✓

What Should I Be Able to Do?

- I can solve common base logarithmic equations that require me to condense logarithms.

Solve:

1) $\log_3(x)-\log_3(5)=4$

$\log_3\left(\frac{x}{5}\right)=4$

$3^4=\frac{x}{5}$

$81=\frac{x}{5}$

$x=405$

Check:

$\log_3 405-\log_3 5=4$ ✓

2) $\ln x+\ln 10=7$

$\ln 10x=7$

$e^7=10x$

$x=\frac{e^7}{10}$

Check:

$\ln\left(\frac{e^7}{10}\right)+\ln 10=7$ ✓

Solve:

$$\log_4(x) + \log_4(x - 12) = 3$$

$\log_4(x^2-12x)=3$

$x^2-12x=4^3$

$x^2-12x=64$

$x^2-12x-64=0$

$(x-16)(x+4)=0$

$x=16$ | ~~$x=-4$~~

Check:

$\log_4 16+\log_4(16-12)=3$ ✓

$\log_4(-4)+\log_4(-4-12)=3$ X

Checkpoint:

Solve the following equation.

$$\log x + \log(x + 15) = 2$$

$\log(x^2+15x)=2$

$10^2=x^2+15x$

$100=x^2+15x$

$x^2+15x-100=0$

$(x+20)(x-5)=0$

~~$x=-20$~~ | $x=5$

Check:

$\log(-20)+\log(-20+15)=2$ X

$\log 5+\log(5+15)=2$ ✓

$x=5$

Do Now: Find the solution to each equation.

1) $\frac{x}{4} = \frac{7}{4}$

$x = 7$

2) $\frac{x}{9} + \frac{x-2}{3} = \frac{8}{9}$

$\cdot 3 \quad \cdot 3$

$\frac{x}{9} + \frac{3(x-2)}{9} = \frac{8}{9}$

$\frac{x+3(x-2)}{9} = \frac{8}{9}$

$x+3(x-2)=8$

$x+3x-6=8$

$4x=14$

$x=3.5$

3) $\log x = \log 72$

$x = 72$

4) $\log_8 10 = \log_8 x$

$10 = x$

5) $\ln 2x = \ln 98$

$2x = 98$

$x = 49$

Check:

$\ln(2 \cdot 49) = \ln 98$ ✓

6) $\log 15 + \log x = \log 360$

$\log(15x) = \log(360)$

$15x = 360$

$x = 24$

Check:

$\log 15 + \log 24 = \log 360$ ✓

Solve:

1) $\log_6(12) - \log_6(x-1) = \log_6(7)$

$\log_6\left(\frac{12}{x-1}\right) = \log_6(7)$

$\frac{12}{x-1} = 7$

$12 = 7(x-1)$

$12 = 7x - 7$

$19 = 7x$

$x = \frac{19}{7}$

Check:

$\log_6(12) - \log_6\left(\frac{19}{7} - 1\right) = \log_6(7)$ ✓

2) $2\ln x + \ln 4 = \ln 9$

$\ln x^2 + \ln 4 = \ln 9$

$\ln(4x^2) = \ln(9)$

$4x^2 = 9$

$4x^2 - 9 = 0$

$(2x+3)(2x-3) = 0$

$2x+3=0 \quad | \quad 2x-3=0$

$x = -3/2 \quad | \quad x = 3/2$

Check:

$2\ln\left(-\frac{3}{2}\right) + \ln 4 = \ln 9$ ✗

$2\ln\left(\frac{3}{2}\right) + \ln 4 = \ln 9$ ✓

$x = \frac{3}{2}$

3) $\log(x-5) = \log(9x+4) - \log(x+4)$

$\log(x-5) = \log\left(\frac{9x+4}{x+4}\right)$

$x - 5 = \frac{9x+4}{x+4}$

$(x-5)(x+4) = 9x+4$

$x^2 + 4x - 5x - 20 = 9x + 4$

$x^2 - x - 20 = 9x + 4$

$x^2 - 10x - 24 = 0$

$(x-12)(x+2) = 0$

$x = 12 \quad | \quad x = -2$

Check:

$\log(12-5) = \log(9(12)+4) - \log(12+4)$ ✓

$\log(-2-5) = \log(9(-2)+4) - \log(-2+4)$ ✗

$x = 12$

Checkpoint:

Solve the following equation.

$$\log_2 x + \log_2(x-5) = \log_2(x+17)$$

$\log_2(x^2-5x) = \log_2(x+17)$

$x^2-5x = x+17$

$x^2-6x-17=0$

$a=1 \quad b=-6 \quad c=-17$

$$= \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{6 \pm \sqrt{(-6)^2-4(1)(-17)}}{2(1)}$$

$$\frac{6 \pm \sqrt{104}}{2}$$

$$\frac{6 \pm 4\sqrt{26}}{2}$$

$3 \pm 2\sqrt{26}$

$3+2\sqrt{26}$

~~$3-2\sqrt{26}$~~

Check:

$\log_2(3-2\sqrt{26}) + \log_2(3-2\sqrt{26}-5) = \log_2(3-2\sqrt{26}+17)$ ✗

$\log_2(3+2\sqrt{26}) + \log_2(3+2\sqrt{26}-5) = \log_2(3+2\sqrt{26}+17)$ ✓

$x = 3+2\sqrt{26}$

Success Criteria

- I can solve logarithmic equations that require me to condense logarithms.

1) $\log x - \log(x-3) = 1$

$\log\left(\frac{x}{x-3}\right) = 1$

$10^1 = \frac{x}{x-3}$

$10(x-3) = x$

$10x - 30 = x$

$-30 = -9x$

$x = \frac{30}{9}$

$x = \frac{10}{3}$

Check:

$\log\frac{10}{3} - \log\left(\frac{10}{3} - 3\right) = 1$ ✓

Explain why you did each step to solve the equation.

For my first step, I condensed the two logs on the left side of the equal sign in order to simplify the equation. I then converted from logarithm form to exponential form so that I could solve for x. I then cross multiplied and continued to solve for x until I found my answer, 10/3.

2) $\ln x + \ln(x - 20) = \ln(7x - 72)$

$\ln(x^2-20x)=\ln(7x-72)$

$x^2-20x=7x-72$

$x^2-27x+72=0$

$(x-24)(x-3)=0$

$x=24 \quad | \quad x=3$

Check:

$\ln 24+\ln(24-20)=\ln(7\times24)-72)$ ✓

$\ln 3+\ln(3-20)=\ln(7(3)-72)$ ✗

x=24

Explain why you did each step to solve the equation.

I first condensed the logs on the left side of the equal sign in order to simplify the equation. Since each side of the equal sign only had a logarithm of the same base, I could set the arguments equal to each other. I then solved for x, checking my solutions at the end. I rejected 3 because when 3 is plugged in for x in the original equation, one or more of the arguments are less than or equal to zero. Therefore my only solution is 24.

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Classwork: Solving Logarithmic Equations

Solve each of the following logarithmic equations.

1) $\log 2 + \log x = \log 14$

$\log 2x = \log 14$

$2x = 14$

$x = 7$

Check

$\log 2 + \log x = \log 14$ ✓

2) $\log_{13} x + \log_{13}(x - 9) = \log_{13} 22$

$x = 11$

$\log_{13}(x^2 - 9x) = \log_{13}(22)$

$x^2 - 9x = 22$

$x^2 - 9x - 22 = 0$

$(x - 11)(x + 2) = 0$

$x = 11 \quad x = -2$

Check:

$\log_{13} 11 + \log_{13}(11 - 9) = \log_{13} 22$ ✓

$\log_{13}(-2) + \log_{13}(-2 - 9) = \log_{13} 22$ ✗

3) $\log x - \log 72 = \log \frac{1}{2}$

$\log \frac{x}{72} = \log \frac{1}{2}$

$\frac{x}{72} = \frac{1}{2}$

$x = 36$

Check:

$\log 36 - \log 72 = \log \frac{1}{2}$ ✓

4) $\ln 6 - \ln x = 4$

$\ln\left(\frac{6}{x}\right) = 4$

$e^4 = \frac{6}{x}$

$xe^4 = 6$

$x = \frac{6}{e^4}$

Check:

$\ln 6 - \ln\left(\frac{6}{e^4}\right) = 4$ ✓

5) $7 + \log x = 5$

$\log x = -2$

$10^{-2} = x$

$x = 0.01$

Check:

$7 + \log(0.01) = 5$ ✓

6) $\log_8 \sqrt[3]{x - 1} = 2$

$8^2 = \sqrt[3]{x - 1}$

$(64)^3 = (\sqrt[3]{x - 1})^3$

$262144 = x - 1$

$262145 = x$

Check:

$\log_8 \sqrt[3]{262145 - 1} = 2$ ✓

7) $2\log_4(x - 1) = \log_4 16 + 7$

$\log_4 (x-1)^2 = 2 + 7$

$\log_4 (x-1)^2 = 9$

$4^9 = (x - 1)^2$

$\sqrt{262144} = \sqrt{(x-1)^2}$

$512 = x - 1$

$x = 513$

Check:

$2\log_4(513 - 1) = \log_4 16 + 7$ ✓

8) $\ln(2x) + \ln(2x + 6) = \ln 16$

$\ln(4x^2 + 12x) = \ln(16)$

$4x^2 + 12x = 16$

$4x^2 + 12x - 16 = 0$

$4(x^2 + 3x - 4) = 0$

$4(x + 4)(x - 1) = 0$

$x = -4 \quad x = 1$

Check:

$\ln(2(-4)) + \ln(2(-4) + 6) = \ln 16$ ✗

$\ln(2 \cdot 1) + \ln(2 \cdot 1 + 6) = \ln 16$ ✓

9) $3\log x = \log 729$

$\log x^3 = \log 729$

$\sqrt[3]{x^3} = \sqrt[3]{729}$

$x = 9$

Check:

$3\log 9 = \log 729$ ✓

10) $\log_3 x + \log_3(x+8) = 2$

$\log_3(x^2+8x) = 2$

$3^2 = x^2+8x$

$9 = x^2+8x$

$x^2+8x-9=0$

$(x+9)(x-1)=0$

$x=-9 \quad | \quad x=1$

Check:

$\log_3(-1)+\log_3(-9+8)=2$ ✗

$\log_3(1)+\log_3(1+8)=2$ ✓

11) $\log(3x) - \log(x+3) = \log(x-1)$

$\log\left(\frac{3x}{x+3}\right) = \log(x-1)$

$\frac{3x}{x+3} = x-1$

$3x = (x-1)(x+3)$

$3x = x^2+3x-x-3$

$3x = x^2+2x-3$

$x^2-x-3=0$

$\frac{1 \pm \sqrt{(-1)^2-4(1)(-3)}}{2(1)}$

$\frac{1 \pm \sqrt{13}}{2}$

$x = \frac{1+\sqrt{13}}{2}$

Check

$\log\left(3\cdot\frac{1-\sqrt{13}}{2}\right) - \log\left(\frac{1-\sqrt{13}}{2}+3\right) = \log\left(\frac{1-\sqrt{13}}{2}-1\right)$ ✗

$\log\left(3\cdot\frac{1+\sqrt{13}}{2}\right) - \log\left(\frac{1+\sqrt{13}}{2}+3\right) = \log\left(\frac{1+\sqrt{13}}{2}-1\right)$ ✓