Name: _____ Date: _____

Notes: Imaginary Numbers

Do Now: Simplify each expression.

1) $\sqrt{64}$

8

2) $\sqrt{24}$

$\sqrt{4}\sqrt{6}$

$2\sqrt{6}$

3) $\sqrt{-81}$

?

4)

Using complete sentences, explain why you cannot simplify $\sqrt{-25}$ into a real number.

Because a number cannot be multiplied by itself to obtain -25.

$5 \times 5 = 25$

$-5 \times (-5) = 25$

$5 \times (-5) = -25$

Two different numbers

What Should I Be Able to Do?

- **I can define the imaginary unit *i*.**
- **I can explain why $i = \sqrt{-1}$.**
- **I can simplify principal square roots.**
- **I can simplify *i* raise to any integer power.**
- **I can simplify one step expressions involving imaginary numbers.**

Imaginary Unit i: The imaginary unit i is defined to be

$$i = \sqrt{-1} \text{ and } i^2 = -1$$

How do we use i?

Simplify the following radicals.

1) $\sqrt{-4}$

$i\sqrt{4}$

$2i$

2) $\sqrt{-32}$

$i\sqrt{32}$

$i\sqrt{16}\sqrt{2}$

$i(4)\sqrt{2}$

$4i\sqrt{2}$

Why is $i = \sqrt{-1}$?

Because a number cannot be negative inside a square root and still be a real number. Therefore we can factor out a $\sqrt{-1}$ and denote the number as imaginary, i.

Simplify the following radicals.

1) $\sqrt{-196}$

$i\sqrt{196}$

$14i$

2) $\sqrt{-192}$

$i\sqrt{192}$

$i\sqrt{64}\sqrt{3}$

$8i\sqrt{3}$

3) $-5\sqrt{-27}$

$-5i\sqrt{27}$

$-5i\sqrt{9}\sqrt{3}$

$-5i(3)\sqrt{3}$

$-15i\sqrt{3}$

Find i^0. Then find the value of the subsequent powers of i (i^1 and i^2 are already found) by rewriting each using factors of that you know the value of.

$i^0 = 1$

$i^1 = i$

$i^2 = -1$

$i^3 = -i$

$i^3 = i^2 \cdot i$

$(-1) \cdot i$

$-i$

$i^4 = 1$

$i^4 = i^2 \cdot i^2$

$(-1)(-1)$

1

$i^5 = i$

$i^4 \cdot i$

$1 \cdot i$

i

$i^6 = -1$

$i^4 \cdot i^2$

$1(-1)$

$i^7 = -i$

$i^4 i^3$

$1(-i)$

$-i$

$i^8 = 1$

$i^4 i^4$

$1 \cdot 1$

1

$i^9 = i$

$i^8 \cdot i$

$1 \cdot i$

i

$i^{10} = -1$

$i^8 i^2$

$1(-1)$

-1

$i^{11} = -i$

$i^8 (i^3)$

$1(-i)$

$-i$

$i^{12} = 1$

$i^8 i^4$

$1 \cdot 1$

1

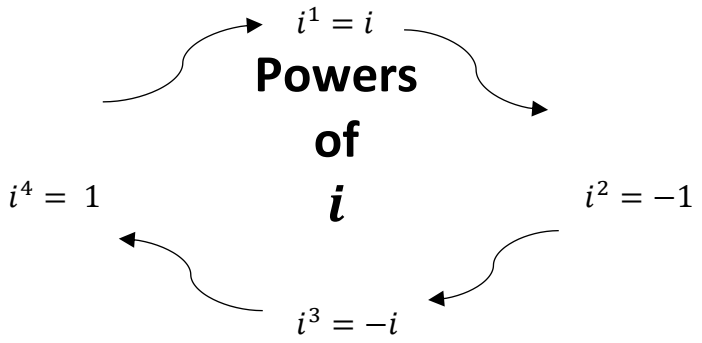
Do you notice a pattern here? Explain the pattern as best as you can.

For i^n, if $\frac{n}{4}$ has a remainder of 1, then $i^n = i$

if $\frac{n}{4}$ has a remainder of 2, then $i^n = -1$

if $\frac{n}{4}$ has a remainder of 3, then $i^n = -i$

if $\frac{n}{4}$ has no remainder, then $i^n = 1$.



Medium Funny Joke:
Why is i the most annoying opponent to lose to?

All he says is "I won, I won, I won, I won, I won!"

Completely simplify the following powers of i.

1) i^{21}

$\frac{21}{4} = 5.25$

i^1

i

2) i^{30}

$\frac{30}{4} = 7.5$

i^2

-1

3) i^{39}

$\frac{39}{4}$

9.75

i^3

$-i$

4) i^{184}

$\frac{184}{4} = 46$

i^0

1

Checkpoint:

Completely simplify the following powers of i.

a) i^{10}

$\frac{10}{4} = 2.5$

-1

b) i^{43}

$\frac{43}{4} = 10.75$

$-i$

c) i^{15}

$\frac{15}{4} = 3.75$

$-i$

d) i^{22}

$\frac{22}{4} = 5.5$

-1

e) i^{100}

$\frac{100}{4} = 25$

1

f) i^{81}

$\frac{81}{4} = 20.25$

i

g) i^{66}

$\frac{66}{4} = 16.5$

-1

h) i^{371}

$\frac{371}{4} = 92.75$

$-i$

k) $i^{1,720}$

$\frac{1720}{4} = 430$

1

1) Find the sum of $7i^2$ and $15i^{15}$.

$\frac{15}{4} = 3.75$

$7i^2 + 15i^{15}$

$7(-1) + 15(-i)$

$-7 - 15i$

2) Find the difference of $7i^2$ and $15i^{15}$.

$7i^2 - 15i^{15}$

$7(-1) - 15(-i)$

$-7 + 15i$

3) Find the product of $7i^2$ and $15i^{15}$.

$(7i^2)(15i^{15})$

$105i^{17}$

$\frac{17}{4} = 4.25$

$105(i)$

$105i$

4) Simplify $(i^{25})^{47}$

$\frac{1175}{4} = 293.75$

i^{1175}

$-i$

OR

$i^{25} = i^1$

$(i^1)^{47}$

$i^{47} = -i$

Success Criteria

- I can define the imaginary unit *i*.

The imaginary unit i is defined by $i = \sqrt{-1}$ and $i^2 = -1$.

- I can explain why $i = \sqrt{-1}$.

A number cannot be negative inside a square root and stay a real number. Therefore we factor out $\sqrt{-1}$ and denote the number as an imaginary number, i.

- I can simplify principal square roots.

Completely simplify the following radicals.

1) $\sqrt{-252}$

$\sqrt{-1}\sqrt{252}$

$i\sqrt{36}\sqrt{7}$

$6i\sqrt{7}$

2) $\sqrt{-2916}$

$\sqrt{-1}\sqrt{2916}$

$i(54)$

$54i$

3) $-3\sqrt{-245}$

$-3\sqrt{-1}\sqrt{-245}$

$-3i\sqrt{49}\sqrt{5}$

$-3i(7)\sqrt{5}$

$-21i\sqrt{5}$

- I can simplify *i* raise to any integer power.

Completely simplify the following powers of i.

1) i^{35}

$\frac{35}{4} = 8.75$

$-i$

2) i^{45}

$\frac{45}{4} = 11.25$

i

3) i^{392}

$\frac{392}{4} = 98$

1

4) $i^{12,906}$

$\frac{12906}{4} = 3226.5$

-1

- I can simplify one step expressions involving imaginary numbers.

Completely simplify the following expressions.

1) $i^{31} + i^{73}$

$-i + i$

0

2) $(i^{30})(i^{7})$

i^{37}

i

3) $2i^{21} - 2i^{2}$

$2i - 2(-1)$

$2i + 2$

$2 + 2i$

4) $(i^{19})^{20}$

i^{380}

1

5) $-3i + 12i^{97}$

$-3i + 12i$

$9i$

6) $48i^{100} - 24i^{50}$

$48(1) - 24(-1)$

$48 + 24$

72

Name: _____ Date: _____

Classwork: Imaginary Numbers

Completely simplify the following radicals.

1) $\sqrt{-772}$

$i\sqrt{193}\sqrt{4}$

$2i\sqrt{193}$

2) $-4\sqrt{-961x^8y^9z}$

$-4i\sqrt{961}\sqrt{x^8}\sqrt{y^8}\sqrt{y}\sqrt{z}$

$-4i(13)x^4y^4\sqrt{yz}$

$-52ix^4y^4\sqrt{yz}$

3) $2xy\sqrt{-800x^{16}y^{19}}$

$2xyi\sqrt{400}\sqrt{2}\sqrt{x^{16}}\sqrt{y^{18}}\sqrt{y}$

$2xyi(20)\sqrt{2}x^8y^9\sqrt{y}$

$40x^9y^{10}i\sqrt{2y}$

Completely simplify the following powers of i.

4) i^{441}

$\frac{441}{4} = 110.25$

$i^1 = i$

5) i^{820}

$\frac{821}{4} = 205$

$i^0 = 1$

6) $i^{8,943,277}$

$\frac{8943277}{4} = 2235819.25$

$i^1 = i$

7) $i^{\frac{345,973,495}{4}} = 864\,93373.75$

$i^3 = -i$

Completely simplify the following expressions.

8) $14\sqrt{-8} + 16i^3$

$14i\sqrt{4}\sqrt{2} + 16i^3$

$14i(2)\sqrt{2} + 16(-i)$

$28i\sqrt{2} - 16i$

9) $-5i^{65}(9i^{102})$

$-5i(9(-1))$

$-5i(-9)$

$45i$

10) $-i^{31} - 2i^{41} - 8i^{109} + 10i^2$

$-(-i) - 2(i) - 8(i) + 10(-1)$

$i - 2i - 8i - 10$

$-10 - 9i$

11) $i(i)$

i^2

-1

12) $(i - 9i)^2$

$(-8i)^2$

$(-8i)(-8i)$

$64i^2$

-64

13) $(3i - 7i^3)(10i - 5i^{19})$

$3i(10i) - 5i^{19}(3i) - 7i^3(10i) - 7i^3(-5i^{19})$

$30i^2 - 15i^{20} - 70i^4 + 35i^{22}$

$30(-1) - 15(1) - 70(1) + 35(-1)$

$-30 - 15 - 70 - 35$

-150

14) Simplify i^{4a} where a is a positive integer.

multiple of 4 $\therefore$ $i^{4a} = 1$

Explain how you arrived at your answer.

Since a is an integer, then 4a is a multiple of 4. There $\frac{4a}{4}$ has no remainder and $i^{4a} = 1$.

15) Express $xi^8 - yi^6$ in simplest form.

$x(1) - y(-1)$

$x + y$

16) Determine the value of n in simplest form:

$i^{13} + i^{18} + i^{31} + n = 0$

$i + (-1) + (-i) + n = 0$

$-1 + n = 0$

$n = 1$

17)

Mrs. Donahue made up a game to help her class learn about imaginary numbers. The winner will be the student whose expression is equivalent to $-i$. Which expression will win the game?

1) i^{46} 2) i^{47} 3) i^{48} 4) i^{49}

18)

What is the greatest possible integral value of x for which $\sqrt{x-5}$ is an imaginary number?

1) 5 2) 6 3) 3 4) 4

$x - 5 < 0$

$+5 \quad +5$

$x < 5$